Theoretical problems in Cause – Specific Mortality forecasting and diagnosis rates.

Solutions and actuarial applications.

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Agenda

- Motivation
- Modelling of cause-specific mortality
- Mitigating the discontinuities
- About the dependence
- Critical Illness Cover
- New proposal for Insured Loan
- Case study
- Conclusions

Motivation

New horizons for Insured Loan

CUSTOMIZING INSURANCE AND FINANCIAL PRODUCTS

INCREASING AVAILABILITY OF SPECIFIC DATA

LONGEVITY RISK

The IDEA is to insert other type of insurance coverage in the financial management

Cause – specific death

Critical Illness

Modelling of cause-specific mortality

Structural Breaks and Dependence

Forecasting the future trend of cause –specific mortality

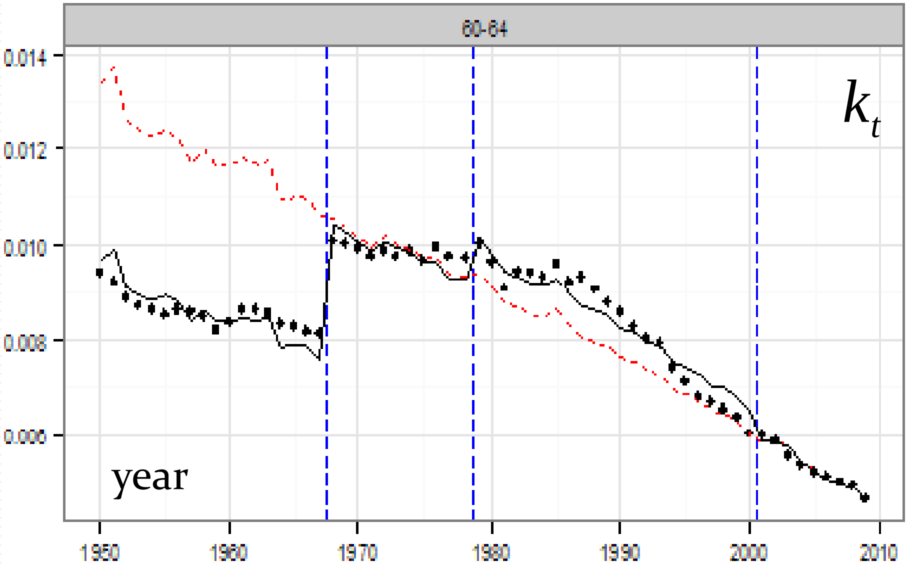
Mitigate the discontinuity points

Consider the dependence

The problem of prediction for causes of death

- **International Classification of Diseases (ICD), dependence**
  - Discontinuities in the time series
  - Dependences among the cause – specific deaths/diseases
- **Important implications on several Insured Loan Proposed**
  - Better estimates and predictions of cause-specific mortality

Central death rate for Circulatory System (age 60-64, female)



Mitigating the discontinuties

The Model of Haberman et al. (2014)

$$\log \mu_{x,t} = \alpha_x + \beta_x k_t + \sum_{i=1}^{h} \delta_x^{(i)} f^{(i)}(t)$$

- α_x: Average age-specific mortality
- β_x: Deviation in mortality
- k_t: Mortality index at year *t*
- $f^{(i)}(t)$: Adjustment for coding changes

Constraints:

$$k_{t_n} = 0$$

$$\sum_x \beta_x = 1$$

Assumption:

$$D_{x,t} \approx Poisson(E_{x,t} \mu_{x,t})$$

where: $\mu_{x,t} = \dfrac{D_{x,t}}{E_{x,t}}$

About the dependence

Vector Error Correction Model

➢Selection the lag order of VAR(p)

Akaike's Information Criteria (AIC), Hannan-Quinn Criterion (HQ), Schwarz Criterion (SC), Final Prediction Error (FPE).

➢Unit root test

With some tests (KPSS, ADF, PP) it is possible to see if the characteristic polynomial has unit root. KPSS tests the null hypothesis that the variable is trend stationary, while ADF and PP test the null hypothesis of a unit root (the null hypothesis of non-stationary).

Vector Error Correction Model

➢Fitting a VAR(p) or VECM

If all the variables are stationary (I(O), integrated of zero order) the fitting with a VAR is appropriate. However, the Johansen's procedure should be applied if some of the variables are I(1) in order to find the number of cointegrating relations. With the trace test and the maximum eigenvalue test we can see the number of cointegrating relations. If there isn't cointegration between the variables it is possible to use a VAR (p-1) on the first difference.

VAR and VECM for Kt

Consider VAR(p)

$$y_t = c + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \varepsilon_t$$

with:

$$E(\varepsilon_i) = 0$$

$$E(\varepsilon_i \varepsilon_l) = \begin{cases} \Omega & \text{for} \quad t = l \\ 0 & \text{for} \quad t \neq l \end{cases}$$

The condition for the Var(p) model to be stationary is:

$$\det\left(I_k - \phi_n \lambda\right) \neq 0, \quad |\lambda| < 1$$

VAR and VECM for Kt

Unit root tests

↓

All causes of death have unit roots

↓

NOT STATIONARY

⇓

ALL VARIABLES ARE I(1)

Cointegration

If all the variables aren't stationary and integrated of the same order, we can find an equilibrium between them in the long – run represented by this equation:

$$\beta' y_t = \beta_1 y_{1t} + \ldots + \beta_k y_{kt} = 0$$

Cointegration

If this relation is stationary (it means that they have the same trend in the long – run), the variables are cointegrated.

Z_t

If the process $\beta' y_t = z_t$ is stationary

ALL CAUSES OF DEATH ARE COINTEGRATED

How many cointegrating relation are there?

- Consider a process with three variables integrated of order 1, $y_t = (y_{1t}, y_{2t}, y_{3t})$ and suppose that it is cointegrated with the cointegrated vector $\beta = (1, \beta_2, \beta_3)'$, ,such that $\beta' y_t = y_{1t} - \beta_2 y_{2t} - \beta_3 y_{3t} \approx I(0)$

- It is shown, with the Granger representation theorem, that if the series are cointegrated, exists a representation ECM.

How many cointegrating relations?

$$\Delta y_{1t} = c_1 + \alpha_1 \left(y_{1t-1} - \beta_2 y_{2t-1} - \beta_3 y_{3t-1} \right) + \gamma_{11} \Delta y_{1t-1} + \gamma_{12} \Delta y_{2t-1} + \gamma_{13} \Delta y_{3t-1} + \varepsilon_{1t}$$

$$\Delta y_{2t} = c_2 + \alpha_2 \left(y_{1t-1} - \beta_2 y_{2t-1} - \beta_3 y_{3t-1} \right) + \gamma_{21} \Delta y_{1t-1} + \gamma_{22} \Delta y_{2t-1} + \gamma_{23} \Delta y_{3t-1} + \varepsilon_{2t}$$

$$\Delta y_{3t} = c_3 + \alpha_3 \left(y_{1t-1} - \beta_2 y_{2t-1} - \beta_3 y_{3t-1} \right) + \gamma_{31} \Delta y_{1t-1} + \gamma_{32} \Delta y_{2t-1} + \gamma_{33} \Delta y_{3t-1} + \varepsilon_{3t}$$

How many cointegrating relations?

In matrix form:

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \varepsilon_t$$

$$\underset{(k*r)}{\alpha} \quad * \quad \underset{(r*k)}{\beta'}$$

Matrix Loading   Coint. Matrix

Representation of the VAR(2) in the form vector error correction model (multivariate version of the Dickey – Fuller test)

How many cointegrating relations?

We can determinate the number of cointegrating relations in the series Yt by the rank of the matrix Π. In particular:

if the $rank(\Pi) = r = 0 \Rightarrow \Delta y_t$ is a VAR(p-1) stationary;

if the $rank(\Pi) = r = k \Rightarrow y_t$ has not unit roots, it is a VAR(p) stationary;

if $0 < rank(\Pi) < k \Rightarrow y_t$ has r cointegrating relations and k-r common trend.

Critical Illness Cover

Stand Alone and Accelerated Benefit

- **Stand Alone Cover**
  - A benefit is paid if the assured suffers for one the following contractual critical conditions (for example Heart Attack, Stroke, Cancer, Respiratory System).

- **Accelerated benefit**
  - Stand Alone + accelerated benefit if the assured dies. A benefit is paid if the assured suffers a particular diseases and an accelerated benefit in case of death.

Insured Loan

SIL: Standard Insured Loan

- **Supposing the borrower/insured's debt is one monetary unit**

$$\sum_{k=0}^{n-1} P_k A^{1}_{x:\overline{k}|} = 1 \quad \text{where} \quad A^{1}_{x:\overline{k}|} = v(0,k)\,{}_k p_x$$

- **The benefit payable if the insured dies**

$$B_h = \frac{1}{a_{\overline{n}|}} \cdot \ddot{a}_{\overline{n-h+1}|}$$

- **The constant actuarial premium the insured pays if alive**

$$_{/m}P_{x,h} = {}_{/m}P_x = \frac{1}{a_{\overline{n}|}} {}_{/m}\pi_x$$

in which

$$_{/m}\pi_x = \frac{1}{\ddot{a}_{\overline{x,m}|}} \sum_{j=0}^{n-1} {}_{j/}a_{\overline{n-j}|} \, {}_{j/1}q_x$$

New proposals for Insured Loan

SpeIL: Death Specific Insured Loan

- **The basic equation is:**

$$\sum_{k=0}^{n-1} \frac{\ddot{a}_{\overline{n-h|}}}{a_{\overline{n|}}} v(0,h+1)_{h/1}q_x^{(c)} = \sum_{h=0}^{n-1} P_h v(0,h)_h P_x$$

- The idea is to design a product in which the loan is saved in case of the borrower's death for a specific cause.

SCILsa: Standard Critical Illness Loan (Stand Alone)

$$\sum_{k=0}^{n-1} \frac{\ddot{a}_{\overline{n-h|}}}{a_{\overline{n|}}} v(0, h+1)_{h/1} w_x^{(d)} = \sum_{h=0}^{n-1} P_h v(0, h)(1 - {}_{h-1/1} w_x^{(d)})$$

- In this case the loan is saved if the insured suffers specified critical diseases.

SCILa: Standard Critical Illness Loan (Accelerated)

$$\sum_{k=0}^{n-1} \frac{\ddot{a}_{\overline{n-h|}}}{a_{\overline{n|}}} v(0,h+1)({}_{h/1}\tilde{q}_x) = \sum_{h=0}^{n-1} P_h v(0,h)(1 - {}_{h-1/1}w_x^{(d)})$$

- in which ${}_{h/1}\tilde{q}_x$ is the probability of the two compatible events, specifically to die for any cause of death and/or to suffer a specified illness.

Case study

Application to the U.K. case

Population data

- U.K. Population (diff. cohort)
- Ages: 25-29,30-34,…,84-89
- Period: 1950-2001

→ **CMI: disease rates**

→ **HMD: aggregated m.r.**

→ **WHO: death-cause m.r.**

Loan Characteristics

- Duration: 10/20 years
- Interest rate = 7%, Issue Time = 2014, C = 200000 euro

Insurance Cover Characteristics

- SIL, SpeIL, SCILsa, SCILa
- Technical actuarial valuation rate = 2%
- Age at entry = 40/60

1) **Mitigating the discontinuities in the mortality time series with an extension of the Lee – Carter Model.**

2) **Capturing the possible dependences among the cause – specific deaths.**

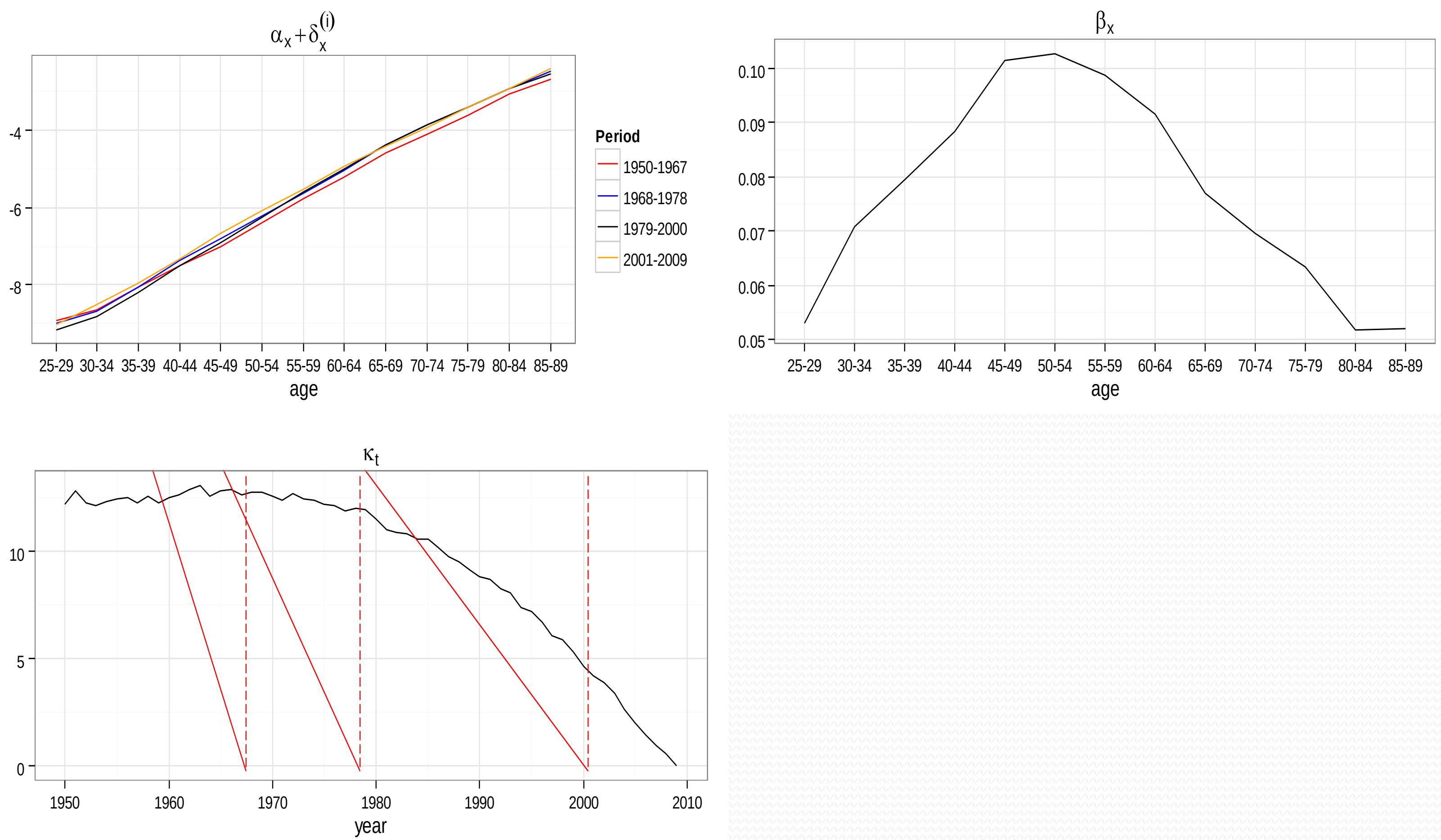
3) **Forecasting them trough the ARIMA models, the VECM (only if there are several stationary cointegrating relations between them) and the Vector Autoregressive Model (if their representation has not unit roots).**

4) **Use the better prevision in order to calculate the future trend of mortality rates.**

5) **Finally, the pricing our proposed contracts.**

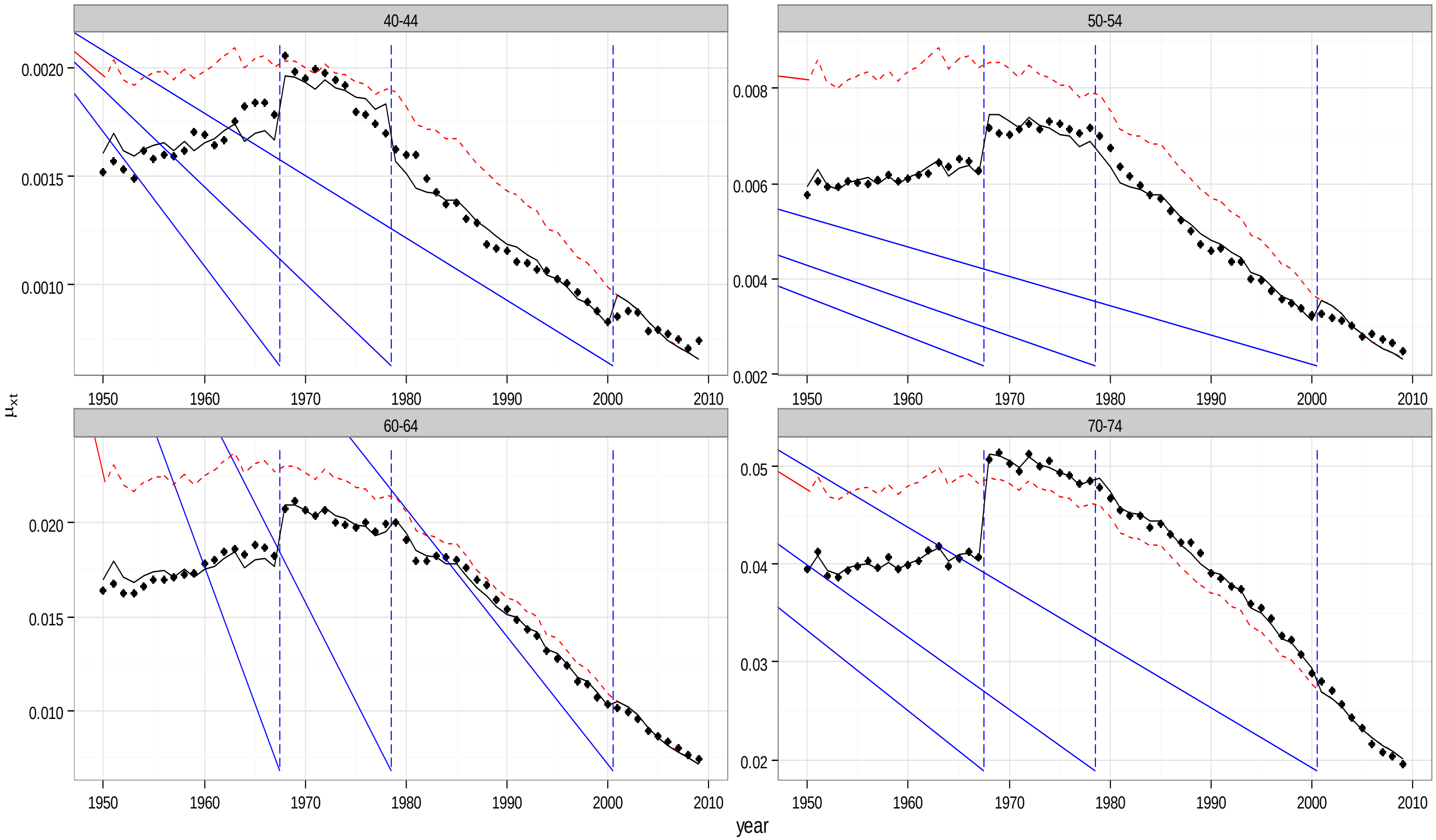
Parameters estimation

Figure 1:
Model parameters (Circulatory System, UK Male population)



Adjusted Kt

Figure 2:
Adjusted mortality index (Circulatory System, UK Male population)



SELECT LAG ORDER

Select lag order VAR(p)

U.K. Male Population				U.K. Female Population			
AIC(n)	HQ(n)	SC(n)	FPE(n)	AIC(n)	HQ(n)	SC(n)	FPE(n)
2	1	1	1	1	1	1	1

- The eigenvalues are bigger than one in absolute value. This means that the VAR could be explode because its characteristic polynomial has unit roots.

UNIT ROOT TESTS: ADF (MALE)

CAUSES OF DEATH	ADF	P - VALUE
I&P	-0.9381	0,9402
Cancer	-0.4784	0,9798
Circulatory System	2.0119	0,99
Respiratory System	-2.33	0,4414
External	-1.2111	0,8936
Other	-1.0549	0,9219

CAUSES OF DEATH	PP	P - VALUE
I&P	-1.7203	0,9737
Cancer	-0.2694	0,99
Circulatory System	1.2455	0,99
Respiratory System	-38.1031	**0,01**
External	-7.3597	0,6793
Other	-2.5195	0,9523

CAUSES OF DEATH	KPSS	P - VALUE
I&P	2.3031	0,01
Cancer	1.163	0,01
Circulatory System	2.6937	0,01
Respiratory System	3.0345	0,01
External	3.0138	0,01
Other	2.4834	0,01

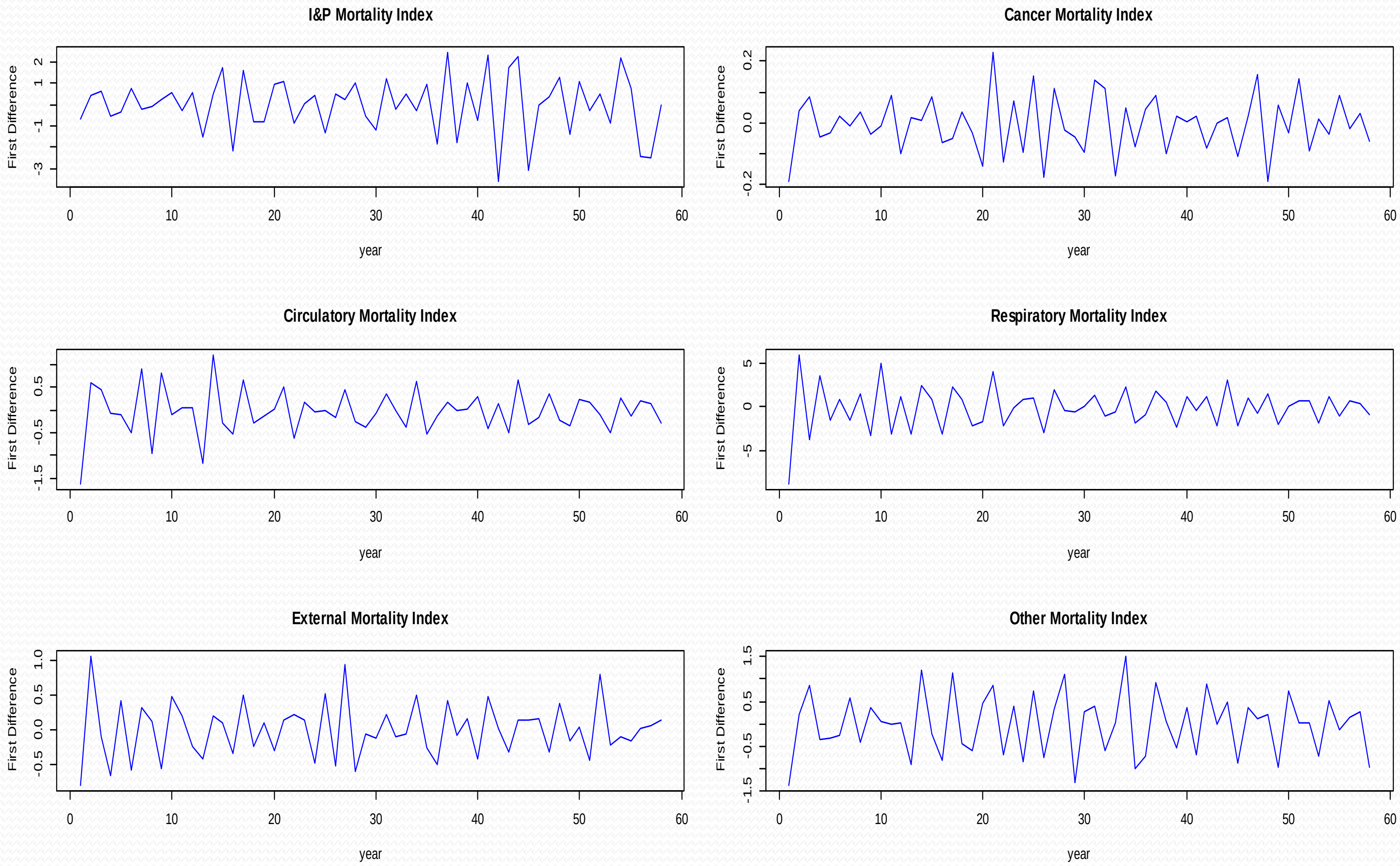
UNIT ROOT TESTS: ADF (FEMALE)

CAUSES OF DEATH	ADF	P - VALUE
I&P	-2.8242	0,2416
Cancer	0.1599	0,99
Circulatory System	1.0873	0,99
Respiratory System	-4.6381	**0,01**
External	-2.1588	0,5106
Other	-1.0109	0,9288

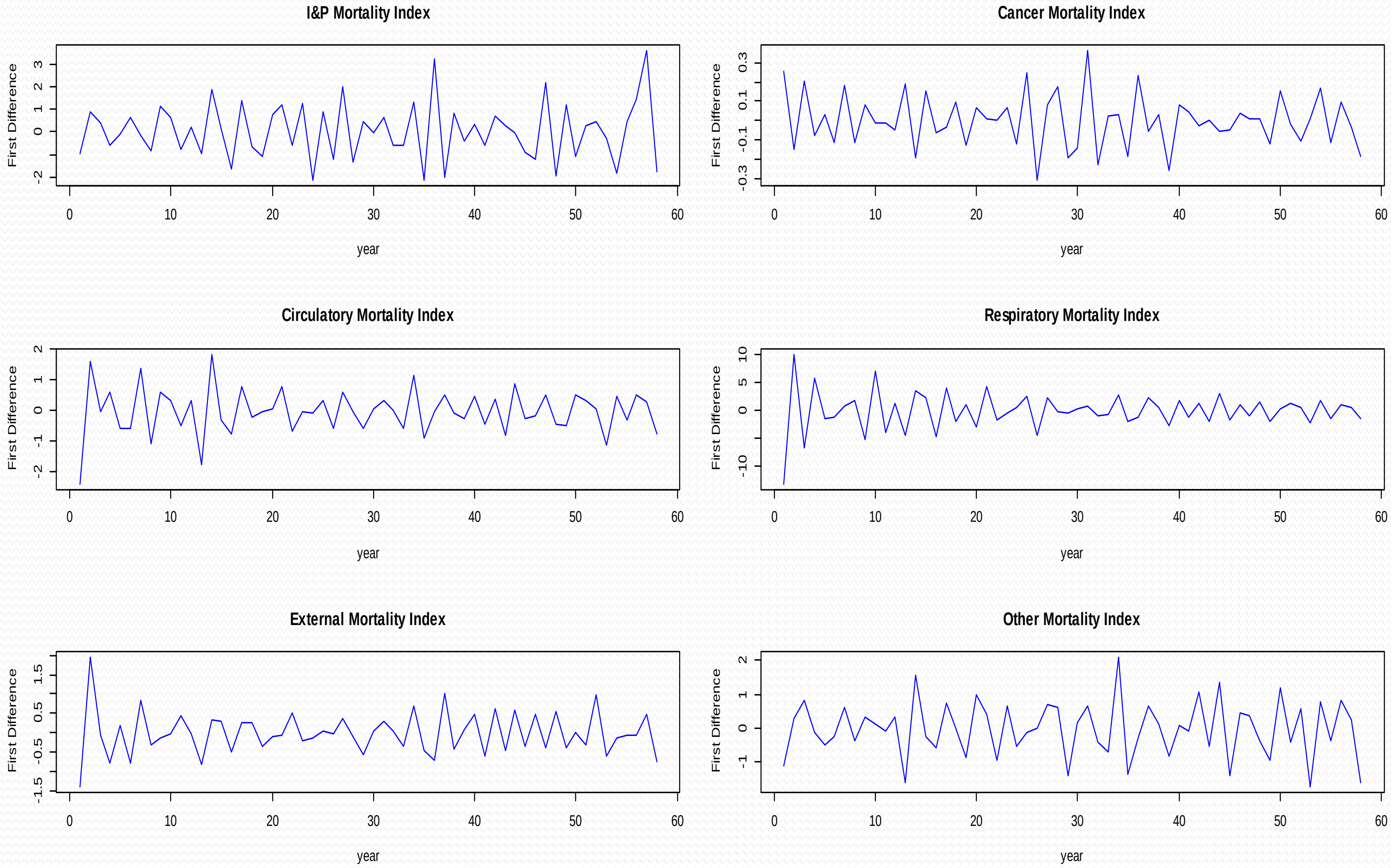
CAUSES OF DEATH	PP	P - VALUE
I&P	-7.5514	0,6677
Cancer	1.362	0,99
Circulatory System	1.3336	0,99
Respiratory System	-74.9722	**0,01**
External	-5.8541	0,7705
Other	-2.6035	0,95

CAUSES OF DEATH	KPSS	P - VALUE
I&P	2.7215	0,01
Cancer	0.5891	0.02363
Circulatory System	2.9883	0,01
Respiratory System	2.1376	0,01
External	2.9435	0,01
Other	1.5903	0,01

Kt First difference, Male



Kt First difference, Female



Trace Test, Maximum Eigenvalue Test

MALE

h	*n-h*	*stat*	*10%*	*5%*	*2.5%*	*1%*
4	1	0.05041928	2.70	3.84	5.25	6.98
3	2	6.18016087	15.74	18.08	20.26	22.40
2	3	24.83020039	31.67	34.27	36.98	40.10
1	**4**	**51.09236539**	**50.62**	**54.02**	**57.01**	**61.03**
0	5	96.15921357	73.73	77.61	81.29	85.56

h	*n-h*	*stat*	*10%*	*5%*	*2.5%*	*1%*
4	1	0.05041928	2.70	3.84	5.25	6.98
3	2	6.12974159	14.64	16.69	18.84	20.88
2	3	18.65003953	21.44	23.75	25.68	28.31
1	**4**	**26.26216499**	**27.39**	**29.93**	**32.22**	**35.57**
0	5	45.06684818	33.45	36.46	39.00	41.87

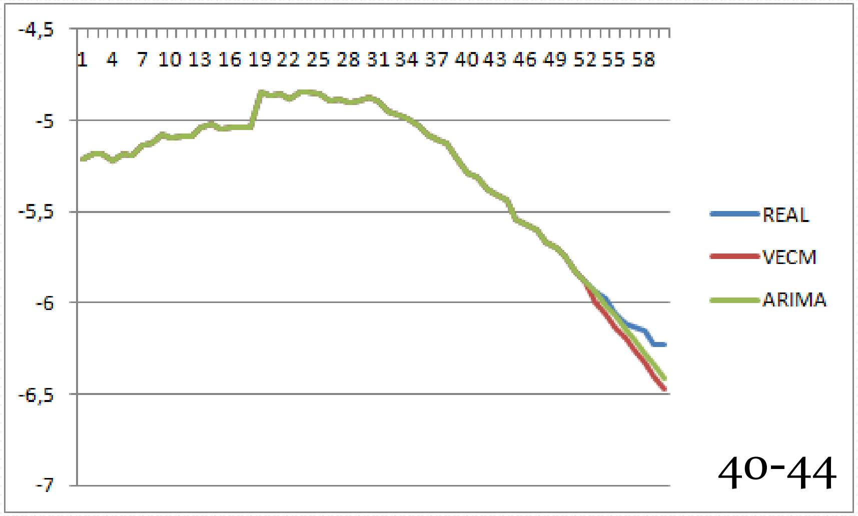
Trace Test, Maximum Eigenvalue Test

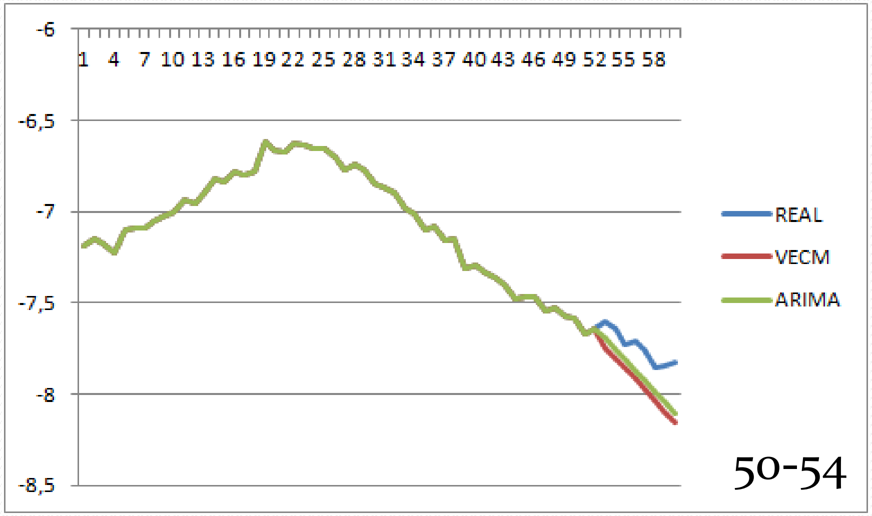
FEMALE

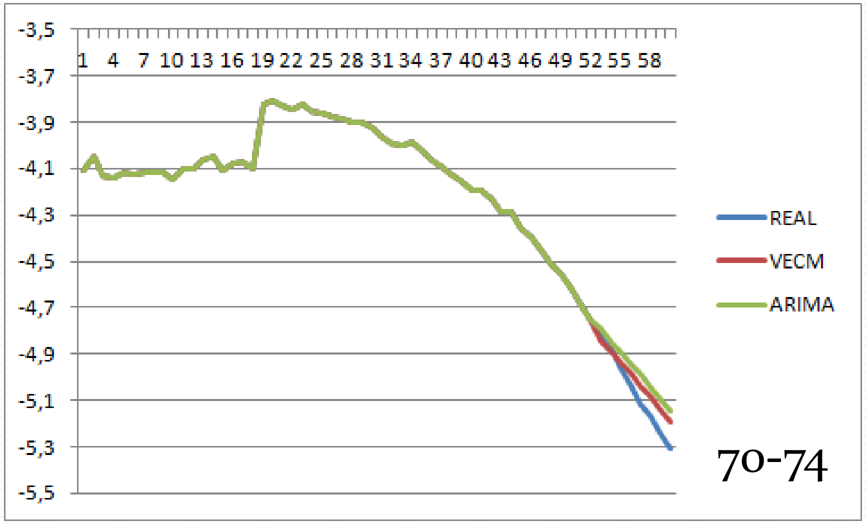
h	*n-h*	*stat*	*10%*	*5%*	*2.5%*	*1%*
4	1	0.2883353	2.70	3.84	5.25	6.98
3	2	9.5286355	15.74	18.08	20.26	22.40
2	**3**	**39.2167916**	**31.67**	**34.27**	**36.98**	**40.10**
1	4	83.8383564	50.62	54.02	57.01	61.03
0	5	155.0295236	73.73	77.61	81.29	85.56

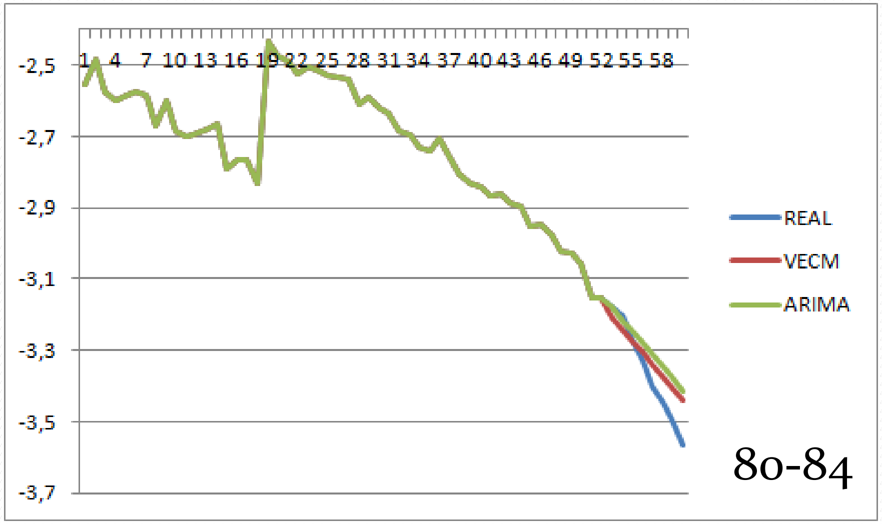
h	*n-h*	*stat*	*10%*	*5%*	*2.5%*	*1%*
4	1	0.2883353	2.70	3.84	5.25	6.98
3	2	9.2403001	14.64	16.69	18.84	20.88
2	**3**	**29.6881561**	**21.44**	**23.75**	**25.68**	**28.31**
1	4	44.6215648	27.39	29.93	32.22	35.57
0	5	71.1911673	33.45	36.46	39.00	41.87

Forecast VECM and ARIMA









MAPE

MALE	*VECM*	*ARIMA*
2002	1,32%	**0,72%**
2003	1,40%	**1,03%**
2004	1,55%	**1,42%**
2005	1,78%	**1,72%**
2006	**2,43%**	2,46%
2007	**2,53%**	2,55%
2008	**3,17%**	3,21%
2009	**3,30%**	3,35%

FEMALE	*VECM*	*ARIMA*
2002	1,08%	**0,76%**
2003	1,63%	**1,31%**
2004	**1,44%**	1,47%
2005	**1,74%**	1,83%
2006	**1,84%**	2,04%
2007	**2,17%**	2,31%
2008	**2,40%**	2,42%
2009	**2,85%**	2,93%

Table 1. Actuarial Periodic Premium – Female. Issue Time 2014, r = 2%, i = 7%, C = 200.000

Table 1.a *Standard Insured Loan – SIL*

Age at entry/Duration	40	60
10	90,19	749,04
20	175,95	1.320,07

Table 1.b *Specific Insured Loan – SpeIL*

Age at entry/Duration	40	60
10	62,65	532,64
20	120,72	981,14

Table 2. Actuarial Periodic Premium
Female non-smokers. Issue Time 2014, r = 2%, i = 7%, C = 200.000

Table 2.a *Stand. C. I. Loan (Stand Alone)- SCILsa*

Age at ent./ Duration	40	60
10	262,4	831,4
20	422,3	1198,6

Table 2.b *Standard C.I. Loan (Accelerated) – SCILa*

Age at ent./ Duration	40	60
10	285,62	925,37
20	456,98	1424,2

Table 3. Actuarial Periodic Premium
Female smokers. Issue Time 2014, r = 2%, i = 7%, C = 200.000

Table 3.a *Stand. C. I. Loan (Stand Alone)- SCILsa*

Age at ent./ Duration	40	60
10	213,8	805
20	273,2	925,8

Table 3.b *Standard C.I. Loan (Accelerated) – SCILa*

Age at ent./ Duration	40	60
10	352,21	1304,3
20	580,7	2034,17

Table 1. Actuarial Periodic Premium – Male. Issue Time 2014, r = 2%, i = 7%, C = 200.000

Table 1.a *Standard Insured Loan – SIL*

Age at entry/Duration	40	60
10	108,23	1.251,55
20	231,06	2.106,08

Table 1.b *Specific Insured Loan – SpeIL*

Age at entry/Duration	40	60
10	64,87	746,67
20	129,59	1.440,78

Table 2. Actuarial Periodic Premium
Male non-smokers. Issue Time 2014, r = 2%, i = 7%, C = 200.000

Table 2.a *Stand. C. I. Loan (Stand Alone)- SCILsa*

Age at ent./ Duration	40	60
10	218,7	1339,6
20	429,7	2049,7

Table 2.b *Standard C.I. Loan (Accelerated) – SCILa*

Age at ent./ Duration	40	60
10	260,27	1515,2
20	498,18	2373,1

Table 3. Actuarial Periodic Premium
Male smokers. Issue Time 2014, r = 2%, i = 7%, C = 200.000

Table 3.a *Stand. C. I. Loan (Stand Alone)- SCILsa*

Age at ent./ Duration	40	60
10	440,6	2378
20	834,2	3590,5

Table 3.b *Standard C.I. Loan (Accelerated) – SCILa*

Age at ent./ Duration	40	60
10	547,7	2975
20	1035,2	4686,7

Amortization Schedule

Table 7.a. Amortization Schedule.

Issue Time 2014, r = 7%, C = 200.000, n = 10

Mat.	Financial Instalment	Payment due in case of insolvency
1	28.475.50	214.000,00
2	28.475,50	198.511,21
3	28.475,50	181.938,21
4	28.475,50	164.205,10
5	28.475,50	145.230,66
6	28.475,50	124.928,02
7	28.475,50	103.204,22
8	28.475,50	79.959,72
9	28.475,50	55.088,10
10	28.475,50	28.475,49

Table 7.a. Amortization Schedule.

Issue Time 2014, r = 7%, C = 200.000, n = 20

Mat.	Financial Instalment	Payment due in case of insolvency	Maturity	Financial Instalment	Payment due in case of insolvency
1	18.878,59	214.000,00	**11**	18.878,59	141.876,95
2	18.878,59	208.779,91	**12**	18.878,59	131.608,26
3	18.878,59	203.194,41	**13**	18.878,59	120.620,75
4	18.878,59	197.217,95	**14**	18.878,59	108.864,10
5	18.878,59	190.823,10	**15**	18.878,59	96.284,51
6	18.878,59	183.980,65	**16**	18.878,59	82.824,33
7	18.878,59	176.659,19	**17**	18.878,59	68.421,95
8	18.878,59	168.825,25	**18**	18.878,59	53.011,41
9	18.878,59	160.442,95	**19**	18.878,59	36.961,41
10	18.878,59	151.473,85	**20**	18.878,59	18.878,59

Global Installment

Table 8.a. *Global annual obligation.*
Standard C.I. Loan (Accelerated) – SCILa
Female non smokers, C=200000, i=7%, r=2%

Age at entry/ Duration	40	60
10	28761.12	29400.87
20	19335.57	20302.75

Table 8.b. *Global annual obligation.*
Standard Insured Loan – SIL
Female non smokers, C=200000, i=7%, r=2%

Age at entry/ Duration	40	60
10	28565.69	29224.54
20	19054.54	20198.66

Table 8.c *Global annual obligation.*
Specific C.I. Loan (Accelerated)- SCILsa
Female non smokers, C=200000, i=7%, r=2%

Age at entry/ Duration	40	60
10	28738.88	29307.03
20	19300.87	20077.33

Table 8.c *Global annual obligation.*
Specific Insured Loan – SpeIL
Female, C=200000, i=7%, r=2%

Age at entry/ Duration	40	60
10	28538.15	29008.14
20	18999.31	19859.73

Conclusions

- The causes of death are competing risk, a dependence exists between them.

- We introduce a new method to better understand the dependence between all causes of death, also mitigating the discontinuity points.

- We can propose tailored contract modelling the cause – specific deaths.

Thanks for your attention

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