



**UNIVERSITY OF SALERNO**  
**ENGINEERING FACULTY**

**PH. D. COURSE**

**“STRUCTURAL ENGINEERING AND URBAN REHABILITATION”**

**(XIII CICLE)**

Thesis submitted for the degree of  
Doctor of Philosophy  
Field of study: Structural Mechanics

**ON THE MECHANICAL MODELING AND THE OPTIMAL DESIGN  
OF TENSEGRITY STRUCTURES**

**ABSTRACT**

**Tutor**

Prof. Fernando Fraternali

**Co-tutor**

Prof. Robert E. Skelton

**Candidate**

Gerardo Carpentieri

Matr. 8881200083

**Coordinator**

Prof. Ciro Faella

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In this thesis, we investigate the use of the most fundamental elements; cables for tension and bars for compression, in the search for the most efficient bridges. Stable arrangements of these elements are called tensegrity structures. We show herein the minimal mass arrangement of these basic elements to satisfy both yielding and buckling constraints. We show that the minimal mass solution for a simply-supported bridge subject to buckling constraints matches Michell's 1904 paper which treats the case of only yield constraints, even though our boundary conditions differ. The necessary and sufficient condition is given for the minimal mass bridge to lie totally above (or below) deck. Furthermore this condition depends only on material properties. If one ignores joint mass, and considers only bridges above deck level, the optimal complexity (number of elements in the bridge) tends toward infinity (producing a material continuum). If joint mass is considered then the optimal complexity is finite. The optimal (minimal mass) bridge below deck has the smallest possible complexity (and therefore cheaper to build), and under reasonable material choices, yields the smallest mass bridge.

We also study a design for a minimal mass, deployable support structure for a solar panel covering of water canals. The results are based upon the minimal mass properties of tensegrity structures. The efficient structure is a tensegrity system which has an optimal complexity for minimal mass. This optimal complexity is derived in this thesis, along with deployable schemes which are useful for construction, repairs, for sun following, and for servicing. It is shown that the minimal structure naturally has deployable features so that extra mass is not needed to add the multifunctional features. The design of bridge structures with tensegrity architecture will show an optimal complexity depending only on material choices and external loads. The minimization problem considers a distributed load (from weight of solar panels and wind loads), subject to buckling and yield constraints. The result is shown to be a Class 1 Tensegrity substructure (support structure only below the deck).

These structures, composed of axially-loaded members (tension and compressive elements), can be easily deployable and have many portable applications for small spans, or they can be easily assembled for prefabricated component parts for large spans. The focus of this work is an application

of these minimal mass tensegrity concepts to design shading devices to prevent or reduce evaporation loss, while generating electric power with solar panels as the cover.

While the economics of the proposed designs are far from finalized, this document shows a technical solution that uses the smallest material resources, and shows the technical feasibility of the concept.

Moreover, we formulate and discuss the relationship between polyhedral stress functions and tensegrity structures in 2D, and a two-mesh technique for the prediction of the stress field associated with such systems.

We generalize classical results concerned with smooth Airy stress functions to polyhedral functions associated with arbitrary triangulations of a simply-connected domain.

We also formulate a regularization technique that smoothly projects the stress function corresponding to an unstructured force network over a structured triangulation. The thesis includes numerical examples dealing with a benchmark problem of plane elasticity, and the stress fields associated with tensegrity models of a cantilever beam and an arch bridge.