Abstract

In the "standard" Gizburg-Landau approach, a phase transition is intimately connected to a local order parameter, that spontaneously breaks some symmetries. In addition to the "traditional" symmetry-breaking ordered phases, a complex quantum system exhibits exotic phases, without classical counterpart, that can be described, for example, by introducing non-local order parameters that preserve symmetries.

In this scenario, this thesis aims to shed light on open problems, such as the local distinguishability between ground states of a symmetry-breaking ordered phase and the classification of one dimensional quantum orders, in terms of entanglement measures, in systems for which the Gizburg-Landau approach fails.

In particular, I briefly introduce the basic tools that allow to understand the nature of entangled states and to quantify non-classical correlations. Therefore, I analyze the conjecture for which the maximally symmetry-breaking ground states (MSBGSs) are the most classical ones, and thus the only ones selected in real-world situations, among all the ground states of a symmetry-breaking ordered phase. I make the conjecture quantitatively precise, by proving that the MSBGSs are the only ones that: i) minimize pairwise quantum correlations, as measured by the quantum discord; ii) are always local convertible, by only applying LOCC transformations; iii) minimize the residual tangle, satisfying at its minimum the monogamy of entanglement.

Moreover, I analyze how evolves the distinguishability, after a sudden change of the Hamiltonian parameters. I introduce a quantitative measure of distinguishability, in terms of the trace distance between two reduced density matrices. Therefore, in the framework of two integrable models that falls in two different classes of symmetries, i.e. XY models in a transverse magnetic field and the N-cluster Ising models, I prove that the maximum of the distinguishability shows a time-exponential decay. Hence, in the limit of diverging time, all the informations about the particular initial ground state disappear, even if a system is integrable.

Far away from the Gizburg-Landau scenario, I analyze a family of fully-analytical solvable one dimensional spin-1/2 models, named the N-cluster models in a transverse magnetic field. Regardless of the cluster size $N+2$, these models exhibit a quantum phase transition, that separates a paramagnetic phase

from a cluster one. The cluster phase coresponds to a nematic ordered phase or a symmetry-protected topological ordered one, for even or odd N respectively. Using the Jordan-Wigner transformations, it is possible to diagonalize these models and derive all their spin correlation functions, with which reconstruct their entanglement properties. In particular, I prove that these models have only a non-vanishing bipartite entanglement, as measured by the concurrence, between spins at the endpoints of the cluster, for a magnetic field strong enough.

Moreover, I introduce the minimal set of nonlinear ground-states functionals to detect all 1-D quantum orders for systems of spin-1/2 and fermions. I show that the von Neumann entanglement entropy distinguishes a critical system from a non critical one, because of the logarithmic divergence at a quantum critical point. The Schmidt gap detect the disorder of a system , because it saturates to a constant value in a paramagnetic phase and goes to zero otherwise. The mutual information, between two subsystems macroscopically separated, identifies the symmetry-breaking ordered phases, because of its dependence on the order parameters. The topological order phases, instead, via their deeply non-locality, can be characterized by analyzing all three functionals.