Università degli Studi di Salerno
DIPARTIMENTO DI SCIENZE ECONOMICHE E STATISTICHE

Rosa Ferrentino*

WELL-POSEDNESS, A SHORT SURVEY

WORKING PAPER 3.183

Department of Economic and Sciences – University of Salerno – via Ponte Don Melillo – 84084 Fisciano (Salerno)

Abstract

In this paper we analyze the property of Tykhonov well-posedness in relation to other well-posedness properties which are ordinal and, as stated in the title, we give a survey on some important results on well-posedness in scalar optimization and in scalar inequalities.

1 Introduction

The notion of well-posedness is significant for several mathematical problems and, in particular, it plays an important role in establishing convergence of algorithms for solving scalar optimization problems.
Two notions of well-posedness, extensively investigated in many papers [4],[10],[16] are known in scalar optimization: one is based on the behaviour of a prescribed class of sequences of approximate solutions, the other on the continuous dependence of the solution on the data of the problem. These notions are named, respectively, Tykhonov and Hadamard well-posedness.
The notion of well-posedness was generalized also to other contexts:variational inequalities, Nash equilibria, saddle point problems and vector optimization problems, all special cases of an equilibrium problem. For istance, Lignola and Morgan [7] investigated well-posedness for optimization problems with constraints defined by variational inequalities; Huang [6] discussed well-posedness for vector optimization problems while Patrone and Margiocco [8], [12], [14] Tykhonov well-posedness for Nash equilibria. Cavazzuti and Morgan [2], at least, given a definition of well-posed saddle point problems and related results. In all these cases the idea is an extension of the concept of minimizing sequences seen as approximate solutions.
The aim of this survey is to collect the two notions of well-posedness, Tykhonov well-posedness and Hadamard well-posedness, and to show some links between these two

different notions. It is organized as follows. In section 2, after notations and definitions needed in addressing our study, we consider the two different definitions of well-posedness while subsequently in section 3 we present some strengthened versions of well-posedness (for istance well-posednes in the sense of Levitin and Polyak) and some notions generalized in case in which there is not uniqueness of solutions. Section 4 contains the relationships between Tykhonov and Hadamard well-posedness while the last section of the paper contains some basics on the notion of well-posedness of a scalar variational inequality. The notion of well-posedness in vector case will be considered separately.

2. Notations, definitions and preliminaries

In scalar optimization the different notions of well-posedness are based either on the behaviour of "appropriate" minimizing sequences (one requires the convergence of such sequences to a solution of the problem) or on the dependence of the optimal solutions on the data of optimization problem. This section is devoted exactly to a study of these notions.

The first notion of well-posedness of an optimization problem was introduced in 1966 by Tykhonov and later took his name. Given a real-valued function $f : R^n \to R$ and a closed, nonempty and convex subset K of R^n, we consider the scalar optimization problem:

$$P(f,K) \qquad \min_{x \in K} f(x)$$

which we denote in the sequel by $P(f,K)$ and which consists in finding $x_0 \in K$ such that

$$f(x_0) = \inf\{f(x) : x \in K\} = \inf_K f(x)$$

The set, possible empty, of the solutions of the minimization problem $P(f,K)$ is denoted by *argmin* $P(f,K)$.

It is well known that usually every numerical method solving the optimization problem $P(f,K)$ produces a sequence.

A sequence $\{x_n\}_{n=1}^{\infty} \subset K$ with the property:

$$f(x_n) \to \inf_K f(x) = f(x_0) \qquad \text{as } n \to +\infty$$

is said *minimizing sequence* for the problem $P(f,K)$. Such sequence is also called *sequence of approximate solutions* for the problem $P(f,K)$. The following classical definition states the notion of well-posedness of a given optimization problem $P(f,K)$ and it implies unique solvability of $P(f,K)$ together with convergence of the numerical methods of minimization for problem $P(f,K)$.

Definition 2.1[4]: *The minimization problem $P(f,K)$ is called Tykhonov well-posed if it has unique solution $x_0 \in X$ and, moreover, every minimizing sequence for $P(f,K)$ converges to x_0.*

More precisely, $P(f,K)$ is Tykhonov well-posed if exists exactly one $x_0 \in K$ such that $f(x_0) \leq f(x)$ for all $x \in K$ and $x_n \to x_0$ whenever $f(x_n) \to \inf_K f(x) = f(x_0)$ and i.e. the Tykhonov well-posedness of the minimization problem $P(f,K)$ requires existence and uniqueness of minimum point x_0 towards which every approximate solution of $P(f,K)$ converges. In other words, to consider well-posedness of Tykhonov type, one introduces the notion of "approximating sequence" for the solutions of optimization problems and requires convergence of such sequences to a solution of the problem. For more details see [4] and [8].

Example: Let $K = R$. If $f(x) = x^2\ e^{-x}$, $P(f,K)$ has a unique minimum $x_0 = 0$ ma it is not Tykhonov well-posed, since the sequences $\{x_n\} = \{n\}$ is minimizing but it does not converges to $x_0 = 0$. If $f(x) = x^2$ then $P(f,K)$ is Tykhonov well-posed.

For convex functions in finite dimensions the uniqueness of the solution is enough to guarantee its Tykhonov well-posedness while this is no longer valid in infinite dimensions [15]. In analogous way the uniqueness of the solution of a minimization problem $P(f,K)$ is enough to guarantee its well-posedness in the case when K is compact. Instead, the uniqueness of the solution is not enough to guarantee Tykhonov well-posedness for continuous functions. A simple example of a problem with a unique solution but which is not Tykhonov well-posed is the following:

$$f(x) = x^2/(x^4+1) \qquad \forall\ x \in X$$

Obviously $P(f,K)$ has a unique solution at zero, i.e. the argmin $P(f,K) = \{0\}$, while $x_n = n$, $n = 1,2,...$ provides a minimizing sequence for $P(f,K)$ which does not converge to this unique solution. Hence $P(f,K)$ is not Tykhonov well-posed. Therefore for continuous functions Tykhonov well-posedness of $P(f,K)$ simply means that every minimizing sequence of $P(f,K)$ is convergent.

The following results are known [4].

Proposition 2.1: *Let $f : K \subseteq R^n \to R$ be a convex function and let K be convex set. If $P(f,K)$ has a unique solution, then $P(f,K)$ is Tykhonov well-posed.*

Proposition 2.2: *If K is a nonempty convex and closed subset of R^n and if f is lower hemicontinuous bounded from below and uniformly quasi convex on K, then $P(f,K)$ is Tykhonov well-posed.*

The next fundamental theorem [4] gives an alternative characterization of Tykhonov well-posed problems; it states that Tykhonov well-posedness of $P(f,K)$ can be characterized by behaviour of $diam[\varepsilon - \arg\min(f,K)]$ as $\varepsilon \to 0$.

Theorem 2.1: *If $P(f,K)$ is Tykhonov well-posed, then*

$$diam[\varepsilon - \arg\min(f,K)] \to 0 \text{ as } \varepsilon \to 0$$

where $\varepsilon - \arg\min(f,k) = \{x \in K : f(x) \le \varepsilon + \inf_K f(x)\}$ is the set of $\varepsilon-$minimizers of f over K.

Conversely, if f is lower semicontinuous and bounded from below on K, $diam[\varepsilon - \arg\min(f,K)] \to 0$ as $\varepsilon \to 0$ implies Tykhonov well-posedness of $P(f,K)$.

The second notion of well-posedness is inspired by the classical idea of J. Hadamard to the beginning of previous century: it requires existence and uniqueness of solution of the optimization problem together with continuous dependence of solution on the problem's data.

Definition 2.2: *The minimization problem $P(f,K)$ is said to be Hadamard well-posed if it has unique solution $x_0 \in K$ and x_0 depends continuously on the data of the problem.*

This is the well-known condition of well-posedness considered in the study of differential equations, translated for minimum problems. The essence of this notion is that a "small" change of the data of the problem yields a "small" change of the solution. In fact, very often the mathematical model of a phenomenon is so complicated that it is necessary to simplify it and replace it by other model which is “near” the original and, at the same time, one wants to be sure that the new problem will have a solution which is “near” the original one. The well-Known variational principle of Ekeland [5] asserts just that a particular optimization problem can be replaced by other which is near the original and has a unique solution.

3. Some generalizations

The idea of the behaviour of the minimizing sequences was used by different authors to extend this concept to two directions: first for strengthened notions and second for the case in which the optimal solution is not unique. These notions are not suitable for numerical methods, where the function f is approximated by a family or a sequence of functions. For this reason new notions of well-posedness have been introduced and studied. Before, however, we consider two generalizations of the notion of minimizing sequence. The first one was introduced and studied by Levitin and Polyak.

Definition 3.1: *Let K be a nonempty subset of R^n. The sequences $\{x_n\}_{n=1}^{\infty} \subset K$ is a Levitin-Polyak minimizing sequences for the minimization problem $P(f,K)$ if $\forall x_n \in K$:*

$$f(x_n) \to \inf_K f(x) \quad \textit{and} \quad d(x_n,K) \to 0$$

where $d(x,K) = \inf\{\|x-y\| : y \in K\}$ is the distance function from the point x to the set K while $\| \ \|$ is the Euclidean norm.

In other words, a sequences $\{x_n\}_{n=1}^{\infty}$ is a *Levitin-Polyak minimizing sequences* for $P(f,K)$ if not only $\{f(x_n)\}_{n=1}^{\infty}$

approaches the greatest lower bound of f over K but also the sequence $\{x_n\}_{n=1}^{\infty}$ tends (with respect to the norm) to K.

Then, the well-posedness concept can be strengthened as follows.

Definition 3.2: *The minimization problem $P(f,K)$ is called Levitin-Polyak well-posed if it has unique solution $x_0 \in K$ and moreover, every Levitin-Polyak minimizing sequence for $P(f,K)$ converges to x_0.*

Of course, this definition is stronger than the Tykhonov one since one wants each sequence from a larger set of minimizing sequences to be convergent to the unique solution, i.e. Levitin-Polyak well-posedness implies Tykhonov well-posedness. The converse is true provided that f is uniformly continuous but not necessarily true if f is only continuous. It is enough to consider $K = R \times \{0\} \subset R^2$ $f(x,y) = x^2 - y^2(x + x^4)$ and generalized minimizing sequences $\{1 \setminus n\}$.

As Tykhonov well-posedness can be characterized by the behaviour of $diam[\varepsilon - \arg\min(f,K)]$, as Levitin Polyak well-posedness can be characterized by the behaviour of the set:

$$L(\varepsilon) = \{x \in K : dist(x,K) \le \varepsilon \quad \text{and} \quad f(x) \le \inf f(K) + \varepsilon\}$$

defined for $\varepsilon > 0$ and for f bounded from below on K. In analogy with the theorem 2.1 the following result gives:

Theorem 3.1 [4]: *If K is closed and f is lower semicontinuous and bounded from below on K, then $diam\, L(\varepsilon) \to 0$ as $\varepsilon \to 0$ implies Levitin-Polyak well-posedness of $P(f,K)$*

A second generalization of the usual notion of minimizing sequences is the following:

Definition 3.3: *A sequence* $\{x_n\}_{n=1}^{\infty} \subset K$ *is said to be a generalized minimizing sequence for the minimization problem* $P(f,K)$ *if are fulfilled both:*

$$d(x_n, K) \to 0 \qquad \textit{and} \qquad \textit{lim sup} \quad f(x_n) \leq \inf_K f(x)$$

Consequently another strengthened version of the well-posedness is:

Definition 3.4: *The minimization problem* $P(f,K)$ *is said strongly well-posed if it has unique solution* $x_0 \in K$ *and, moreover, every generalized minimizing sequences for* $P(f,K)$ *converges to* x_0.

Obviously, in general strong well-posedness of the problem $P(f,K)$ implies Levitin-Polyak one, which in its turn implies Tykhonov well-posedness. It is important underline that, unlike def. 2.2, each of the previous definitions (def 2.1, def 3.2, def 3.4), widely studied in many papers [8],[9],[10], [15], is based on the behaviour of a certain set of minimizing sequences.
While the requirement of existence in the previous definitions is crucial, the uniqueness condition is more debatable. Many problems in linear programming or multicriteria optimization problems are in fact usually considered as well-posed problems, although they do not satisfy this uniqueness requirement. More precisely the different notions of well-posedness admit generalizations which do not require uniqueness of the solution. In particular, the concept of Tykhonov well posedness can be extend to minimum problems without uniqueness of the optimal solutions. This new definition requires existence, but not uniqueness, of

solution of $P(f,K)$, and, for every minimizing sequences, the convergence of some subsequence towards some optimal solution.

Definition 3.5: *The problem $P(f,K)$ is called Tykhonov well-posed in the generalized sense if every its minimizing sequence has some subsequence converging to a solution of a $P(f,K)$, i.e. to an element of $\arg\min(f,K)$.*

More precisely the problem $P(f,K)$ is called Tykhonov well-posed in the generalized sense if argmin $(f,K)\neq\emptyset$ and every sequence $x_n \in K$ such that $f(x_n) \to \inf f(K)$ has some subsequence $y_n \to y$ with $y \in \arg\min(f,K)$.

Obviously, if the problem $P(f,K)$ is Tykhonov well-posed in the generalized sense then it has a non-empty compact set of solutions, i.e. *argmin* (f,K) is nonempty and compact.
When *argmin* (f,K) is a singleton, the previous definition reduces to the classical notion of Tykhonov well-posedness or rather the problem $P(f,K)$ is Tykhonov well-posedness if it is Tykhonov well-posed in the generalized Tykhonov sense and *argmin* (f,K) is a singleton; thus generalized well-posedness is really a generalization of Tykhonov well-posedness.
The corresponding generalization of Levitin-Polyak well-posedness is:

Definition 3.7: *The minimization problem $P(f,K)$ is called generalized Levitin-Polyak well-posed if every Levitin-Polyak minimizing sequence $\{x_n\} \subset K$ for $P(f,K)$ has a subsequence converging to a solution of $P(f,K)$.*

Any of the notions of generalized well-posedness together with the uniqueness of the solution is equivalent obviously to corresponding non generalized notion.

4. Relations between Hadamard and Tykhonov well-posedness

Almost all the literature deals with different notions of well-posedness, even if especially with Tykhonov types well-posedness. Some researchers have investigated the relations between these notions of well-posedness but there is no general research to such relations. At first sight the two notions seem to be independent but, at least in the convex case, there are some works showing a connection between the two properties: for instance [8],[19]. The two notions (Tykhonov and Hadamard well-posedness) are equivalent at least for continuous objective functions. The links between Hadamard and Tykhonov well-posedness have been studied in [8], [10], and in [15], [16]. There, besides uniqueness, additional structures are involved: in [8], for example, basic ingredient is convexity.
The object of this section is to describe generally the relations between Hadamard and Tykhonov well-posedness: a central role is provided by the well-Known Hausdorff convergence.

Definition 4.1: *Let* $\{A_n\}$ *be a sequence of subsets of* R^n*.We say that* A_n *converges according to Hausdorff to* $A \subset R^n$ *when*

$$e(A_n, A) \to 0$$

where $e(A_n, A) = \sup_{a \in A_n} d(a, A)$ with $d(a, A) = \inf_{b \in A} \|a - b\|$

The following theorems show the relations between Tykhonov and Hadamard well-posedness.

Theorem 4.1 [8]: *Let K be a closed convex subset of R^n and let $f: K \to R$ be a convex continuous function with a unique minimum point on every closed and convex subset of K. If $P(f, K)$ is Hadamard well-posed, with respect to the Hausdorff convergence, then $P(f, K)$ is Tykhonov well-posed on every closed and convex subset of K.*

Theorem 4.2 [8]: *Let $f: R^n \to R$ be a convex function uniformly continuous on every bounded set. If $P(f, K)$ is Tykhonov well-posed on every closed and convex set, then $P(f, K)$ is Hadamard well-posed, with respect to the Hausdorff convergence.*

Tykhonov well-posedness does not, in general, imply Hadamard well-posedness if the objective function is only continuous.

5. Well-posed variational inequalities

In this section, some notions of well-posedness for scalar variational inequalities and their connections with optimization problems are presented.
In recent years variational inequality theory, which was introduced in the sixties of previous century, has emerged as a powerful tool for handling optimization problems and its importance is not only from the theoretical point of view but also provides an unified and efficient framework for a wide spectrum of applied problems. In other words the theory of variational inequalities provides a convenient mathematical apparatus for obtain result relating to a large number of problems with a wide range of applications in economics, social, pure and applied sciences. In fact, it is well known that many equilibrium problems arising in finance, economics, transportation science and contact problems in

elasticity can be formulated in terms of the variational inequalities [13].

Given an operator $F : K \to R^n$ with K a nonempty, closed and convex subset of R^n, we consider the problem which consist in finding a vector $x^* \in K$ such that:

$$SVI(F,K) \qquad < F(x^*), y - x^* > \geq 0 \qquad \forall y \in K$$

where $< \cdot,\cdot >$ denotes the usual inner product in R^n.

The previous problem is known as the *Stampacchia variational inequality* (for short $SVI(F,K)$) and was introduced and studied mainly by Stampacchia in 1964 [17].

Definition 5.1: *The variational inequality* $SVI(F,K)$ *is well-posed when:*

i) $G(\varepsilon) \neq \emptyset$ $\qquad \forall \varepsilon > 0$

ii) $diam\, G(\varepsilon) \to 0$ $\qquad$ *as* $\varepsilon \to 0$

where $G(\varepsilon) = \{x \in K :< F(x), x - y > \leq \varepsilon \|y - x\| \quad \forall y \in K\}$.

Some results concerning the existence and uniqueness of a solution of variational inequality are investigated.

Under very weak hypotheses, if the variational inequality is well-posed, the solution of variational inequality exist and is unique. Before, we recall the following well Known concepts that will be useful later on*:*

Definition 5.2: *The operator* F *is said to be*

i) *monotone on* K *if:* $\forall x, y \in K \;\; < F(x) - F(y), x - y > \geq 0$

ii) *pseudomonotone on* K *if:* $\forall x, y \in K$

$$< F(x), x - y > \geq 0 \Rightarrow < F(y), x - y > \geq 0$$

iii) *hemicontinuous on* K *if* $x_n \to x$ *implies* $F(x_n) \to F(x)$

It is well known that monotonicity implies pseudomonotonicity but the converse is not true.

Proposition 5.1 [4],[9]: *Let F be an hemicontinuous and monotone function. If the variational inequality $SVI(F,K)$ is well-posed, then $SVI(F,K)$ has an unique solution.*

Proof: The set of solution of $SVI(F,K)$ is exactly $\bigcap_{\varepsilon \to 0} T(\varepsilon)$. If the variational inequality is well-posed, then $\bigcap_{\varepsilon \to 0} T(\varepsilon)$ is nonempty and shrinks to a single point.

In [4] can be found the converse of previous proposition:

Proposition 5.2: *Let F be an hemicontinuous and monotone function. If $SVI(F,K)$ has exactly one solution, then $SVI(F,K)$ is well-posed.*

Now, let us turn briefly our attention to scalar variational inequalities of differential type (i.e. in which the operator F involved is the gradient of a given function f). Assume that $f : R^n \to R$ is differentiable on an open set containing K, and denote by f' the gradient of f. We recall that:

- a point $x^* \in K$ is a solution of a Stampacchia variational inequality of differential type when:

$$SVI(f',K) \qquad < f'(x^*), y - x^* > \geq 0 \qquad \forall\, y \in K$$

- an optimization problem can be expressed by means of a variational inequality involving the gradient of f.

The links between variational inequalities of differential type and optimization problems have been deeply studied [9], [11],[18]. More precisely, in [4], the following characterizations for Tykhonov well-posed problem are given:

Proposition 5.3: *Let K be a closed convex subset of R^n and let be $Z(\varepsilon) = \{ x \varepsilon K : f(x) \leq f(y) + \varepsilon \|x - y\| \quad \forall\, y \in K \}$.*

If $Z(\varepsilon) \neq \emptyset \quad \forall \varepsilon > 0$ *and* $diam\, Z(\varepsilon) \to 0$ *as* $\varepsilon \to 0$*, then* $P(f,K)$ *is Tykhonov well-posed. The converse holds provided* f *is convex on* K*.*

Proposition 5.4: *Let* f *be differentiable and bounded from below on* K*. Then,* $\forall \varepsilon > 0$*:*

$$T(\varepsilon) = \{x \in K : < f'(x), x - y > \leq \varepsilon \|y - x\| \quad \forall\, y \in K\} \neq 0$$

If $diam\, T(\varepsilon) \to 0$ *as* $\varepsilon \to 0$*, then* $P(f,K)$ *is Tykhonov well-posed. The converse holds if moreover* f *is convex on* K*.*

Since it is well Known [9] that the minimization problem is equivalent to a variational inequality involving the operator f', the following definition that gives the notion of well-posedness for a variational inequality of differential type follows.

Definition 5.3 [4]: *Let* f *be a function convex, lower semi-continuous, differentiable and bounded from below on a closed convex set* K *. The variational inequality* $SVI(f',K)$ *is well-posed when:*

i) $T(\varepsilon) \neq \emptyset \quad \forall \varepsilon > 0$

ii) $diam\, T(\varepsilon) \to 0$ *as* $\varepsilon \to 0$

An important extension call "well-posed" those variational inequalities (governed by an operator that is not necessarily a derivative) such that the diameter of the level set of the operator decreases to zero.

The following theorem gives the link between Tykhonov well-posedness of $P(f,K)$ and well-posedness of $SVI(f',K)$:

Theorem 5.1 [4], [9]: *Let* f *be bounded from below and differentiable on an open set containing* K*. If* $SVI(f',K)$ *is* *well-posed, then problem* $P(f, K)$ *is Tykhonov well-posed. The converse is true if* f *is convex.*

Bibliography

[1]Bednarczuk E.-Penot J.P.:*On the position of the notions of well-posed minimization problems*,BollettinoU.M.I.vol.7, 1992

[2]Cavazzuti E.-Morgan J.:*Well-posed saddle point problems* in Optimization, Theory and Algorithms, (eds. Hirriat-Urruty, Oettli and Stoer), Proc. Conf. Confolant/France 1981,1983.

[3] Dentcheva D. - Helbig S.: *On variational princeples, level sets*, *well-posedness and* ε *- solutions in vector optimization,* Journal of Optimization Theory and Applications vol.89, 1996

[4]Dontchev A.-Zolezzi T.:*Well-posed optimization problems*, Lecture Notes in Mathematics, vol.1543, Springer Verlag, Berlin, 1993.

[5] Ekeland T.: *On the variational principle*. Journal Math. Anal. Appl. 47,1974.

[6] Huang X.X.:*Extended well-posedness properties of vector optimization problems.* Journal of Optimization Theory and Applications 106, 2000.

[7] Lignola M.B. - J. Morgan: *Well-posedness for optimization problems with constraints defined by variational inequalities having a unique solution.* Journal of Global Optimization 16, 2000.

[8] Lucchetti R. - Patrone F. : *Hadamard and Tykhonov well-posedness of certain class of convex functions*. Journal of Mathematical Analysis and Application Vol.88, 1982.

[9] Lucchetti R. - Patrone F: *A characterization of Tykhonov well-posedness for minimum problems, with applications to variational inequalities,* Numer. Funct. Analysis and Optimiz. Vol.3, 1981.

[10] Lucchetti R.: *Some aspects of the connection between Hadamard and Tykhonov well-posedness of convex problems,* Bollettino U.M.I. serie VI, vol.1, sez. C ,1982.

[11] Lucchetti R. - Patrone F.: *Some properties of well-posed variational inequalities governed by linear operators,* Numer. Funct. Analysis and Optimiz., vol.5, 1982-83.

[12] Margiocco M.- Patrone F-Pusillo C.L.:*A new approach to Tykhonov well-posedness for Nash equilibria,* Optimization 40,1997

[13] Noor M. Aslam - Noor K. Inayat - Rassias Th. M.: *Some aspect of variational inequalities,* Journal of Comput. and Appl. Math. 47,1993.

[14] Patrone F.: *Well-posedness for Nash Equilibria and related topics* in "Recent Development in well-posed variational problems" (Lucchetti and Revalski eds.), Kluwer, Dordrecht, 1995.

[15] Revalski J.P.: *Generic well-posedness in some classes of optimization problems,* Acta University Carolinae: Mathematica et Physica, vol. 28, 1987

[16] Revalski J.P.: *Various aspects of well-posedness of optimization problems.* Recent Developments in well-posed

variational problems, (Lucchetti and Revalski eds), Kluwer Academic Publishers, Dordrecht, 1995.

[17] Stampacchia G.: *Formes bilinéaires coercitives sur les ensembles convexes*, C.R. Acad. Sci. Paris Sér I. Math. 258,1964.

[18] Stampacchia G. – Kinderlehrer D.: *An introduction to variational inequalities and their applications,* Accademic Press, New York, 1980.

[19] Zolezzi T.: *A characterization of well-posed optimal control systems.* Siam Journal Control and Optimization Vol.19, 5, pag.604-616, 1981.

1990, 3.15 Emanuele SALSANO
Investimenti, valore aggiunto e occupazione in Italia in contesto biregionale: una prima analisi dei dati 1970/1982.

1990, 3.16 Alessandro PETRETTO- Giuseppe PISAURO
Uniformità vs selettività nella teoria della ottima tassazione e dei sistemi tributari ottimali.

1990, 3.17 Adalgiso AMENDOLA
Inflazione, disoccupazione e aspettative. Aspetti teorici dell'introduzione di aspettative endogene nel dibattito sulla curva di Phillips.

1990, 3.18 Pasquale PERSICO
Il Mezzogiorno e le politiche di sviluppo industriale.

1990, 3.19 Pasquale PERSICO
Priorità delle politiche strutturali e strategie di intervento.

1990, 3.20 Adriana BARONE - Concetto Paolo VINCI
La produttività nella curva di Phillips.

1990, 3.21 Emiddio GALLO
Varianze ed invarianze socio-spaziali nella transizione demografica dell'Italia post-industriale.

1991, 3.22 Alfonso GAMBARDELLA
I gruppi etnici in Nicaragua. Autonomia politica ed economica.

1991, 3.23 Maria SCATTAGLIA
La stima empirica dell'offerta di lavoro in Italia: una rassegna.

1991, 3.24 Giuseppe CELI
La teoria delle aree valutarie: una rassegna.

1991, 3.25 Paola ADINOLFI
Relazioni industriali e gestione delle risorse umane nelle imprese italiane.

1991, 3.26 Antonio e Bruno PELOSI
Sviluppo locale ed occupazione giovanile: nuovi bisogni formativi.

1991, 3.27 Giuseppe MARIGLIANO
La formazione del prezzo nel settore dell'intermediazione commerciale.

1991, 3.28 Maria PROTO
Risorse naturali, merci e ambiente: il caso dello zolfo.

1991, 3.29 Salvatore GIORDANO
Ricerca sullo stato dei servizi nelle industrie del salernitano.

1992, 3.30 Antonio LOPES
Crisi debitoria e politiche macroeconomiche nei paesi in via di sviluppo negli anni 80.

1992, 3.31 Antonio VASSILLO
Circuiti economici semplici, complessi, ed integrati.

1992, 3.32 Gaetano Fausto ESPOSITO
Imprese ed istituzioni nel Mezzogiorno: spunti analitici e modalità di relazione.

1992, 3.33 Paolo COCCORESE
Un modello per l'analisi del sistema pensionistico.

1994, 3.34 Aurelio IORI
Il comparto dei succhi di agrumi: un caso di analisi interorganizzativa.

1994, 3.35 Nicola POSTIGLIONE
Analisi multicriterio e scelte pubbliche.

1994, 3.36 Adriana BARONE
Cooperazione nel dilemma del prigioniero ripetuto e disoccupazione involontaria.

1994, 3.37 Adriana BARONE
Le istituzioni come regolarità di comportamento.

1994, 3.38 Maria Giuseppina LUCIA
Lo sfruttamento degli idrocarburi offshore tra sviluppo economico e tutela dell'ambiente.

1994, 3.39 Giuseppina AUTIERO
Un'analisi di alcuni dei limiti strutturali alle politiche di stabilizzazione nei LCDs.

1994, 3.40 Bruna BRUNO
Modelli di contrattazione salariale e ruolo del sindacato.

1994, 3.41 Giuseppe CELI
Cambi reali e commercio estero: una riflessione sulle recenti interpretazioni teoriche.

1995, 3.42 Alessandra AMENDOLA, M. Simona ANDREANO
The TAR models: an application on italian financial time series.

1995, 3.43 Leopoldo VARRIALE
Ambiente e turismo: Parco dell'Iguazù - Argentina.

1995, 3.44 A. PELOSI, R. LOMBARDI
Fondi pensione: equilibrio economico-finanziario delle imprese.

1995, 3.45 Emanuele SALSANO, Domenico IANNONE
Economia e struttura produttiva nel salernitano dal secondo dopoguerra ad oggi.

1995, 3.46 Michele LA ROCCA
Empirical likelihood and linear combinations of functions of order statistics.

1995, 3.47 Michele LA ROCCA
L'uso del bootstrap nella verosimiglianza empirica.

1996, 3.48 Domenico RANESI
Le politiche CEE per lo sviluppo dei sistemi locali: esame delle diverse tipologie di intervento e tentativo di specificazione tassonomica.

1996, 3.49 Michele LA ROCCA
L'uso della verosimiglianza empirica per il confronto di due parametri di posizione.

1996, 3.50 Massimo SPAGNOLO
La domanda dei prodotti della pesca in Italia.

1996, 3.51 Cesare IMBRIANI, Filippo REGANATI
Macroeconomic stability and economic integration. The case of Italy.

1996, 3.52 Annarita GERMANI
Gli effetti della mobilizzazione della riserva obbligatoria. Analisi sull'efficienza del suo utilizzo.

1996, 3.53 Massimo SPAGNOLO
A model of fish price formation in the north sea and the Mediterranean.

1996, 3.54 Fernanda MAZZOTTA
RTFL: problemi e soluzioni per i dati Panel.

1996, 3.55 Angela SPAGNUOLO
Concentrazione industriale e dimensione del mercato: il ruolo della spesa per pubblicità e R&D.

1996, 3.56 Giuseppina AUTIERO
The economic case for social norms.

1996, 3.57 Francesco GIORDANO
Sulla convergenza degli stimatori Kernel.

1996, 3.58 Tullio JAPPELLI, Marco PAGANO
The determinants of saving: lessons from Italy.

1997, 3.59 Tullio JAPPELLI
The age-wealth profile and the life-cycle hypothesis: a cohort analysis with a time series of cross sections of Italian households.

1997, 3.60 Marco Antonio MONACO
La gestione dei servizi di pubblico interesse.

1997, 3.61 Marcella ANZOLIN
L'albero della qualità dei servizi pubblici locali in Italia: metodologie e risultati conseguiti.

1997, 3.62 Cesare IMBRIANI, Antonio LOPES
Intermediazione finanziaria e sistema produttivo in un'area dualistica. Uno studio di caso.

1997, 3.63 Tullio JAPPELLI
Risparmio e liberalizzazione finanziaria nell'Unione europea.

1997, 3.64 Alessandra AMENDOLA
Analisi dei dati di sopravvivenza.

1997, 3.65 Francesco GIORDANO, Cira PERNA
Gli stimatori Kernel per la stima non parametrica della funzione di regressione.

1997, 3.66 Biagio DI SALVIA
Le relazioni marittimo-commerciali nell'imperiale regio litorale austriaco nella prima metà dell'800.
I. Una riclassificazione delle Tafeln zur Statistik der Öesterreichischen Monarchie.

1997, 3.67 Alessandra AMENDOLA
Modelli non lineari di seconda e terza generazione: aspetti teorici ed evidenze empiriche.

1998, 3.68 Vania SENA
L'analisi econometrica dell'efficienza tecnica. Un'applicazione agli ospedali italiani di zona.

1998, 3.69 Domenico CERBONE
Investimenti irreversibili.

1998, 3.70 Antonio GAROFALO
La riduzione dell'orario di lavoro è una soluzione al problema disoccupazione: un tentativo di analisi empirica.

1998, 3.71 Jacqueline MORGAN, Roberto RAUCCI
New convergence results for Nash equilibria.

1998, 3.72 Rosa FERRENTINO
Niels Henrik Abel e le equazioni algebriche.

1998, 3.73 Marco MICOCCI, Rosa FERRENTINO
Un approccio markoviano al problema della valutazione delle opzioni.

1998, 3.74 Rosa FERRENTINO, Ciro CALABRESE
Rango di una matrice di dimensione K.

1999, 3.75 Patrizia RIGANTI
L'uso della valutazione contingente per la gestione del patrimonio culturale: limiti e potenzialità.

1999, 3.76 Annamaria NESE
Il problema dell'inefficienza nel settore dei musei: tecniche di valutazione.

1999, 3.77 Gianluigi COPPOLA
Disoccupazione e mercato del lavoro: un'analisi su dati provinciali.

1999, 3.78 Alessandra AMENDOLA
Un modello soglia con eteroschedasticità condizionata per tassi di cambio.

1999, 3.79 Rosa FERRENTINO
Su un'applicazione della trasformata di Laplace al calcolo della funzione asintotica di non rovina.

1999, 3.80 Rosa FERRENTINO
Un'applicazione della trasformata di Laplace nel caso di una distribuzione di Erlang.

1999, 3.81 Angela SPAGNUOLO
Efficienza e struttura degli incentivi nell'azienda pubblica: il caso dell'industria sanitaria.

1999, 3.82 Antonio GAROFALO, Cesare IMBRIANI, Concetto Paolo VINCI
Youth unemployment: an insider-outsider dynamic approach.

1999, 3.83 Rosa FERRENTINO
Un modello per la determinazione del tasso di riequilibrio in un progetto di fusione tra banche.

1999, 3.84 DE STEFANIS, PORZIO
Assessing models in frontier analysis through dynamic graphics.

1999, 3.85 Annunziato GESUALDI
Inflazione e analisi delle politiche fiscali nell'U.E..

1999, 3.86 R. RAUCCI, L. TADDEO
Dalle equazioni differenziali alle funzioni e^x, $\log x$, a^x, $\log_a x$, x^α.

1999, 3.87 Rosa FERRENTINO
Sulla determinazione di numeri aleatori generati da equazioni algebriche.

1999, 3.88 C. PALMISANI, R. RAUCCI
Sulle funzioni circolari: una presentazione non classica.

2000, 3.89 Giuseppe STORTI, Pierluigi FURCOLO, Paolo VILLANI
A dynamic generalized linear model for precipitation forecasting.

2000, 3.90 Rosa FERRENTINO
Un procedimento risolutivo per l'equazione di Dickson.

2000, 3.91 Rosa FERRENTINO
Un'applicazione della mistura di esponenziali alla teoria del rischio.

2000, 3.92 Francesco GIORDANO, Michele LA ROCCA, Cira PERNA
Bootstrap variance estimates for neural networks regression models.

2000, 3.93 Alessandra AMENDOLA, Giuseppe STORTI
A non-linear time series approach to modelling asymmetry in stock market indexes.

2000, 3.94 Rosa FERRENTINO
Sopra un'osservazione di De Vylder.

2000, 3.95 Massimo SALZANO
Reti neurali ed efficacia dell'intervento pubblico: previsioni dell'inquinamento da traffico nell'area di Villa S. Giovanni.

2000, 3.96 Angela SPAGNUOLO
Concorrenza e deregolamentazione nel mercato del trasporto aereo in Italia.

2000, 3.97 Roberto RAUCCI, Luigi TADDEO
Teoremi ingannevoli.

2000, 3.98 Francesco GIORDANO
Una procedura per l'inizializzazione dei pesi delle reti neurali per l'analisi del trend.

2001, 3.99 Angela D'ELIA
Some methodological issues on multivariate modelling of rank data.

2001, 3.100 Roberto RAUCCI, Luigi TADDEO
Nuove classi di funzioni scalari quasiconcave generalizzate: caratterizzazioni ed applicazioni a problemi di ottimizzazione.

2001, 3.101 Adriana BARONE, Annamaria NESE
Some insights into night work in Italy.

2001, 3.102 Alessandra AMENDOLA, Marcella NIGLIO

Predictive distributions of nonlinear time series models.

2001, 3.103 Roberto RAUCCI
Sul concetto di certo equivalente nella teoria HSSB.

2001, 3.104 Roberto RAUCCI, Luigi TADDEO
On stackelberg games: a result of unicity.

2001, 3.105 Roberto RAUCCI
Una definizione generale e flessibile di insieme limitato superiormente in $\Re^n$

2001, 3.106 Roberto RAUCCI
Stretta quasiconcavità nelle forme funzionali flessibili.

2001, 3.107 Roberto RAUCCI
Sugli insiemi limitati in $\Re^m$ *rispetto ai coni.*

2001, 3.108 Roberto RAUCCI
Monotonie, isotonie e indecomponibilità deboli per funzioni a valori vettoriali con applicazioni.

2001, 3.109 Roberto RAUCCI
Generalizzazioni del concetto di debole Kuhn-Tucker punto-sella.

2001, 3.110 Antonia Rosa GURRIERI, Marilene LORIZIO
Le determinanti dell'efficienza nel settore sanitario. Uno studio applicato.

2001, 3.111 Gianluigi COPPOLA
Studio di una provincia meridionale attraverso un'analisi dei sistemi locali del lavoro. Il caso di Salerno.

2001, 3.112 Francesco GIORDANO
Reti neurali per l'analisi del trend: un approccio per identificare la topologia della rete.

2001, 3.113 Marcella NIGLIO
Nonlinear time series models with switching structure: a comparison of their forecast performances.

2001, 3.114 Damiano FIORILLO
Capitale sociale e crescita economica. Review dei concetti e dell'evidenza empirica.

2001, 3.115 Roberto RAUCCI, Luigi TADDEO
Generalizzazione del concetto di continuità e di derivabilità.

2001, 3.116 Marcella NIGLIO
Ricostruzione dei dati mancanti in serie storiche climatiche.

2001, 3.117 Vincenzo VECCHIONE
Mutamenti del sistema creditizio in un'area periferica.

2002, 3.118 Francesco GIORDANO, Michele LA ROCCA, Cira PERNA
Bootstrap variable selection in neural network regression models.

2002, 3.119 Roberto RAUCCI, Luigi TADDEO
Insiemi debolmente convessi e concavità in senso generale.

2002, 3.120 Vincenzo VECCHIONE
Know how locali e percorsi di sviluppo in aree e settori marginali.

2002, 3.121 Michele LA ROCCA, Cira PERNA
Neural networks with dependent data.

2002, 3.122 Pietro SENESI
Economic dynamics: theory and policy. A stability analysis approach.

2002, 3.123 Gianluigi COPPOLA
Stima di un indicatore di pressione ambientale: un'applicazione ai comuni della Campania.

2002, 3.124 Roberto RAUCCI
Sull'esistenza di autovalori e autovettori positivi anche nel caso non lineare.

2002, 3.125 Maria Carmela MICCOLI
Identikit di giovani lucani.

2002, 3.126 Sergio DESTEFANIS, Giuseppe STORTI
Convexity, productivity change and the economic performance of countries.

2002, 3.127 Giovanni C. PORZIO, Maria Prosperina VITALE
Esplorare la non linearità nei modelli Path.

2002, 3.128 Rosa FERRENTINO
Sulla funzione di Seal.

2003, 3.129 Michele LA ROCCA, Cira PERNA
Identificazione del livello intermedio nelle reti neurali di tipo feedforward.

2003, 3.130 Alessandra AMENDOLA, Marcella NIGLIO, Cosimo VITALE
The exact multi-step ahead predictor of SETARMA models.

2003, 3.131 Mariangela BONASIA
La dimensione ottimale di un sistema pensionistico: means tested vs programma universale.

2003, 3.132 Annamaria NESE
Abitazione e famiglie a basso reddito.

2004, 3.147 Gianluigi COPPOLA
In corso di pubblicazione.

2004, 3.148 Massimo Spagnolo
The Importance of Economic Incentives in Fisheries Management

2004, 3.149 F. Salsano
La politica monetaria in presenza di non perfetta osservabilità degli obiettivi del banchiere centrale.

2004, 3.150 A. Vita
La dinamica del cambiamento nella rappresentazione del territorio. Una mappa per i luoghi della Valle dell'Irno.

2004, 3.151 Celi
Empirical Explanation of vertical and horizontal intra-industry trade in the UK: a comment.

2004, 3.152 Amendola – P. Vitale
Self-Assessment and Career Choices: An On-line resource for the University of Salerno.

2004, 3.153 A. Amendola – R. Troisi
Introduzione all'economia politica dell'organizzazione: nozioni ed applicazioni.

2004, 3.154 A. Amendola – R. Troisi
Strumenti d'incentivo e modelli di gestione del personale volontario nelle organizzazioni non profit.

2004, 3.155 Lavinia Parisi
La gestione del personale nelle imprese manifatturiere della provincia di Salerno.

2004, 3.156 Angela Spagnuolo – Silvia Keller
La rete di accesso all'ultimo miglio: una valutazione sulle tecnologie alternative.

2005, 3.157 Davide Cantarelli
Elasticities of Complementarity and Substitution in Some Functional Forms. A Comparative Review.

2005, 3.158 Pietro Coretto – Giuseppe Storti
Subjective Sxpectations in Economics: a Statistical overview of the main findings.

2005, 3.159 Pietro Coretto – Giuseppe Storti
Moments based inference in small samples.

2005, 3.160 Massimo Salzano
Una simulazione neo-keynesiana ad agenti eterogeni.

2005, 3.161 Rosa Ferrentino
Su alcuni paradossi della teoria degli insiemi.

2005, 3.162 Damiano Fiorillo
Capitale sociale: uno o molti? Pochi.

2005, 3.163 Damiano Fiorillo
Il capitale sociale conta per outcomes (macro) economici?.

2005, 3.164 Damiano Fiorillo – Guadalupi Luigi
Attività economiche nel distretto industriale di Nocera inferiore – Gragnano. Un'analisi su Dati Tagliacarne.

2005, 3.165 Rosa Ferrentino
Pointwise well-posedness in vector optimization and variational inequalities.

2005, 3.166 Roberto Iorio
La ricerca universitaria verso il mercato per il trasferimento tecnologico e rischi per l'"Open Science": posizioni teoriche e filoni di indagine empirica.

2005, 3.167 Marisa Faggini
The chaotic system and new perspectives for economics methodology. A note.

2005, 3.168 Francesco Giordano
Weak consistent moving block bootstrap estimator of sampling distribution of CLS estimators in a class of bilinear models

2005, 3.169 Edgardo Sica
Tourism as determinant of economic growth: the case of south-east asian countries.

2005, 3.170 Rosa Ferrentino
On Minty variational inequalities and increasing along rays functions.

2005, 3.171 Rosa Ferrentino
On the Minty and Stampacchia scalar variational inequalities

2005, 3.172 Destefanis - Storti
A procedure for detecting outliers in frontier estimation

2005, 3.173 Destefanis - Storti
Evaluating business incentives trough dea. An analysis on capitalia firm data

2005, 3.174 Nese – O'Higgins
In and out of the capitalia sample: evaluating attrition bias.

2005, 3.175 Maria Patrizia Vittoria
Il Processo di terziarizzazione in Campania. Analisi degli indicatori principali nel periodo 1981-2001

2005, 3.176 Sergio Destefanis – Giuseppe Mastromatteo
Inequality and labour-market performance. A survey beyond an elusive trade-off.

2006, 3.177 Giuseppe Storti
Modelling asymmetric volatility dynamics by multivariate BL-GARCH models

2006, 3.178 Lucio Valerio Spagnolo – Mario Cerrato
No euro please, We're British!

2006, 3.179 Maria Carmela Miccoli
Invecchiamento e seconda transizione demografica

2006, 3.180 Maria Carmela Miccoli – Antonio Cortese
Le scuole italiane all'estero: una realtà poco nota

2007, 3.181 Rosa Ferrentino
Variational inequalities and optimization problems

2007, 3.182 Lavinia Parisi
Estimating capability as a latent variable: A Multiple Indicators and Multiple Causes Approach. The example of health

Stampa a cura della C.U.S.L. Cooperativa Universitaria Studio e Lavoro, Via Ponte Don Melillo, Fisciano per conto Del Dipartimento di Scienze Economiche e Statistiche
Finito di stampare il 10 aprile 2007