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ELASTICITIES OF COMPLEMENTARITY AND SUBSTITUTION IN SOME FUNCTIONAL FORMS. A COMPARATIVE REVIEW*

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Abstract

This paper presents and discusses the results of a comparative review of the elasticities of complementarity and substitution which may be obtained from seven functional forms, inflexible and flexible. Every functional form is examined in its four versions: production, cost, profit, and distance. The elasticities of complementarity were obtained from the production and distance functions (elasticities of Hicks, Antonelli and Morishima), and those of substitution from the cost and profit functions (elasticities of Allen-Uzawa and Lau). The results of the calculations of the various elasticities are compared and evaluated.

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production, cost, profit and distance functions; input substitutability and complementarity; derived, gross and net, demand functions for inputs.

1. Introduction

Many, reading the title of this paper, will object that there is no need for such a survey. The main task of this introduction is therefore to justify this work.

The contemporary substitutability and complementarity of production factors is certainly one of the most important grounds of theoretical and applied research on the phenomena of production and distribution. To the initial analytical instrument, i.e., the production function, thanks to the development of the duality theory, functions of cost, profit and distance have been added, and applied to numerous functional forms with equally numerous "variants" and extensions, mainly put forward in the twenty-year period 1970-1990. It is well-known that the main reason for proposing flexible functional forms (FFF) was the wish (and also, objectively, the need) to use them to increase the possibility of variation of the elasticity of substitution among the factors (i.e., what was defined – in our view, improperly – as a "second-order parameter") with respect to the elasticity permitted by inflexible functional forms (IFF). Theoretical contributions and relative discussions on the elasticity of substitution have also covered the capacity of its various versions to express the nature, directions and intensity of the phenomenon[1]. One very significant example of these discussions was the contested validity of Allen's elasticity and above all its exten-

[1] See Mundlak (1968) and Chambers (1988).

sion from a production function with two factors to one with *n* factors[2]. A very different fate was that of the elasticity of complementarity, which has received far less attention - even less than that justified by its more specific and limited field of application[3]. After the work by J.R. Hicks (1970), in which the author credited Joan Robinson with having created the elasticity of substitution while Hicks maintained that of complementarity, there were remarkably few – in absolute and not only relative terms – theoretical and applicative contributions devoted to the elasticity of complementarity until the end of the last century (and millennium), when interest was renewed[4]. Together with this, three more facts characterise the current situation of the theory of production: first, the growing use of the distance function, which allows us to treat cases in which there is more than one output; second, the current cessation of new proposed functional forms; and third, the development and spread of the "non-parametric" analysis of production.

This set of conditions suggested the idea of surveying a certain number of functional forms in their quadruple aspects of production, cost, profit, and distance, and the elasticities of substitution and complementarity deriving from them. By highlighting particular aspects which may escape a superficial examination, we hope to make the survey more interesting – not just a manual or a "reference paper" for the future. We do not consider here the aspects and problems of the econometric application of the functional forms examined here, since they are beyond both the author's capacity and competence and the proper dimensions of a paper.

This work has six sections. Section 2 presents the functional forms examined here, their layout, and the list of uniform symbols which are used. Section 3 presents and discusses IFF formulas, and sections 4 and 5 FFF formulas. Section 6 makes conclusive considerations.

[2] See Blackorby and Russel (1989).

[3] It has been rightly noted (Kim, 2000) that the elasticity of complementarity is useful when there are constraints to the quantities of inputs, so that their prices are distorted or even non-existent. It measures the substitutability of inputs in terms of their quantities.

[4] From 1970 to 1991, only about a dozen theoretical and applicative contributions to Hicks' elasticity of complementarity appeared in leading journals, i.e., a number not comparable with contributions regarding the elasticity of substitution. See Kim (2000) for references.

2. *Functional Forms*

The functional forms examined in this survey are three IFF, i.e., Cobb-Douglas (CD), CES-ACMS (Arrow, Chenery, Minhas, Solow 1961) and CMC-CES[5], and four FFF, i.e., Translog, Diewert's Generalized Leontiev (DGL 1971), Lau's Quadratic (LQ 1978), and Diewert's Generalized Cobb-Douglas (1973), corrected and extended by Magnus (MEDGCD) (1979). The survey opens with the Cobb-Douglas IFF, with all its constant and unitary elasticities, as an act of homage to and respect for a cornerstone in the construction of the theory of production and as an analytical tool which – rightly or wrongly – is still used[6]. The CES-ACMS has constant elasticities but they are not always equal for the four functions. The CMC-CES is homothetic, like the previous two, but not homogeneous, and it has properties which are partly equal and partly different. Among the FFF, there are, of course, Translog and DGL, because they are the forms mostly frequently applied with respect to the other two, Lau's Quadratic and MEDGCD.

MEDGCD is presented in place of the original function (1973), because the latter has the defect that estimations of its primary and secondary parameters (and thus also their elasticities) are not invariant with respect to the multiplicative scaling of input prices[7]. Every function form is accompanied by its formulas for the functions of: production, cost, profit, distance, elasticity of substitution or of complementarity, demand or quota of input cost, scale economies and size economies[8]. It should be recalled that the

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[5] On the characteristics and properties of the CMC-CES function, see Cantarelli, Di Fonzo and Giacomello (1994).

[6] Its use remains extended to the macro level even if it non is limited to the micro level.

[7] The modified function of Magnus (1979) is immune from this defect. In a letter to the present author, Prof. Diewert stated that it was precisely this defect, pointed out to him by Prof. E. Berndt, which led him not to submit his 1973 working paper containing the GLD function to any journal.

[8] Since the distance function, of the four is definitely the least known, it is appropriate to recall its definition. Starting from the simplest case of a one input-one output production function, output y_o is produced with an increase of input x_o^+, while – given function production $y = f(x)$ - quantity $x_o < x_o^+$ would be sufficient, and with x_o^+ $y_o^+ > y_o$ could be pro-

functions of the elasticity of complementarity have as their arguments the physical quantities of productive inputs and are thus obtained from the production and distance functions, whereas those of the elasticity of substitution have as their arguments costs (normalised, or less so, in various ways) of inputs. The elasticities of complementarity presented here are those of Hicks, Antonelli and Morishima (1967). While Hicks' elasticity of complementarity is obtained from the production function, that yielded by the distance function is called "Antonelli's elasticity" (1886), to recall both that he was the first to define the concept of inverse demand functions (Cornes (1992) and Kim (2000)), and that Antonelli's substitution matrix is precisely the matrix of second partial derivatives of the distance function in the consumer theory (Deaton (1979) and thus of the producer. Morishima's elasticities of complementarity, proposed by Kim (2000), are also derived from the distance function.

The elasticities of substitution are those of Allen-Uzawa (1962) and Morishima (1967), taken from the cost functions, and that of Hotelling-Lau, taken from the profit function[9]. Allen's (1938) elasticity of substitution, yielded by the production function with *n* inputs, is not examined here, because the author shares the opinion of authoritative experts that it does not give a correct measure of the

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duced. The ratio x_o^+ / x_o is the maximum scalar λ for which x_o may be divided and still produce y_o. In symbols, $\lambda = D_I\left(x_o^+, y_o\right) \equiv max\left\{\lambda | f\left(x_o^+ / \lambda\right) \geq y_o\right\}$. The maximum value of λ depends on the choice of x_o^+ and y_o, and the distance function precisely expresses this dependence. In symbols, $D_I\left(y, x\right) \equiv max\left\{\lambda | (1/\lambda)x, y\text{) } possible\right\}$. This is the input distance function, which must not be confused with the output distance function, which also exists and which mirrors it. The input distance function may easily be extended to cases of several inputs and outputs. In this case, if $D_I\left(\boldsymbol{y},\boldsymbol{x}\right)$ may be differentiated twice everywhere, $\partial D_I\left(\boldsymbol{y},\boldsymbol{x}\right) / \partial x_i \equiv w_i\left(\boldsymbol{y},\boldsymbol{x}\right)$ is the marginal product or inverse demand function for input x_i. For further details, see Cornes (1992). It is perhaps also appropriate to recall that the elasticity of scale measures how output varies moving along a scale line from the origin in the input space; the elasticity of size measures how cost varies along the expansion path (locus of the minimum cost) in the input space. If the production function is homothetic, the two elasticities are equivalent.

[9] The name Hotelling-Lau is due to Bertoletti (2001) who refers to Chapter I.3, "Applications of Profit Functions", of Lau's "Production Economics", in which Lau defines this elasticity by means of the profit function (page 197).

relation between inputs when there are more than two of them, and that this relation can be obtained with Morishima's elasticity.

The elasticity formulas shown in the tables are mostly calculated autonomously (like all the elasticities of complementarity and Morishima's elasticity) and partly taken from authors who have proposed functional forms (e.g., the elasticities of Translog production and cost functions) and are all presented with uniform symbols. Autonomous proposals and adaptations "by analogy" have also been advanced, in cases in which one of the cost, profit or distance functions is not explicitly proposed by experts on the subject[10].

Given the generic function $y = g(z_1 ... z_n)$ of production, cost, profit or distance, non-Morishima elasticities e_{ij} are calculated by means of the formula:

(1) $$e_{ij} = \frac{g(z_1 ... z_n) \cdot \partial^2 g / \partial z_i \partial z_j}{(\partial g / \partial z_i)(\partial g / \partial z_j)} = e_{ji};\ i \neq j$$

and those of Morishima by means of:

(2) $$e_{ij} = \frac{\partial^2 g / \partial z_i \partial z_j}{\partial g / \partial z_j} \cdot z_i - \frac{\partial^2 / \partial^2 z_i}{\partial g / \partial z_i} \cdot z_i$$

and

$$e_{ji} = \frac{\partial^2 g / \partial z_i \partial z_j}{\partial g / \partial z_i} \cdot z_j - \frac{\partial^2 / \partial^2 z_j}{\partial g / \partial z_j} \cdot z_j$$

For the FFF, the four functions are maintained in their simplest forms, i.e., uni-product, without fixed inputs, and without the addition of variables in a multiplicative relation with the main arguments of the functions themselves, i.e., inputs or their costs, which may enter (complicating them) into the elasticity formulas through apposite derivatives.

As symbols for the parameters appearing in each of the four functions of every functional form, letters α and β are always

[10] In Chapter II.1 of Production Economics, "A Survey of Functional Forms in the Economic Analysis of Production" by McFadden, Fuss and Mundlak, the functional forms shown on pages 238 and 239, according to the authors, may be production, cost or profit functions, according to whether the variables are quantities of inputs, costs or prices. No mention is made of distance functions.

used, and γ appears as an efficiency parameter. Obviously, these letters take on differing values for each of the functions to which they refer. The characteristics of the parameters are shown next to the production functions and are to be understood as extended and valid also for the other three, unless otherwise indicated. Simplification of the expressions resulting from the application of formulas (1) and (2) was carried out with varying intensity (greater for (2)), bearing in mind conditions sufficient for their easy identification.

As the number of symbols used is quite high, they are listed, for readers' convenience, at the end of this Section and before the tables containing the functional formulas.

Symbols

y	*output in physical units*
x_i	*quantity of input i*
w_i	*nominal unit cost of input i*
p_y	*nominal unit price of output y*
$q_i = w_i / p_y$	*unit "normalized" cost of input i*
$y(\boldsymbol{x})$	*production function*
$C(y,\boldsymbol{w})$	*total cost function*
$G(\boldsymbol{q})$	*normalized profit function*
$D(y,\boldsymbol{x})$	*distance function*
μ	*scale coefficient in production function*
${}_{y}S_i$	*output's share of input i in Translog production function*
${}_{C}S_i$	*cost's share of input i in Translog cost function*
${}_{CM}S_i$	*cost's share of input i in MEDGCD cost function*
$\varepsilon(y,\boldsymbol{x})$	*elasticity of scale from production function*
$\varepsilon^*(y,\boldsymbol{w})$	*elasticity of size from cost function*
σ^A	*Allen's elasticity of substitution from production function*
σ	*constant elasticity of substitution in CES functions*
$\theta = (1-\sigma)/\sigma$	
ρ_{ij}^{HEC}	*Hicks' elasticity of complementarity from production function*
ρ_{ij}^{AEC}	*Antonelli's elasticity of complementarity from distance function*
$\rho_{ij}^{MEC} \neq \rho_{ji}^{MEC}$	*Morishima's elasticities of complementarity from distance function*
σ_{ij}^{AUES}	*Allen-Uzawa's elasticity of substitution from cost function*
$\sigma_{ij}^{MES} \neq \sigma_{ji}^{MES}$	*Morishima's elasticities of substitution from cost function*
σ_{ij}^{HLES}	*Hotelling-Lau's elasticity of substitution from profit function*

3. *Inflexible Functional Forms*

As already noted in section 2, Table 1 (Cobb-Douglas) does not require much comment. All the elasticities, including σ^A are positive, constant, and equal to *1*. The inputs are thus always and only *q*-complement and *p*-substitutes. The total passus coefficient $\varepsilon(y,x)=\mu$ is assumed to be <*1*, in order to write the profit function, which would otherwise be annulled, as the formula clearly shows. The demand functions of inputs – like all the functional forms considered here – are taken from the cost function according to Shephard's lemma, and from the profit function according to Hotelling's lemma. The two functions are different because the one obtained from the cost function is conditional or "compensated" by the constancy of output, while that obtained from the profit function is not. Therefore, σ_{ij}^{AUES} expresses net *p*-substitutability and σ_{ij}^{HLES} gross *p*-substitutability.

Table 2, showing the CES-ACMS functions, presents a slightly more varied situation. Production is written in the way Arrow, Chenery, Minhas and Solow wrote it in their original work of 1961, i.e., with negative exponents inside or outside square brackets[11].

Also in this case, and for the same reason as for the Cobb-Douglas function, μ is assumed to be <*1.* One first aspect to be noted is that, although coefficient μ enters into the calculation of ρ_{ij}^{HEC}, it does not in σ_{ij}^{A}. However, if μ is assumed to be equal to *1*, then ρ_{ij}^{HEC} is the reciprocal of σ^A. In this regard, the hypothesis may be advanced that, whereas in the Cobb-Douglas function (and, as we shall see later, also in the CMC-CES) the value of μ is, as it were, "incorporated" in the function (see Table 1), and obtainable from it, in CES-ACMS it is a constant introduced from the outside.

[11] In the same pages of "Production Economics" (see previous note), the CES function is written with positive exponents and without the γ constant. Simulations reveal that the version with negative exponents supplies values for output y which are systematically lower and in (modest) constant proportion with respect to those obtained with positive exponents. The difference in sign of the exponents of the production function affects those of the profit function.

Table 1 – *Cobb-Douglas Functions*

Production
$y(\boldsymbol{x}) = \gamma \prod_{i=1}^{n} x_i^{\alpha_i}; \quad \gamma > 0; \quad 0 < \alpha_i < 1; \quad x_i > 0$
$\varepsilon(y,\boldsymbol{x}) = \sum_{i=1}^{n} \alpha_i = \mu < 1; \quad \sigma^{A} = 1; \quad \rho^{HEC} = 1$
Cost
$C(y,\boldsymbol{w}) = \left(\frac{y}{\gamma}\right)^{\frac{1}{\mu}} \cdot \prod_{i=1}^{n} \left(\frac{w_i}{\alpha_i}\right)^{\frac{\alpha_i}{\mu}}; \quad \varepsilon^{*}(y,\boldsymbol{w}) = \mu; \quad \sigma_{ij}^{AUES} = \sigma_{ji}^{AUES} = 1$
$\sigma_{ij}^{MES} = 1; \quad \sigma_{ji}^{MES} = 1$
demand for input $x_j = \frac{\partial C(y,\boldsymbol{w})}{\partial w_j} = \left(\frac{Y}{\gamma}\right)^{\frac{1}{\mu}} \cdot \prod_{i=1}^{n} \left(\frac{w_i}{\alpha_i}\right)^{\frac{\alpha_i}{\mu}} \cdot \frac{1}{\mu} \cdot \frac{w_j}{\alpha_j}$
Profit
$G(\boldsymbol{q}) = (1-\mu)\gamma \prod_{\lambda=1}^{n} \left(\frac{q_i}{\alpha_i}\right)^{\frac{-\alpha_i}{(1-\mu)^{-1}}}; \quad q_i = \frac{w_i}{p_y}; \quad \sigma_{ij}^{HLES} = 1$
supply of output $y = y(\boldsymbol{q}) = \gamma \prod_{i=1}^{n} \left(\frac{q_i}{\alpha_i}\right)^{-\alpha_i(1-\mu)^{-1}}$
demand for input $x_i = -\frac{q_j}{\alpha_j} \gamma \prod_{i=1}^{n} \left(\frac{q_i}{\alpha_i}\right)^{-\alpha_i(1-\mu)^{-1}}$
Distance
$D(y,\boldsymbol{x}) = \gamma \prod_{i=1}^{n} x_i^{\alpha_i/\mu} y^{-1/\mu}; \quad \rho_{ij}^{AEC} = \rho_{ji}^{AEC} = 1; \quad \rho_{ij}^{MEC} = 1; \quad \rho_{ji}^{MEC} = 1$
(inverse) demand for input $x_j = w_j = \frac{\partial D(y,\boldsymbol{x})}{\partial x_j} = D(y,x) \cdot \frac{\alpha_j}{x_j} \cdot \frac{1}{\mu}$

Table 2 *– CES-ACMS Functions*

Production

$$y(\boldsymbol{x}) = \gamma\left[\sum_{i=1}^{n}\alpha_i x_i^{-\theta}\right]^{-\mu/\theta};\quad \gamma > 0;\quad 0 < \alpha_i < 1;\quad x_i > 0;\quad \sum_{i=1}^{n}\alpha_i = 1$$

$$\varepsilon(y,\boldsymbol{x}) = \mu < 1;\quad \sigma_{ij}^{A} = \frac{1}{1+\theta} = \sigma;\quad \rho_{ij}^{HEC} = 1 + \frac{\theta}{\mu} = \rho_{ji}^{HEC}$$

Cost

$$C(y,\boldsymbol{w}) = \frac{y^{1/\mu}}{\gamma}\left[\sum_{i=1}^{n}\alpha_i^{\sigma} w_i^{\sigma\theta}\right]^{1/\sigma\theta};\quad \varepsilon^{*}(y,\boldsymbol{w}) = \mu;\quad \sigma_{ij}^{AUES} = \sigma_{ji}^{AUES} = \sigma$$

$$\sigma_{ij}^{MES} = \sigma;\ \sigma_{ji} = \sigma$$

demand for input $x_j = \dfrac{\partial C(y,\boldsymbol{w})}{\partial w_j} = \dfrac{y^{1/\mu}}{\gamma}\left[\sum_{i=1}^{n}\alpha_i^{\sigma} w_i^{\sigma\theta}\right]^{\left(\frac{1}{\sigma\theta}-1\right)} \cdot \alpha_j^{\sigma} w_j^{\sigma\theta-1}$

Profit

$$G(\boldsymbol{q}) = \mu^{\frac{\mu}{\mu-1}}(1-\mu)\left[\sum_{i=1}^{n}\alpha_i\left(\frac{q_i}{\alpha_i}\right)^{1-\sigma}\right]^{\frac{\mu(\theta+1)}{\theta(\mu-1)}};$$

$$q_i = \frac{w_i}{p_y};\quad \sigma_{ij}^{HLES} = \frac{\mu+\theta}{\mu(\theta+1)} = \sigma_{ji}^{HLES}$$

supply of output $y = (1-\mu)^{-1}G(\boldsymbol{q})$

demand for input $x_j = -\mu^{\frac{1}{1-\mu}}\left[\sum_{i=1}^{n}\alpha_i\left(\dfrac{q_i}{\alpha_i}\right)^{\frac{\theta}{\theta+1}}\right]^{\frac{\mu+\theta}{\theta(\mu-1)}} \cdot \left(\dfrac{q_j}{\alpha_j}\right)^{-\frac{1}{\theta+1}}$

Distance

$$D(y,\boldsymbol{x}) = \gamma^{\frac{1}{\mu}}\left[\sum_{i=1}^{n}\alpha_i\left(\frac{x_i}{y^{1/\mu}}\right)^{-\theta}\right]^{-1/\theta};\quad \rho_{ij}^{AEC} = \rho_{ji}^{AEC} = \rho_{ij}^{MEC} = \rho_{ji}^{MEC} = \frac{1}{\sigma}$$

(inverse) demand for input

$$x_j = \frac{\partial D(y,\boldsymbol{x})}{\partial x_j} = \left(\frac{\gamma}{y}\right)^{1/\mu}\cdot\left[\sum_{i=1}^{n}\alpha_i x_i^{-\theta}\right]^{-\frac{1+\theta}{\theta}}\cdot\alpha_j x_j^{-(\theta+1)}$$

When this happens, it does not modify σ_{ij}^{A} but enters into the calculation of ρ_{ij}^{HEC} and does not leave it. Instead, the reciprocal of σ_{ij}^{A} is ρ^{AEC}, respecting the rule that this must happen for a function which is homogeneous and not simply homothetic. The two ρ_{ij}^{MEC} are also reciprocals of σ_{ij}^{A}. The constancy of σ^{A} is also transmitted to σ_{ij}^{AUES} and σ_{ij}^{MES}. Instead, a constant but different value results for σ_{ij}^{HLES}, in which (as in ρ_{ij}^{MEC}), the value of μ enters. This contemporary presence matches the duality which, it is stated, exists between the production and profit functions (Bertoletti, 2001), and the circumstance that ρ^{HEC} and σ_{ij}^{HLES} respectively express a gross *q*-complementarity and a gross *p*-substitutability.

A very different situation arises with the CMC-CES functional form. Its production function is homothetic but not homogeneous, σ^{A} is constant and equal to $1/s$, and its dual cost function is also homothetic. Although the production function is not homogeneous, its Hicks' elasticity of complementarity, ρ^{HEC}, is equal to 1, as in the Cobb-Douglas function – a result which was not expected *a priori*. The elasticities of substitution $\sigma_{ij}^{AUES}, \sigma_{ij}^{MES}$ and σ_{ji}^{MES} obtainable by means of the cost function, are all equal to $1/s$ as expected. The CMC-CES function is a special case, comparatively simpler, of the more general CMC non-homogeneous function, in which each input x_i has its own s_i, and its elasticity σ^{A} is variable within limits defined by the relations $>,<,=$ among the s_i.

Table 3 - *CMC-CES Functions*

Production
$y(\boldsymbol{x}) = \gamma \prod_{i=1}^{n} e^{\frac{\alpha_i}{1-s} x_i^{1-s}}$; $\gamma > 0$; $\alpha_i > 0$; $s \gg 1$
$\varepsilon(y, \boldsymbol{x}) = \sum_{i=1}^{n} \alpha_i x_i^{1-s}$; $\sigma_{ij}^{A} = \frac{1}{s}$; $\rho_{ij}^{HEC} = 1$
Cost
$C(y, \boldsymbol{w}) = \left[\sum_{i=1}^{n} \left(\frac{\alpha_i}{s-1} \right)^{1/s} w_i^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}} \cdot \left(-ln \frac{y}{\gamma} \right)^{\frac{1}{1-s}}$
$\varepsilon^{*}(y, \boldsymbol{w}) = (s-1)\left(-ln \frac{y}{\gamma} \right)$; $\sigma_{ij}^{AUES} = \sigma_{ij}^{MES} = \sigma_{ji}^{MES} = \frac{1}{s}$
demand for input x_j
$x_j = \left[\sum_{i=1}^{n} \left(\frac{\alpha_i}{s-1} \right)^{1/s} \cdot w_i^{-1/s} \right]^{\frac{1}{s-1}} \cdot \left(\frac{\alpha_j}{s-1} \right)^{1/s} \cdot w_j^{-1/s} \cdot \left(-ln \frac{y}{\gamma} \right)^{\frac{1}{1-s}}$

The general CMC is a somewhat complex function, which yields the dual cost function only in very particular cases, or else by means of more or less satisfactory approximations[12]. Now, for such a function with a variable σ^{A} [13], ρ^{HEC} is constant and equal to *1*, as in the simpler Cobb-Douglas function!

Although the presence of a single s in the CMC-CES allows its dual cost function to be rendered in explicit form, this is not enough for the present writer to obtain the profit function. Lau's procedure in the case of two preceding IFF to yield the $G(\boldsymbol{q})$ functions shown in Tables 1 and 2 is – at least, in the writer's opinion – inapplicable[14], because it is based on the constancy of coefficient

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[12] On the approximated forms of the non-homothetic CMC cost function, see Cantarelli (1999) and (2003).

[13] On σ^{A} in the CMC (not CES) function, see Cantarelli and Giacomello (1995).

[14] See Lau (1978), pp. 190-192. Lau starts from the first-order condition necessary in order to have maximum profit, $(\partial y / \partial x_i) = w_i / p_y$ (which, for example, in the Cobb-Douglas

$\varepsilon(\boldsymbol{x}) = \mu$ (see Tables 1 and 2). In the CMC-CES, $\varepsilon(\boldsymbol{x}) = \sum_i^n \alpha_i x_i^{1-s}$ decreases variably with the increase in the quantity of inputs, passing from an initial part, in which it is >*1*, to the following part, in which it becomes <*1*. The profit and function $G(\boldsymbol{q})$ may thus exist only starting from $\mu < 1$. Some simple simulations confirmed that the value of μ at which maximum profit is reached is one and only one, and that it determines the maximum value of G at the same time[15]. In other words, μ is not only a variable but also an unknown, to be defined before or together with function $G(\boldsymbol{q})$, if we know how to determine it. In fact, by means of simulations it is possible to obtain the value of G_{max} with every degree of approximation desired, as explained in note (14), but they do not supply the equations of function $G(\boldsymbol{q})$ and of σ^{HLES} which – at least for the moment – we must abandon.

The variability of μ also comes into play in the distance function, again making it difficult to define. The CMC-CES function, with an equational form only slightly more complex than that of CD and CES-ACMS, confirms the opinion of experts (cf. for example,

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function) becomes $(\alpha_i y / x_i) = q_i$) and, using the mathematical procedure called "Legendre transformation", y is substituted by its value in terms of profit function G and x_i by its corresponding $-\partial G / \partial q_i$. He thus obtains for input x_i a first-order differential equation with separable variables, making part of the compound system of similar equations for all n inputs. He treats this system as an ordinary differential equation and, integrating it, obtains function $G(\boldsymbol{q})$. The writer dispelled his initial doubts on the legitimacy of treating the system as a single equation by means of a series of control simulations. They confirmed the result: given parameters α_i and γ, and q_i with constant μ and <1, functions $G(\boldsymbol{q})$ of the Cobb-Douglas and CES-ACMS forms do give G values realizing maximum profit.

[15] The simulation is effected in the following way. If there are, for instance, 3 inputs, x_1, x_2 and x_3, with given values of x_1, the functions of the expansion paths yield the levels of x_2 and x_3. These may be used to calculate output y, total cost $C(y,\boldsymbol{w}) = \sum_{i=1}^{n} x_i w_i$, total revenue $R = y p_y$, profit $P = R - C(y,\boldsymbol{w})$ and, proceeding by trial and error, P_{max} and, thus, the x_i which define μ. In this way, $G(\boldsymbol{q}) = y - \sum q_i x_i$ turns out to be an increasing function up to a maximum when even P is maximum, and then a decreasing one, and $P_{max} = G_{max} \cdot p_y$.

Cornes, 1992) regarding the difficulty of defining it outside really simple cases. For example, in the case of CMC-CES, relation $D(y,\boldsymbol{x})=f(\boldsymbol{x})^{-1}$ cannot be used (as it is later), because it is applicable only in the case of constant returns to scale which, as already mentioned, are variable in the CMC-CES function[16].

Intuition, the fact that the other elasticities of the functional form are constant, tests carried out with some "invented" functions, and the meaning of "efficient complementarity" [17] which parameter s takes on in the production function, are all circumstances which lead us to conclude that elasticities of complementarity ρ^{HEC} and ρ^{MES} must be equal to s.

However, apart from hypotheses and inferences and the real or presumed impossibility of determining the profit and distance functions of CMC-CES or any other function, it should be recalled that a way of obtaining σ^{HLES} directly from σ^{AUES} and ρ^{HEC} from ρ^{AEC} and vice versa was recently proposed, not resorting to the functions themselves but to given elasticities, such as those regarding the output of conditional or unconditional demands of inputs and those of the total or marginal cost (Bertoletti, 2001, formulas 13 and 17). "Filling in" and checking these formulas by means of the above-mentioned elasticities for the functional forms

[16] In this regard, see Syrquin and Hollender (1982) and Kim (2000).

[17] As regards the efficient complementarity expressed by s, see Cantarelli, Di Fonzo and Giacomello (1994) and Cantarelli (1999). Out of curiosity and as proof, recourse to the equation $D(y,\boldsymbol{x})=f(\boldsymbol{x})^{-1}$ gives $\rho^{AEC}=\rho^{HEC}=1$ and $\rho_{ij}^{MES}=\rho_{ji}^{MES}=s$. The result, $\rho^{AEC}=\rho^{HEC}=1$, is not acceptable in the case of variable returns to scale, because ρ^{HEC} is an index of gross complementarity, which incorporates an output effect next to the substitution effect – contrary to what happens to the other forms, which only express the second effect (cf. note 16). Alternative versions were also tried out, which, variously, mimic the Cobb-Douglas function, exploiting the circumstance of the identity of work undertaken by exponents α_i of CD and $\alpha_i x_i^{1-s}$ of CMC-CES, i.e., $\varepsilon(y,x)=\sum\alpha_i$ and $\varepsilon(y,x)=\sum\alpha_i x_i^{1-s}$ (cf. Table 1). Mile-long expressions were obtained for ρ^{AEC}: however, possibilities of achieving further simplifications - which the computer could not or would not carry out - could not be found. Instead, for ρ^{MEC}, on similarly mile-long expressions, the computer carried out the simplifications and, for all of them, supplied s as a result, the reciprocal of σ^{AUES} and σ^{MES} obtained from the cost function, which is a dual of the distance function.

examined here, and for others, together with a practical application, would certainly be an interesting subject for a special research.

4. Flexible Functional Forms: a) Diewert's GL and Translog

Undeniably, Diewert's GL has the positive characteristic of simple path-breaking. It does determine linear demand functions of inputs, the econometric estimation of which is thereby facilitated, and elasticities of substitution and complementarity whose values may range over wide intervals. This appreciation is particularly and specifically true for the cost rather than for the production function[18]. This, in our opinion, is because the former can be subjected to extensions of various types, which may transform it from homothetic into non-homothetic, thus making it more realistic, as econometric applications show. If we examine some extensions of it (Table 4), we can see how simple they are, easy but nonetheless significant after modification-substitution of the generic function $h(y)$. Instead, as regards the production function, it is more difficult to imagine what content and significance function h may take on (note the difference in writing it used by Diewert!). The quantities of inputs must be excluded, because they already appear in the function. In the explicit words of the author, quoted in Table 4[19], h must be understood as an increasing function and not as a constant, similar to the μ which appears in the CES-ACMS, capable of transforming constant returns to scale into increasing or decreasing ones.

[18] Looking at the space devoted to the cost function over the production function in Diewert's pioneering paper of 1971, perhaps the author's main interest was in the former, and that in the latter residual and formally complementary.
[19] See Diewert (1971), p. 506.

Table 4 *- Diewert's Generalized Leontiev (DGL) Functions*

Production

$$y = h\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}x_i^{1/2}x_j^{1/2}; \quad i \neq j; \quad \beta_{ij} = \beta_{ji} \geq 0;$$

$$h\text{=continuous, monotonic, increasing function } \rightarrow +\infty; \quad h(0) = 0;$$

$$\varepsilon(y, \boldsymbol{x}) = 1$$

$$\rho_{ij}^{HEC} = \frac{1}{2}\beta_{ij}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}x_i^{1/2}x_j^{1/2}\right)\left(\frac{1}{\sum_{j=1}^{n}\beta_{ij}x_j^{1/2}}\cdot\frac{1}{\sum_{i=1}^{n}\beta_{ij}x_i^{1/2}}\right)$$

Cost

$$C(y, \boldsymbol{w}) = h(y)\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}w_i^{1/2}w_j^{1/2}; \quad i \neq j; \quad \beta_{ij} = \beta_{ji} \geq 0;$$

$$h(y) = \text{continuous, monotonic, increasing function of } y,$$
$$\rightarrow +\infty \text{ with } y \rightarrow +\infty; \quad h(0) = 0$$

$$\sigma_{ij}^{AUES} = \frac{1}{2}\beta_{ij}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}w_i^{1/2}w_j^{1/2}\right)\left(\frac{1}{\sum_{j=1}^{n}\beta_{ij}w_j^{1/2}}\cdot\frac{1}{\sum_{i=1}^{n}\beta_{ij}w_i^{1/2}}\right)$$

$$\sigma_{ij}^{MES} = \frac{1}{2}\left(1 + w_i^{1/2}\left(\frac{\beta_{ij}}{\sum_{i=1}^{n}\beta_{ij}w_i^{1/2}} - \frac{\beta_{ii}}{\sum_{j=1}^{n}\beta_{ij}w_j^{1/2}}\right)\right)$$

$$\sigma_{ji}^{MES} = \frac{1}{2}\left(1 + w_j^{1/2}\left(\frac{\beta_{ij}}{\sum_{j=1}^{n}\beta_{ij}w_j^{1/2}} - \frac{\beta_{jj}}{\sum_{j=1}^{n}\beta_{ij}w_i^{1/2}}\right)\right)$$

demand function for input $x_i = \dfrac{\partial C(y, \boldsymbol{w})}{\partial \boldsymbol{w}_i} = h(y)\sum_{j=1}^{n}\beta_{ij}w_i^{-1/2}w_j^{1/2}$

Profit

$$G(\boldsymbol{w}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} w_i^{1/2} w_j^{1/2}; \quad i \neq j; \quad \beta_{ij} = \beta_{ji}$$

$$\sigma_{ij}^{HLES} = \frac{1}{2}\beta_{ij}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} w_i^{1/2} w_j^{1/2}\right)\left(\frac{1}{\sum_{j=1}^{n}\beta_{ij} w_j^{1/2}} \cdot \frac{1}{\sum_{i=1}^{n}\beta_{ij} w_i^{1/2}}\right)$$

demand function for input $x_i = \frac{\partial G(\boldsymbol{w})}{\partial w_i} = -\sum_{j=1}^{n}\beta_{ij} w_i^{-1/2} w_j^{1/2}$

Distance

$D(y,\boldsymbol{x}) = y^{-1} h\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} x_i^{1/2} x_j^{1/2}\right); \quad i \neq j; \quad \beta_{ij} = \beta_{ji}; \quad h =$ *see production function*

$$\rho_{ij}^{AEC} = \frac{1}{2}\beta_{ij}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} x_i^{1/2} x_j^{1/2}\right)\left(\frac{1}{\sum_{j=1}^{n}\beta_{ij} x_j^{1/2}} \cdot \frac{1}{\sum_{i=1}^{n}\beta_{ij} x_i^{1/2}}\right)$$

$$\rho_{ij}^{MEC} = \frac{1}{2}\left(\frac{\beta_{ij} x_i^{1/2}}{\sum_{i=1}^{n}\beta_{ij} x_i^{1/2}} + \frac{\sum_{k=1;k\neq i}^{n}\beta_{ik} x_k^{1/2}}{\sum_{j=1}^{n}\beta_{ij} x_j^{1/2}}\right)$$

$$\rho_{ji}^{MEC} = \frac{1}{2}\left(\frac{\beta_{ij} x_j^{1/2}}{\sum_{j=1}^{n}\beta_{ij} x_j^{1/2}} + \frac{\sum_{k=1;k\neq j}^{n}\beta_{jk} x_k^{1/2}}{\sum_{i=1}^{n}\beta_{ij} x_i^{1/2}}\right)$$

(inverse) demand function for input

$$x_i = \frac{\partial D(y,\boldsymbol{x})}{\partial x_i} = y^{-1} h \sum_{j=1}^{n}\beta_{ij} x_i^{-1/2} x_j^{1/2}$$

Extensions of the DGL cost function

1. Homothetic function with constant returns to scale: $h(y)=y$ *(Parks 1971)*

$$C(y,\boldsymbol{w}) = y\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} w_i^{1/2} w_j^{1/2}; \quad i \neq j; \quad \beta_{ij}=\beta_{ji}; \quad \varepsilon^*(y,\boldsymbol{w})=1$$

$\sigma_{ij}^{AUES}, \sigma_{ij}^{MES}, \sigma_{ji}^{MES}$ are the same as the original DGL

demand function for input $x_i = \dfrac{\partial C(y,\boldsymbol{w})}{\partial w_i} = y\sum_{j=1}^{n}\beta_{ij} w_i^{-1/2} w_j^{1/2}$

2. Non-homothetic function with variable returns to scale (Parks 1971)

$$C(y,\boldsymbol{w}) = y\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} w_i^{1/2} w_j^{1/2} + y^2\sum_{i=1}^{n}\alpha_i w_i; \quad i \neq j; \quad \beta_{ij}=\beta_{ji}$$

$$\varepsilon^*(y,\boldsymbol{w}) = 1 - \frac{y\sum_{i=1}^{n}\alpha_i w_i}{\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} w_i^{1/2} w_j^{1/2} + 2y\sum_{i=1}^{n}\alpha_i w_i}$$

$$\sigma_{ij}^{AUES} = \frac{\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} + y\sum_{i=1}^{n}\alpha_i w_i\right)\left(\frac{1}{2}\beta_{ij} w_i^{-1/2} w_j^{-1/2}\right)}{\left(\sum_{j=1}^{n}\beta_{ij} w_i^{-1/2} w_j^{1/2} + \alpha_i y\right)\left(\sum_{i=1}^{n}\beta_{ij} w_i^{1/2} w_j^{-1/2} + \alpha_j y\right)}$$

$$\sigma_{ij}^{MES} = \frac{1}{2}\left(\frac{\beta_{ij} w_i^{1/2}}{\sum_{i=1}^{n}\beta_{ij} w_i^{1/2} + \alpha_j y w_j^{1/2}} + \frac{\sum_{k=L;k\neq i}^{n}\beta_{ik} w_k^{1/2}}{\sum_{j=1}^{n}\beta_{ij} w_j^{1/2} + \alpha_i y w_i^{1/2}}\right)$$

$$\sigma_{ji}^{MES} = \frac{1}{2}\left(\frac{\beta_{ij} w_j^{1/2}}{\sum_{j=1}^{n}\beta_{ij} w_j^{1/2} + \alpha_i y w_i^{1/2}} + \frac{\sum_{k=1;k\neq j}^{n}\beta_{jk} w_k^{1/2}}{\sum_{i=1}^{n}\beta_{ij} w_i^{1/2} + \alpha_j y w_j^{1/2}}\right)$$

demand function for input $x_i = \dfrac{\partial C(y,\boldsymbol{w})}{\partial w_i} = y\sum_{j=1}^{n}\beta_{ij} w_i^{-1/2} w_j^{1/2} + \alpha_i y^2$

3. Non-homothetic function with variable returns to scale (Guilkey, Lovell and Sickles, 1983)

$$C(y,\boldsymbol{w}) = y\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} w_i^{1/2} w_j^{1/2} + y^2\sum_{i=1}^{n}\alpha_i w_i + \sum_{i=1}^{n}\beta_i w_i; \quad i \neq j; \quad \beta_{ij}=\beta_{ji}$$

$$\sigma_{ij}^{AUES} = \frac{C\left(y,\boldsymbol{w}\right)\cdot\frac{1}{2}\beta_{ij} y w_i^{-1/2} w_j^{-1/2}}{\left(y\sum_{j=1}^{n}\beta_{ij} w_i^{-1/2} w_j^{1/2} + y^2\alpha_i + \beta_i\right)\left(y\sum_{i=1}^{n}\beta_{ij} w_i^{1/2} w_j^{-1/2} + y^2\alpha_j + \beta_j\right)}$$

$$\sigma_{ij}^{MES} = \frac{1}{2} y \left(\frac{\beta_{ij} w_i^{1/2}}{y\sum_{i=1}^{n}\beta_{ij} w_i^{1/2} + y^2\alpha_j w_j^{1/2} + \beta_j w_j^{1/2}} + \frac{\sum_{k=1;k\neq i}^{n}\beta_{ik} w_k^{1/2}}{y\sum_{j=1}^{n}\beta_{ij} w_j^{1/2} + y^2\alpha_i w_i^{1/2} + \beta_i w_i^{1}} \right.$$

$$\sigma_{ji}^{MES} = \frac{1}{2} y \left(\frac{\beta_{ij} w_j^{1/2}}{y\sum_{j=1}^{n}\beta_{ij} w_j^{1/2} + y^2\alpha_i w_i^{1/2} + \beta_i w_i^{1/2}} + \frac{\sum_{k=1;k\neq j}^{n}\beta_{jk} w_k^{1/2}}{y\sum_{i=1}^{n}\beta_{ij} w_i^{1/2} + y^2\alpha_j w_j^{1/2} + \beta_j w_j^{1}} \right.$$

demand function for inputs

$$x_i = \frac{\partial C\left(y,\boldsymbol{w}\right)}{\partial w_i} = y\sum_{j=1}^{n}\beta_{ij} w_i^{-1/2} w_j^{1/2} + y^2\alpha_i + \beta_i$$

In any case, observing the formulas of ρ^{HEC} and σ^{AUES} of Diewert's original functions, we see that they are in an identical equational form, with the obvious difference that the quantities of inputs appear in the formula of ρ^{HEC} and their unitary costs in that of σ^{AUES}. In the application of formula (1) of Section 2, h and $h(y)$ cancel out, as was to be expected with homothetic functions. Comparing σ^{AUES} and σ^{HLES}, we see that their formulas are not formally similar, as above, but substantially equal[20]. Diewert does not resort, like Lau, to the normalization procedure $q_i = w_i / p_y$ for the profit function, so that the equality between the two formulas is immediately apparent. The result is that this profit function yields a σ^{HLES} formula which prevents us from distinguishing the gross substitutability it expresses from the net substitutability, expressed by σ^{AUES}. If we then observe the formulas of ρ^{AEC} and ρ^{HEC}, we see that they too are equal, as must happen with production func-

[20] The formula of Diewert's profit function in the multi-output form (see Diewert [1973a], p. 300) is reduced here to one-output.

tions with constant returns to scale like the GL. The latter characteristic, together with homotheticity, removes the differences between gross and net complementarity and substitutability. Clearly, no comparison is possible between extended non-homothetic cost functions, and Diewert's original production function[21].

As regards the Translog production and cost functions, it is very difficult, if not impossible, to find something that has not already been written[22]. However, these functions are shown in Table 5, in order to make proper comparisons with the profit and distance functions.

The profit function was taken from Lau[23]. Its analytical form is formally similar to that of the production function, with q_i in the place of x_i, and thus the formulas of ρ^{HEC} and σ^{HLES} are equally similar. Apart from this analogy, the two elasticities have the common feature of being gross.

The distance function was obtained by means of the reduction to a single output of the multi-output function proposed and used by R. Färe and S. Grosskopf, unanimously recognised as experts on the subject[24]. The reduction to a single output and the elimination of fixed inputs, as well as obviously reducing the number of terms in the function, means that it takes on a significant formal similarity with the cost function. Apart from the presence of inputs x_i in the place of their unitary costs w_i, the difference between it and the cost function is due to the absence of the term $\alpha_{yy}\, ln(y)^2$. However, its presence or absence does not influence calculation of

[21] There are, of course, more sophisticated extensions and variants than those presented here (see, for instance, Nakamura, 1990). However, those in the text are the most frequently used.

[22] Although it is quite well-known, the only circumstance which is perhaps worth noting here is that there are substantial differences in the formal equality of the formulas of ρ^{HEC} and σ^{AUES}, which influence their values. In effect, there is nothing which imposes and/or guarantees that coefficients β_{ij}, obtained from the estimates of the cost function, are not significantly different from those of the production function. This is why the independent variables are different, and why the term $\beta_{iy}\, ln\, y$ appears in the equations of the s_i of the cost function but not in those of the production function.

[23] See Lau 1978, pag. 194.

[24] See Grosskopf, Hayes and Hirschberg (1995). It should be recalled that these authors, on page 257, write the function as $ln(1) = \alpha_0 + \alpha\, ln\, y ...$.

the partial derivatives (of an additive function in the logarithms) with respect to w_i or x_i, which are necessary to calculate the elasticities of substitution and complementarity. If the uni-product version of the distance function is accepted, its partial derivatives are a mix of those of the production function (due to the presence of inputs x_i) and of those of the cost function (due to the presence of output y). The different values of the elasticities are thus entrusted to the differing combinations of the variables according to which they are defined[25].

***Table 5** – Translog Functions*

Production

$$ln\, y(\boldsymbol{x}) = ln\,\alpha_0 + \sum_{i=1}^{n}\alpha_i\, ln\, x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\, ln\, x_i\, ln\, x_j\,; \quad i \neq j; \quad \beta_{ij} = \beta_{ji}$$

$$\varepsilon(y,\boldsymbol{x}) = \sum_{i=1}^{n}\alpha_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\, ln\, x_j$$

$$\rho_{ij}^{HEC} = 1 + \frac{\beta_{ij}}{\left(\alpha_i + \sum_{j=1}^{n}\beta_{ij}\, ln\, x_j\right)\left(\alpha_j + \sum_{i=1}^{n}\beta_{ij}\, ln\, x_i\right)} = 1 + \frac{\beta_{ij}}{{}_yS_i\;{}_yS_j}$$

$${}_yS_i = \frac{\partial y}{\partial x_i}\cdot\frac{x_i}{y} = \frac{\partial\, ln\, y}{\partial\, ln\, x_i}; \quad if \quad \frac{\partial y}{\partial x_i} = \frac{w_i}{p_y} \Rightarrow {}_yS_i = \frac{w_i x_i}{p_y y}$$

Cost

$$ln(y,\boldsymbol{w}) = ln\,\alpha_0 + \alpha_y\, ln\, y + \frac{1}{2}\alpha_{yy}\,(ln\, y)^2 + \sum_{i=1}^{n}\alpha_i\, ln\, w_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\, ln\, w_i\, ln\, w_j + \\ + \sum_{i=1}^{n}\beta_{iy}\, ln\, y\, ln\, w_i$$

$$\varepsilon^*(y,\boldsymbol{w}) = \frac{1}{\alpha_y + \alpha_{yy} + \sum_{i=1}^{n}\beta_{iy}\, ln\, w_i}$$

[25] In order to obtain ρ^{AEC}, Kim (2000, p. 252) does not use a distance function but a Translog production function, stating that the distance function is “less well known”, and proposes a procedure which, starting from ρ^{HEC} in the case of a production technology of a given degree, achieves definition of the ρ^{AEC}. We must confess that this procedure seems neither clear nor pertinent.

$$\sigma_{ij}^{AUES} = 1 + \frac{\beta_{ij}}{\left(\alpha_i + \beta_{iy}\, ln\, y + \sum_{j=1}^{n} \beta_{ij}\, ln\, w_j\right)\left(\alpha_j + \beta_{jy}\, ln\, y + \sum_{i=1}^{n} \beta_{ij}\, ln\, w_i\right)} =$$

$$= 1 + \frac{\beta_{ij}}{{}_cS_i\; {}_cS_j}$$

$${}_cS_i = \frac{\partial\, ln\, C}{\partial\, ln\, w_i} = \frac{\partial C}{\partial w_i} \cdot \frac{w_i}{C}; \quad x_i = \frac{\partial C}{\partial w_i} \Rightarrow {}_cS_i = \frac{w_i x_i}{C}$$

$$\sigma_{ij}^{MES} = 1 + \frac{\beta_{ij}}{\alpha_j + \beta_{iy}\, ln\, y + \sum_{i=1}^{n} \beta_{ij}\, ln\, w_i} - \frac{\beta_{ii}}{\alpha_i + \beta_{iy}\, ln\, y + \sum_{j=1}^{n} \beta_{ij}\, ln\, w_j} =$$

$$= 1 + \frac{\beta_{ij}}{{}_cS_j} - \frac{\beta_{ii}}{{}_cS_i}$$

$$\sigma_{ji}^{MES} = 1 + \frac{\beta_{ij}}{\alpha_i + \beta_{iy}\, ln\, y + \sum_{j=1}^{n} \beta_{ij}\, ln\, w_j} - \frac{\beta_{jj}}{\alpha_j + \beta_{jy}\, ln\, y + \sum_{i=1}^{n} \beta_{ij}\, ln\, w_i} =$$

$$= 1 + \frac{\beta_{ij}}{{}_cS_i} - \frac{\beta_{ii}}{{}_cS_j}$$

demand function input $x_i =$ cost share of input x_i

$$\frac{w_i x_i}{C} = \alpha_i + \beta_{iy}\, ln\, y + \sum_{i=1}^{n} \beta_{ij}\, ln\, w_i$$

Profit

$$ln\, G(\boldsymbol{q}) = ln\, \alpha_0 + \sum_{i=1}^{n} \alpha_i\, ln\, q_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij}\, ln\, q_i\, ln\, q_j; \quad i \neq j; \quad \beta_{ij} = \beta_{ji}; \quad q_i = \frac{w_i}{p_y}$$

$$\sigma_{ij}^{HLES} = 1 + \frac{\beta_{ij}}{\left(\alpha_i + \sum_{j=1}^{n} \beta_{ij}\, ln\, q_i\right)\left(\alpha_j + \sum_{i=1}^{n} \beta_{ij}\, ln\, q_j\right)}$$

demand function input x_i

$$\frac{q_i x_i}{G(\boldsymbol{q})} = -\left(\alpha_i + \sum_{j=1}^{n} \beta_{ij}\, ln\, q_j\right)$$

Distance

$$ln\,D(y,\boldsymbol{x}) = ln\,\alpha_0 + \alpha_y\,ln\,y + \sum_{i=1}^{n}\alpha_i\,ln\,x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}ln\,x_i\,ln\,x_j + \sum_{i=1}^{n}\beta_{iy}\,ln\,x_i\,ln\,y$$

$$\rho_{ij}^{AEC} = \frac{1}{2}\beta_{ij}\frac{2\left(ln\,\alpha_0 + \alpha_y\,ln\,y + \sum_{1}^{n}\alpha\,ln\,x\right) + \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\,ln\,x_i\,ln\,x_j + 2y\sum_{i=1}^{n}\beta_{iy}\,ln\,x_i}{\left(\alpha_i + \beta_{iy}y + \sum_{j=1}^{n}\beta_{ij}\,ln\,x_j\right)\left(\alpha_j + \beta_j y + \sum_{i=1}^{n}\beta_{ij}\,ln\,x_i\right)}$$

$$\rho_{ij}^{MEC} = 1 + \frac{\beta_{ij}}{\alpha_j + \beta_{jy}\,ln\,y + \sum_{i=1}^{n}\beta_{ij}\,ln\,x_i} - \frac{\beta ii}{\alpha_i + \beta_{iy}\,ln\,y + \sum_{j=1}^{n}\beta_{ij}\,ln\,x_j}$$

$$\rho_{ji}^{MEC} = 1 + \frac{\beta_{ij}}{\alpha_i + \beta_{iy}\,ln\,y + \sum_{j=1}^{n}\beta_{ij}\,ln\,x_j} - \frac{\beta_{jj}}{\alpha_j + \beta_{jy}\,ln\,y + \sum_{i=1}^{n}\beta_{ij}\,ln\,x_i}$$

(inverse) demand function for inputs x_i

$$x_i = \frac{\partial D(y,\boldsymbol{x})}{\partial x_i} = \frac{1}{x_i}\left(\alpha_i + \beta_{iy}\,ln\,y + \sum_{j=1}^{n}\beta_{ij}x_j\right)$$

5. *Flexible Functional Forms: b) Lau's Quadratic and Diewert's Generalized Cobb-Douglas, Extended by Magnus*

If we compare Lau's Quadratic production function with Translog, we see that the latter repeats its form, with natural rather than logarithmic values. Lau, who is one of the most important contributors to the theory of the profit function, did not formulate any function, of either cost or distance. Both have been proposed by us, exploiting the analogy with Translog, i.e. using its form in natural values. However, from production and cost functions formulas of ρ^{HEC} and σ^{AUES} result, which do not repeat the form of the Translog ones, because the second mixed derivative, necessary for their calculation, is reduced in both cases only to term β_{ij}**,** a thing which does not allow the processing which, in the Translog function, eliminates respectively y and $C(y,\boldsymbol{w})$ from the formulas. Lau states that the production function is "self dual" and thus its convex conjugate, i.e., the profit function, is also quadratic and is written with $G(\boldsymbol{q})$ in place of y, and q_i in place of x_i[26]. The consequence – *mutatis mutandis* – is that the formula of σ^{HLES} is formally similar to that of ρ^{HEC}.

Table 6 *– Lau's Quadratic Functions*

Production

$$y(\boldsymbol{x}) = \alpha_0 + \sum_{i=1}^{n} \alpha_i x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} x_i x_j; \quad i \neq j; \quad \beta_{ij} = \beta_{ji}; \quad \alpha_0 \geq 0$$

$$\varepsilon(y,\boldsymbol{x}) = \frac{1}{y}\left(\sum_{i=1}^{n} \alpha_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} x_i x_j\right)$$

$$\rho_{ij}^{HEC} = \frac{y\beta_{ij}}{\left(\alpha_i + \sum_{j=1}^{n} \beta_{ij} x_j\right)\left(\alpha_j + \sum_{i=1}^{n} \beta_{ij} x_i\right)}$$

Cost

$$C(y,\boldsymbol{w}) = \alpha_0 + \alpha_y y + \frac{1}{2}\alpha_{yy} y^2 + \sum_{i=1}^{n} \alpha_i w_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} w_i w_j + \sum_{i=1}^{n} \beta_i y w_i; \quad \alpha_0 \geq 0$$

$$\varepsilon^*(y,\boldsymbol{w}) = \frac{C(y,\boldsymbol{w})}{y}\left(\alpha_y + \alpha_{yy} y + \sum_{i=1}^{n} \beta_i w_i\right)^{-1}$$

$$\sigma_{ij}^{AUES} = \frac{C(y,\boldsymbol{w}) \cdot \beta_{ij}}{\left(\alpha_i + \beta_i y + \sum_{j=1}^{n} \beta_{ij} w_j\right) \cdot \left(\alpha_j + \beta_j y + \sum_{i=1}^{n} \beta_{ij} w_i\right)}$$

$$\sigma_{ij}^{MES} = w_i\left(\frac{\beta_{ij}}{\alpha_j + \beta_j y + \sum_{i=1}^{n} \beta_{ij} w_i} - \frac{\beta_{ii}}{\alpha_i + \beta_i y + \sum_{j=1}^{n} \beta_{ij} w_j}\right)$$

[26] See Lau (1978), p. 194. However, his statement regarding self-duality is not accompanied by any explanation or reference to another publication.

$$\sigma_{ji}^{MES} = w_j \left(\frac{\beta_{ij}}{\alpha_i + \beta_i y + \sum_{j=1}^{n} \beta_{ij} w_j} - \frac{\beta_{jj}}{\alpha_j + \beta_j y + \sum_{i=1}^{n} \beta_{ij} w_i} \right)$$

demand function for input $x_i = \frac{\partial C(y, \boldsymbol{w})}{\partial w_i} = \alpha_i + \beta_i y + \sum_{j=1}^{n} \beta_{ij} w_j$

Profit

$$G(\boldsymbol{q}) = \alpha_0 + \sum_{i=1}^{n} \alpha_i q_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} q_i q_j ; q_i = \frac{w_i}{p_y}; \quad \alpha_0 \geq 0$$

$$\sigma_{ij}{}^{HLES} = \frac{G(\boldsymbol{q}) \beta_{ij}}{\left(\alpha_i + \sum_{j=1}^{n} \beta_{ij} q_j \right) \left(\alpha_j + \sum_{i=1}^{n} \beta_{ij} q_i \right)}$$

demand function for input $x_i = \frac{\partial G(\boldsymbol{q})}{\partial q_i} = -\left(\alpha_i + \sum_{j=1}^{n} \beta_{ij} q_i \right)$

Distance

$$D(y, \boldsymbol{x}) = \alpha_y y + \sum_{i=1}^{n} \alpha_i x_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} x_i x_j + \sum_{i=1}^{n} \beta_{iy} x_i y$$

$$\rho_{ij}^{AEC} = \frac{D(y, \boldsymbol{x}) \beta_{ij}}{\left(\alpha_i + \beta_{iy} y + \sum_{j=1}^{n} \beta_{ij} x_j \right) \left(\alpha_j + \beta_{jy} y + \sum_{i=1}^{n} \beta_{ij} x_i \right)}$$

$$\rho_{ij}^{MEC} = x_i \left(\frac{\beta_{ij}}{\alpha_j + \beta_{jy} y + \sum_{i=1}^{n} \beta_{ij} x_i} - \frac{\beta_{ii}}{\alpha_i + \beta_{iy} y + \sum_{j=1}^{n} \beta_{ij} x_j} \right)$$

$$\rho_{ji}^{MEC} = x_j \left(\frac{\beta_{ij}}{\alpha_i + \beta_{iy} y + \sum_{j=1}^{n} \beta_{ij} x_j} - \frac{\beta_{jj}}{\alpha_j + \beta_{jy} y + \sum_{i=1}^{n} \beta_{ij} x_i} \right)$$

inverse demand for input $x_i = \frac{\partial D(y, \boldsymbol{x})}{\partial x} = \alpha_i + \beta_{iy} y + \sum_{j=1}^{n} \beta_{ij} x_j$

Among the elasticities taken from the distance and cost functions, the relationships of formal analogy described above for the same functions in Translog are repeated.

The functions of the generalized Cobb-Douglas form, shown in logarithmic terms for convenience, do not present any especially distinctive features. As the production function is subject to constant returns to scale, the equality between ρ^{HEC} and ρ^{AEC} turns out to be confirmed, as expected (Kim, 2000).

The cost function shown in non-homothetic form is due to Guilkey et al. (1983). The presence of terms $ln\, y$ and $(ln\, y)^2$ - as in Translog and GL – means that σ^{AUES} loses its formal analogy with ρ^{HEC} and reduces the possibility of simplifying the formula. In the σ^{MES} formulas, the partial own prime and second derivatives are shown with the symbol d (for reasons of space) and appear in extended form in the Appendix. The presence in the formulas of negative signs in front of the two terms should not mislead readers, causing perplexity. That of the first term comes from the second mixed derivative of the function, whereas the second term is annulled by the negativity of its own second derivatives.

The demand function of input i is presented in Translog form as the quota of its total cost, multiplying the prime derivative of the cost function by its unitary cost w_i.

The normalised profit function, proposed by the writer, does not present any particular difficulty.

Table 7 - *Magnus Extended, Diewert's Generalized Cobb-Douglas Functions*

Production

$$ln\, y = ln\,\gamma + \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\, ln\left(\alpha_i x_i + \alpha_j x_j\right); \quad \alpha_i \geq 0; \quad \gamma > 0; \quad \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} = 1; \quad \beta_{ij} = \beta_{ji}$$

$$\varepsilon\left(y, \boldsymbol{x}\right) = \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} = 1$$

$$\rho_{ij}^{HEC}=1-\frac{1}{2}\cdot\frac{\beta_{ij}}{\left(\alpha_i x_i+\alpha_j x_j\right)^2\left(\sum_{j=1}^{n}\frac{\beta_{ij}}{\alpha_i x_i+\alpha_j x_j}\right)\left(\sum_{i=1}^{n}\frac{\beta_{ij}}{\alpha_i x_i+\alpha_j x_j}\right)}$$

Cost

$$ln\,C\left(y,\boldsymbol{w}\right)=ln\,\alpha_0+\alpha_y\,ln\,y+\frac{1}{2}\alpha_{yy}\left(ln\,y\right)^2+$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\,ln\left(\alpha_i w_i+\alpha_j w_j\right)+\sum_{i=1}^{n}\alpha_{iy}\,ln\,y\,ln\,w_i$$

$$\varepsilon^*\left(y,\boldsymbol{w}\right)=\left(\alpha_y+\alpha_{yy}\,ln\,y+\sum_{i=1}^{n}\alpha_{iy}\,ln\,w_i\right)^{-1}$$

$$\sigma_{ij}^{AUES}=1-\frac{2\alpha_i\alpha_j\beta_{ij}}{\left(\alpha_i w_i+\alpha_j w_j\right)^2\left(\frac{\alpha_{iy}\,ln\,y}{w_i}+2\alpha_i\sum_{j=1}^{n}\frac{\beta_{ij}}{\alpha_i w_i+\alpha_j w_j}\right)}\cdot$$

$$\cdot\frac{1}{\left(\frac{\alpha_{iy}\,ln\,y}{w_j}+2\alpha_j\sum_{i=1}^{n}\frac{\beta_{ij}}{\alpha_i w_i+\alpha_j w_j}\right)}$$

$$\sigma_{ij}^{MES}=w_i\left(-\frac{2\alpha_i\alpha_j\beta_{ij}}{\left(\alpha_i w_i+\alpha_j w_j\right)^2}-\frac{d_{ii}}{d_i}\right)$$

$$\sigma_{ji}^{MES}=w_j\left(-\frac{2\alpha_i\alpha_i\beta_{ij}}{\left(\alpha_i w_i+\alpha_j w_j\right)^2}-\frac{d_{jj}}{d_j}\right)$$

demand function input $x_i=$ cost share of input x_i

$$\frac{w_i x_i}{C\left(y,\boldsymbol{w}\right)}=2\alpha_i\alpha_j\beta_{ij}w_i\left(\alpha_i w_i+\alpha_j w_j\right)^{-1}+\alpha_{yi}\,ln\,y$$

Profit

$$ln\,G\left(\boldsymbol{q}\right)=ln\,\alpha_0+\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\cdot ln\left(\alpha_i q_i+\alpha_j q_j\right)$$

$$\sigma_{ij}{}^{HLES}=1-\frac{1}{2}\frac{\beta_{ij}}{\left(\alpha_i q_i+\alpha_j q_i\right)^2\left(\sum_{j=1}^{n}\frac{\beta_{ij}}{\left(\alpha_i q_i+\alpha_j q_j\right)}\right)\left(\sum_{i=1}^{n}\frac{\beta_{ij}}{\left(\alpha_i q_i+\alpha_j q_j\right)}\right)}$$

demand function for input $x_i = -\frac{\partial G(\boldsymbol{q})}{\partial q_i} = -2\alpha_i \sum_{j=1}^{n} \frac{\beta_{ij}}{\left(\alpha_i q_i + \alpha_j q_j\right)}$

Distance

$$ln\, D\left(y, \boldsymbol{x}\right) = ln\, \gamma + \alpha_y\, ln\, y^{-1} + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij}\, ln\left(\alpha_i x_i + \alpha_j x_j\right)$$

$$\rho_{ij}^{AEC} = 1 - \frac{1}{2} \frac{\beta_{ij}}{\left(\alpha_i x_i + \alpha_j x_j\right)^2 \left(\sum_{j=1}^{n} \frac{\beta_{ij}}{\left(\alpha_i x_i + \alpha_j x_j\right)}\right)\left(\sum_{i=1}^{n} \frac{\beta_{ij}}{\left(\alpha_i x_i + \alpha_j x_j\right)}\right)}$$

$$\rho_{ij}^{MEC} = -\frac{\beta_{ij}\alpha_i x_i}{\left(\alpha_i x_i + \alpha_j x_j\right)^2 \sum_{i=1}^{n} \frac{\beta_{ij}}{\left(\alpha_i x_i + \alpha_j x_j\right)}} + \frac{\beta_{ii}}{2\alpha_i x_i \sum_{j=1}^{n} \frac{\beta_{ij}}{\left(\alpha_i x_i + \alpha_j x_j\right)}}$$

$$\rho_{ji}^{MEC} = -\frac{\beta_{ij}\alpha_j x_j}{\left(\alpha_i x_i + \alpha_j x_j\right)^2 \sum_{j=1}^{n} \frac{\beta_{ij}}{\left(\alpha_i x_i + \alpha_j x_j\right)}} + \frac{\beta_{jj}}{2\alpha_j x_j \sum_{i=1}^{n} \frac{\beta_{ij}}{\left(\alpha_i x_i + \alpha_j x_j\right)}}$$

inverse demand function for input

$$x_i = \frac{\partial D\left(y, \boldsymbol{x}\right)}{\partial x_i} = 2\alpha_i \sum_{j=1}^{n} \frac{\beta_{ij}}{\left(\alpha_i x_i + \alpha_j x_j\right)}$$

6. Final Considerations

At the conclusion of a research which produced in the author a mixture of disappointment and interest – work which he would not have undertaken if he had seen the general picture clearly right from the beginning – some comments may be made. First, although admitting the modesty of the aim of the research and of its mathematical and theoretical standard, we hope that the “parade” of the seven inflexible and flexible functional forms, may be useful for formulating evaluations which are not simple but more articulated.

A first consideration regards IFF and FFF as separate groups. Due to their functions and correlated elasticities, IFF present solutions which may be defined as clear and distinct within their limited scope, as shown by unitary or constant but in any case positive values. For these solutions, inputs are always and only q-complements and p-substitutes. They are clear solutions because, due to their self-dual nature, the rules of economic and mathematical logic proceed along the same lines; and they are distinct because the four functions have specific and proper equation forms. The non-determination of the profit function for the CMC-CES form and the hypothetical conclusions on the elasticity of the distance function are certainly due only to the writer's limited capacity to face the analytical difficulties which others will doubtless find easy to carry out.

The overall picture of the FFF is very different from that of the IFF, but not less unitary from other aspects. The deliberate abandon of self-duality between the functions and the acceptance of an approximation as second best, involves more or less similar consequences for all functions. Once the equation form of the production function has been chosen and defined while respecting established canons, the others must be "invented". Emblematic here is the case of the cost function of the functional forms DGL, TL and MEDGCD, of which the first and third have production functions with constant returns to scale. Now, the terms y and y^2 were inserted in the three cost functions in order to have variable economies of size and to presume *a priori* – and more realistically – variable (increasing) average and marginal costs. If this was done by clearly authoritative experts, then it seems to us that the "invention by analogy" on our part of the cost function for Lau's Quadratic, in which we introduced y and y^2, should not be blamed.

Of the profit functions, three out of four have the mark of the inventors of production functions, i.e., Diewert and Lau. In this case too, can the extension we made for the MEDGCD profit function be condemned *a priori*? Among the distance functions, the situation is less complex. "Inventions" are limited to Translog and Lau's Quadratic, because DGL and MEDGCD have $\varepsilon(y,\boldsymbol{x})=1$ and therefore the relationship $D(y,\boldsymbol{x})=Y(\boldsymbol{x})y^{-1}$ may be used. The two inventions have very different parentage: Färe and Grosskopf for Translog, and the present writer for Lau's Quadratic.

Secondly, an overall view of the functional forms helps us to assess better and understand the reasons which led econometricians to prefer certain functional forms and, within them, to choose the most suitable among the four functions. Among the FFF, the preference for Translog is explained by its generality and the fact that it can be extended[27]. Its production function is presumed to be neither homogeneous nor homothetic, its elasticities are not constant, and technological, physico-natural and social variables may be added without difficulty. As regards the three IFF, readers will not be surprised if the writer views CMC-CES as progress with respect to CD and CES-ACMS, at least as regards the variable trends (first increasing, then decreasing) of average and marginal productivities.

Considering the set of four FFF, the cost function is the most frequently applied, and we believe this is because non-homogeneity and non-homotheticity may be assumed *a priori* (empirical data reject homogeneity with increasing frequency), as long as they can be tested *a posteriori*[28].

Application of the distance function still does not seem to have reached the level of the cost and production functions, although – for several reasons - this is definitely to be hoped for. The main reasons are its capacity to treat the multi-output case and to be fruitfully applicable in conditions of limited profitability with respect to ends or means[29]. The distance function also seems to be the parametric instrument best able to sustain the challenge with non-parametric analysis, which is becoming more popular. The poor application of the profit function is very probably due to competition conditions in the production markets which it must assume. This circumstance is confirmed by the fact that its application covers

[27] The preference is not hampered by the danger of falling into the "concavity trap", a danger shared by other FFF like DGL, which is also largely applied. Some researches regard the checking of the concavity conditions for all data of the available sample as a positive feature of the Translog and DGL functions.

[28] From this viewpoint too, Translog is in a better position with respect to the others, because its production function may also sometimes perform better than the cost function in terms of fitting empirical data, and with different but yet significant estimates of the parameters.

[29] See Grosskopf et al. (1995), and references therein.

agricultural production, in which the above conditions are judged to be quite satisfied[30].

As regards elasticities, we believe that their characteristics and mutual relations have been sufficiently described in the previous Sections. There is only one question remaining open: is Morishima's or Allen's form (in which all the others are shown) to be preferred? The writer has reflected on this point for some time and the preparation of this survey reinforced his views, although he is in a minority: with more than two inputs and non-homothetic functions, Morishima's elasticities are more appropriate. For this reason, they were calculated and presented together with σ^{AUES} and ρ^{AEC}. Without going back about fifteen years or so to the words of Blackorby and Russell[31], which are still valid, we observe that extension of the application to the distance function of Morishima's elasticities as operated by Kim (2000, p. 257) may be performed for all the others. The reason for this extension, which does not prevent us from using traditional versions when suitable, is that σ^{MES} and ρ^{MEC}, together with non-homogeneous (or at most homothetic) production functions and distance functions are, in our opinion, the main analytical tools capable of continuing development of the parametric theory of production.

[30] See Lau e Yotopoulos (1972), Sidhu e Baanante (1981), Shumway (1983), Antle (1984) e Lopez (1984).
[31] Cf. note (2).

Appendix

The derivatives of the MEDGCD cost function in the σ_{ij}^{MES} and σ_{ji}^{MES} formulas are:

$$d_i = \left(\frac{\alpha_{iy}\, ln\, y}{w_i} + 2\alpha_i \sum_{j=1}^{n} \frac{\beta_{ij}}{\alpha_i w_i + \alpha_j w_i} \right)$$

$$d_{ij} = \left[\begin{array}{l} -\dfrac{2\alpha_i \alpha_j \beta_{ij}}{\left(\alpha_i w_i + \alpha_j w_j\right)^2} + \left(\dfrac{\alpha_{iy}\, ln\, y}{w_i} + 2\alpha_i \displaystyle\sum_{j=1}^{n} \dfrac{\beta_{ij}}{\alpha_i w_i + \alpha_j w_j} \right) \cdot \\ \cdot \left(\dfrac{\alpha_{jy}\, ln\, y}{w_j} + 2\alpha_j \displaystyle\sum_{i=1}^{n} \dfrac{\beta_{ij}}{\alpha_i w_i + \alpha_j w_j} \right) \end{array} \right]$$

$$d_{ii} = \left[\left(-\frac{\alpha_{iy}\, ln\, y}{w_i^2} - 2\alpha_i^2 \sum_{j=1}^{n} \frac{\beta_{ij}}{\alpha_i w_i + \alpha_j w_j} \right) + \left(\frac{\alpha_{iy}\, ln\, y}{w_i} + 2\alpha_i \sum_{j=1}^{n} \frac{\beta_{ij}}{\alpha_i w_i + \alpha_j w_j} \right)^2 \right]$$

d_{jj} - *mutatis mutandis* – is defined in the same way as d_{ii}

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