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ON MINTY VARIATIONAL INEQUALITIES AND INCREASING ALONG RAYS FUNCTIONS

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Abstract

In this paper we investigate the link between a Minty variational inequality and the notion of increasing along rays function. In particular we show that vector increasing along rays functions can be characterized in term of existence of a solution of a generalized Minty variational inequality.

Introduction

Scalar and vector variational inequalities provide a mathematical model for scalar and vector equilibrium problems, problems which include, as special cases, beside variational inequalities, also optimization and complementary problems. Therefore variational inequalities are used to model static equilibria of several economies, such as Cournot oligopoly, spatial oligopoly, general economic equilibrium and characterize also the existence and stability of equilibria of a Dynamical System; in this contest it has been proved that existence of a solution of Stampacchia variational inequality is equivalent to existence of an equilibrium while the existence of a solution of Minty variational inequality ensures the stability of equilibria [3],[19]. In other words, the application of variational inequalities to dynamical systems allows to unify static and dynamic aspects in the study of economic phenomena.
We are mainly concerned with Minty type variational inequalities.
The paper is organized as follows. In section 1 we give some preliminaries on Minty variational inequality and on Minty variational inequality of differentiable type and we point out its links, with an scalar optimization problem. In section 2 we consider the convexity-type properties of Minty variational

inequality of differential type, while in section 3 we briefly recall the notion of scalar increasing along rays functions and its basic properties. In section 4 we present the notion of vector increasing along rays function and we investigate some properties with respect to vector optimization. Here we show a general approach through a nonlinear scalarization, the so called oriented distance function from a point to a set, and we give some basic facts on this concept and the main relations between a vector minimization problem and its scalarized counterpart. Section 5, finally, deals of vector variational inequalities and of their link with vector increasing along rays function.

1 Preliminaries

Let K be a nonempty subset of R^n and let F be a mapping from K into R^n.

Definition 1.1: *The Minty variational inequality (for short $MVI\ (F,K)$) is the problem to find a vector $x^* \in K$ such that*

$$MVI(F;K) \qquad < F(y),\ x^* - y > \leq 0 \qquad \forall\, y \in K$$

where $< \cdot,\cdot >$ denotes the inner product defined on R^n.

Since K is often polyhedral in applications, one generally assumes that K is closed and convex.

There are not, so far, results on existence of a solution of Minty variational inequality. It is well known that, in contrast to the Stampacchia variational inequalities to which, instead, exists Hartman-Stampacchia theorem, convexity and compactness of K and continuity assumptions on F do not ensure the existence of a solution to $MVI(F;K)$. In other words even if F is continuous and quasimonotone and is defined on a compact and convex set, a Minty variational inequality solution may not exist. The relationships between solutions set of $SVI(F;K)$ and of $MVI(F;K)$ are given in the Known Minty Lemma which asserts that

$M(F,K) \subseteq S(F,K)$ where $M(F,K)$ denotes the set of solutions of the Minty variational inequality while $S(F,K)$ the solution set of Stampacchia variational inequality. The reverse inclusion holds if F is pseudomonotone.
In particular, for the case of a continuous map F defined on an open convex domain, it has been shown [14] that the solutions of $MVI(F,K)$ can be considered as the subset of stable equilibria within the set of all equilibria (represented by the Stampacchia solutions) of a dynamical system associated with F. Moreover, if $LM(F,K)$ denotes the set of local solutions of the Minty variational inequality, it holds clearly $M(F,K) \subseteq LM(F,K)$. We recall that $x \in K$ is a local solution of the Minty variational inequality if there exists a neighbourhood U of x such that $x \in M(F, K \cap U)$.
Has been widely studied, mainly in relation with the minimization of function f over the set K, the particular case in which K is a convex set and the function F has a primitive $f : R^n \to R$, defined and differentiable on an open set containing K (i.e. the operator F is a gradient of a function f). It can consider, in this contest, the following variational problem: *find a point* $x^* \in K$ *such that*

$$MVI(f',K) \qquad < f'(y), x^* - y > \leq 0 \qquad \forall y \in K$$

where f' *is the derivative of the function* $f : K \subseteq R^n \to R$.
This problem is called usually *Minty variational inequality of differentiable type* and is denoted with $MVI(f',K)$.
$MVI(f',K)$ has a geometric interpretation that is at the base of so-called Minty variational principle [10]: for the function f, no matter in which point of K one is, if one moves toward a minimizer x^*, then the directional derivative must be non positive.
Interesting for economics studies is the relation between Minty variational inequalities of differential type and scalar

minimization problem in which the objective function to minimize over the set K is a primitive of operator involved in the inequality itself. $MVI(f',K)$ can be related, that is, to minimization problem:

$P(f,K)$ $$\min_{x\in K} f(x)$$

A point $x^* \in K$ is a (global) solution of $P(f,K)$ when

$$f(x) - f(x^*) \geq 0 \qquad \forall\, x \in K$$

A point $x^* \in K$ is a strict solution of $P(f,K)$ if

$$f(x) - f(x^*) > 0 \qquad \forall\, x \in K \setminus \{0\}$$

The set of solutions of $P(f,K)$ is denoted by $\arg\min(f,K)$. A Known result states, in the scalar case, some relations between Minty variational inequality of differential type and the underlying minimization problem and, that is, if $x^* \in K$, with K convex and nonempty subset of R^n, is a solution of the Minty variational inequality, then x^* is a solution also of $P(f,K)$. If, then, f is convex, the converse hold true [4],[16]. The following proposition states, that is, that $MVI(f',K)$ is a sufficient condition for the minimization of f over the set K, condition which becomes necessary if f is a convex.

Proposition 1.1: *Let K be a convex set subset of R^n and let $f : R^n \to R$ be differentiable on an open set containing K.*

i) If $x^* \in K$ is a solution of $MVI(f',K)$, then x^* solves $P(f,K)$.

ii) If f is convex and x^* is a solution of $P(f,K)$, then x^* solves $MVI(f',K)$.

Remark 1: If, in point i) of proposition 1.1, we suppose that x^* is a "strict solution" of $MVI(f',K)$, i.e. $< f'(y), y - x^* > < 0 \quad \forall y \in K, y \neq x^*$, then it is possible to prove that x^* is the unique solution of $P(f,K)$. Moreover, the convexity of f in point ii) can be weakened with the pseudoconvexity.

2. Minty variational inequality of differential type and convexity-type properties.

The result of proposition (1.1) leads to some deeper relationships between the solutions of $MVI(f',K)$ and the corresponding minimization problem. It seems that an " equilibrium" modelled throught a $MVI(f',K)$ is more regular than one modelled throught a $SVI(f',K)$ [14]. In other words, the Minty variational inequality of differential type characterizes a stronger notion of equilibrium than the Stampacchia variational inequality. In fact, under regularity assumptions, the solutions of $MVI(f',K)$ are a subset of those of $SVI(f',K)$ [7]. This conclusion leads to argue that, in relation with the minimization of the function f over the set K, if a $MVI(F,K)$ admits a solution and the operator F admits a primitive f (i.e. the function f to minimize is such that $f' = F$), then f has some regularity property, e.g. convexity or generalized convexity [7].

More precisely, the solution of Minty variational inequality of differential type

only if f obeys some convexity-type properties.

In the case of function of one variable (n=1), f must be quasi-convex . Is well know, in fact, the following proposition:

Proposition 2.1 [6]: *Let* $K \subseteq R^n$ *be convex and let* $f : R^n \to R$. *If there exist a solution* x^* *of* $MVI(f', K)$, *then* f *is quasi-convex.*

Definition 2.1 : *Let* $K \subseteq R^n$ *be convex. A function* $f : K \to R$ *is quasi-convex when the level sets of* f *:*

$$lev_c f := \{x \in K : f(x) \leq c\}$$

are convex for each $c \in R$.

Proposition 2.2: Let $K \subseteq R^n$ *be convex. A differentiable function* $f : R^n \to R$ *is quasi-convex if and only if , for any couple* $x_1, x_2 \in K$ *such that* $f(x_1) \leq f(x_2)$, *we have:* $\langle f'(x_2), x_1 - x_2 \rangle \leq 0$

In the case of several variables ($n \geq 2$) the existence of a solution of $MVI(f', K)$ does not imply necessarily quasi-convexity of the function f but implies, instead, the star-shapedness of the level sets of the function f at a point which is a solution of $MVI(f', K)$. But we, first, recall:

Definition 2.2: *Let* K *be a subset of* R^n.
i)The set $Ker\ K = K^\Diamond = \{x^* \in K : [x^*, x] \subseteq K,\ \forall x \in K\}$ *is called the Kernel of* K *;*
ii)The nonempty set K *is said star-shaped if* $Ker\ K \neq \emptyset$.

Proposition 2.3 [6] **:** *Let* K *be a nonempty subset of* R^n. *If* $f : K \to R$ *is such that there exists a solution* x^* *of* $MVI(f', K)$ *and* K *is star-shaped at* x^*, *then all nonempty level sets of* f *are star-shaped at* x^*.

From previous proposition, that can be regarded, in some sense, as a convexity-type condition, follow easily the next propositions; they show that, under convexity of the set K, a necessary condition for existence of a solution of $MVI(f', K)$ is that the intersection of the kernels of the level sets is nonempty.

Proposition 2.4: *Let* $x^* \in K$ *be a solution of* $MVI(f', K)$ *and let* $\bar{c} = \min_K f(x) = f(x^*)$. *If* $K \subseteq R^n$ *is star-shaped at* x^*, *then:*

$$x \in \bigcap_{c \geq \bar{c}} (level_c f)^{\Diamond}$$

Proposition 2.5: *Let* $\bar{c} = \inf_K f(x)$. *If* $K \subseteq R^n$ *is convex and nonempty and* $\cap_{c > \bar{c}} (level_c f)^{\Diamond} = \emptyset$, *then* $MVI(f', K)$ *has not solution.*

3. $MVI(f', K)$ and scalar increasing along rays functions

Some relations can be established among solutions of Minty variational inequalities of differential type and increasing along rays functions, studied in the field of Generalized Convexity and Optimization. To show that we shall consider a generalization of MVI of differential type. First, but, we recall the notion of increasing along rays scalar function and some of its basic properties. This notion arises mainly in the study of abstract convexity [20] and can be viewed as a generalization of the concept of quasiconvex function.

Definition 3.1: *Let* $K \subseteq R^n$ *be a nonempty star-shaped set and* $x^* \in Ker\, K$. *A function* f *defined on* K, $f : K \to R$, *is called increasing in* K *along rays starting* *at the* x^* *if the restriction of this function on the intersection* $R_{x^*,x} \cap K$ *is increasing, for each* $x \in K$ *.*

The class of increasing along rays functions is denoted by $IAR(K, x^*)$. If f fulfils definition 3.1, we write $f \in IAR(K, x^*)$.

If $K = R^n$, then f is said increasing along ray at the point x^*.

Definition 3.2: *A function* f *defined on* R^n *is called increasing along rays at the point* x^* (for short, $f \in IAR(x^*)$) *if the restriction of this function on the ray starting at* x^**, i.e.* $R_{x^*,x} = \{x^* + \alpha x / \alpha \geq 0\}$*, is an increasing function of* α*, for each* $x^* \in R^n$*.*

It is clear that when K is an interval of R (n=1), and f is a scalar function of one variable, $f \in IAR(K, x^*)$ if and only if it is quasiconvex with a global minimum over K at x^*. Instead, when $n \geq 2$ and K is a convex set, the class of quasiconvex functions with a global minimizer at x^* is a strict subset of that of IAR functions. The following result is quoted, without proof, from [6].

Proposition 3.1 : *If: i)* $f : R^n \to R$ *is quasiconvex and ii)* $x^* \in Ker$ *be a global minimizer for* $VP(f, K)$*, then* $f \in IAR(K, x^*)$.

In other word the class of functions $f \in IAR(K, x^*)$ is broader then the class of quasiconvex functions with a global minimum at x^* [6], as shows the following:

Example 1: *Let* $K = R^2$ *and* $f(x, y) = x^2 y^2$. *Then, for* $x^* = (0,0)$, *it is easily seen that* $f \in IAR(K, x^*)$, *but* f *is not quasi-convex.*

We now define, with regard to optimization problems, some basic properties for scalar functions increasing along rays, which can be considered as extension of analogous properties holding for convex function. For instance, functions increasing along rays starting at some x^* have a global minimizer at the some x^*
The following results are classical and motivate some of interest for the class $IAR(K, x^*)$. They give some basic properties of increasing along rays function and can be quoted from various papers, among the others [6],[7].

Proposition 3.2: *Let* $K \subseteq R^n$ *be a star-shaped set,* $x^* \in Ker\ K$ *and* $f \in IAR(K, x^*)$. *Then:*
a) x^* *is a solution of* $P(f, K)$.
b) no point $x \in K$, $x \neq x^*$, *can be a strict local solution of* $P(f, K)$.
c) $x^* \in Ker$ *arg min*(f, K).

Proposition 3.3 : *Let* $K \subseteq R^n$ *be a star-shaped set,* $x^* \in Ker\ K$ *and* f *be a function defined on* K. *Then* $f \in IAR(K, x^*)$ *if and only if for each* $c \in R$ *with* $c \geq f(x^*)$, *we have* $x^* \in Ker\ lev_c f$.

In particular, the previous proposition allows to consider the increasing along rays scalar function as generalized convex functions, since it is well known that level sets of quasiconvex functions are convex sets.

The increasing along rays scalar functions can be characterized by means of a generalized Minty variational inequality [18]. We consider, now, a generalization of MVI of differential type and we introduce, therefore, the following problem that consist in finding $x^* \in Ker\ K$ for which is satisfied the inequality:

$GMVI(f', K)$ $\qquad f'_-(x, x^* - x) \le 0 \qquad \forall x \in K$

where

$$f'_-(x,u) = \liminf_{t \to +0} \frac{f(x+tu) - f(x)}{t}$$

is the lower Dini directional derivative at the point $x^* \in K$ in the direction $u \in R^n$ of real function f defined an on open set containing K. This problem, that somehow generalizes the MVI of differential type, obviously reduces to the usual Minty variational inequality of differential type $MVI(f', K)$ when f is differentia-ble an on open set containg K [18].

The following proposition links, in the case of K star-shaped, the existence of a solution x^* of $GMVI(f', K)$ to increasing along-rays starting at x^* of f [7]. In fact it states that $GMVI(f', K)$, with f radially lower semicontinuous function in K on the rays starting at x^*, has a solution $x^* \in Ker\, K$ if and only if f is increasing along such rays (for short $f \in IAR(K, x^*)$). First, we recall:

Definition 3.3: *Let $K \subseteq R^n$, $x^* \in Ker\ K$ and let f be a function defined on an open set containing K. The function*

f is said to be radially lower semicontinuous over K along rays starting at x^*, if for each $x \in K$, the restriction of f on the interval $R_{x^*,x} \cap K$ is lower semicontinuous.

If f satisfied the previous definition, we write

$$f \in RLSC(K, x^*)$$

Proposition 3.4 [6],[7]: *Let $K \subseteq R^n$ be a star-shaped set and let $f : K \to R$. If $x^* \in Ker\, K$ is solution of $GMVI(f', K)$ and $f \in RLSC(K, x^*)$, then $f \in IAR(K, x^*)$. Conversely, if $x^* \in Ker\, K$ and $f \in IAR(K, x^*)$, then x^* is a solution of $GMVI(f', K)$.*

The following proposition, quoted in [7], shows that if $GMVI(f', K)$ has a solution $x^* \in K$, then x^* is a global minimizer of the problem $P(f, K)$. This result estends the one classical according to which, if K *is a* convex set, any solution of $MVI(f', K)$ solves $P(f, K)$.

Proposition 3.5 [7]**:** *Let $K \subseteq R^n$ be a star-shaped set and let $f \in RLSC(K, x^*)$. If $x^* \in Ker\, K$ is a solution of $GMVI(f', K)$, then x^* solves $P(f, K)$.*

Proof : According to proposition 3.4, f increases along $R_{x^*,x} \cap K$, where $x \in K$ and $x^* \in Ker\, K$; whence $f(x^*) \leq f(x)$ and therefore x^* is a global minimizer of $P(f, K)$ [7] .

Proposition 3.5 states that any solution of $GMVI(f', K)$ is a solution of the related minimization problem; this is not true when we considered $MVI(f', K)$.

Example 2 [7]: *We consider the set*

$$K = \{ [(-1,0),(0,1)] \cup [(0,1),(0,4)] \}$$

and the function:

$$f : R^2 \to R, \qquad f(x, y) = xy(x + y) - \frac{1}{3}(x + y - 1)^2$$

The set K is star-shaped while $Ker\ K = \{(0,1)\}$. The unique solution of $MVI(f', K)$ is $x^* = (-1,0) \notin Ker\ K$ and it does not belong to the set of the global minimizers of $P(f, K)$ which is the singleton $\{(0,4)\}$.

The existence of solutions of the $GMVI(f', K)$ implies the star-shapedness of the level sets of f, as shows the following proposition:

Proposition 3.6: *Let $K \subseteq R^n$ be a star-shaped set and let $f : K \to R$. If there exists a solution $x^* \in Ker\ K$ of $GMVI(f', K)$ and if $f \in RLSC(K, x^*)$, then all the level sets of f are star-shaped. In particular, the set of the global minimizers of $P(f, K)$ is star-shaped.*

Proof*:* According to proposition 3.5, x^* solves $P(f, K)$. Consider, now, the level set $lev_c f$, fixed $c \in R$. If $c < f(x^*)$, then $lev_c f$ is empty and hence is star-shaped. If $c \geq f(x^*)$, fixed $x \in lev_c f$, consider: $x(t) = (1-t)x^* + tx$. According to theorem 3.4, $f \in IAR(K, x^*)$. This implies that $f(x(t)) \le f(x(1)) = f(x) \le c$.

This condition can be regarded as a convexity-type condition [6]. Since a function f is quasi convex if and only if its level sets are convex, the functions with level sets star-shaped can be considered as a generalization of quasi convex functions. That does not allows to state, however, that when K is convex and $GMVI(f', K)$ has a solution, f is quasi convex.

The problem $GMVI(f', K)$ is equivalent to problem $MVI(f', K)$ if K is star-shaped and if only solutions $x^* \in Ker\, K$ are considered. The equivalence is understood in the sense of coincidence of the solution sets. The two problems are equivalent, also, in the case in which K is convex (which hold, in particular, if f is quasi-convex on K) [7].

Remark 3.1: If $MVI(f', K)$ admits at least one solution $x^* \in Ker\, K$, then each solution of $MVI(f', K)$ is a solution of the related minimization problem**.**

In fact, since $x^* \in Ker\, K$ solves $MVI(f', K)$, then x^* solves $GMVI(f', K)$ and so $f \in IAR(K, x^*)$. Hence, f is increasing along the ray $R_{x^*, x} \cap R$. We assume, now, that $\bar{x} \neq x^*$ is a solution of $MVI(f', K)$. Since also $\bar{x}$ solves $MVI(f', K)$ then it is easily seen that f is increasing along the ray $R_{x^*, x} \cap K$. This implies that all the points on the segment $[x^*, x]$ are minimizes of f over K **.**

Moreover in the case that K is star-shaped, if x^* solves $MVI(f',K)$, then f is increasing along rays starting at x^* (for short $f \in IAR(K,x^*)$).

Proposition 3.7 [6]: *If $x^* \in Ker\ K$ and f is differential on an open set containing K, then x^* solves $MVI(f',K)$ if and only if $f \in IAR(K,x^*)$.*

For the sake of completeness, we recall that it has been observed, also, that the existence of a solution of $MVI(f',K)$ is somehow related to the well-posedness of primitive optimization problem $P(f,K)$ [6]. More precisely, optimization problem with objective function increasing along rays have several well-posedness properties [6]. In fact, as it happens for convex function, the functions $f \in IAR(K,x^*)$ enjoy some well-posedness properties and relations with well-posedness can be established [6]. Before, recall:

Definition 3.4: *i) A sequence $x_n \subseteq K$ is a minimizing sequence for $P(f,K)$*

when $f(x_n) \to \inf_K f(x)$ *as* $n \to +\infty$

ii) A sequence x_n is a generalized minimizing sequence for $P(f,K)$ when

$f(x_n) \to \inf_K f(x)$ *and* $d(x_n,K) \to 0$

where $d(x,K)$ denotes the distance from the point x to the set K.

Definition 3.5: *Problem $P(f,K)$ is Tykhonov well-posed when it admits a unique solution x^* and every minimizing sequence for $P(f,K)$ converges to x^*.*

Definition 3.6: *Problem $P(f,K)$ is said Tykhonov well-posed in the generalized sense when $\arg\min(f,K) \neq 0$ and every minimizing sequence for $P(f,K)$ has some subsequence that converges to an element of $\arg\min(f,K)$, that is to an solution of $P(f,K)$.*

The following results extend to $IAR(K,x^*)$ functions some classical well-posedness properties of convex functions.

Proposition 3.8 [6]: *Let K be a closed subset of R^n, $x^* \in Ker\ K$ and let $f \in IAR(K,x^*)$ be a lower semicontinuous. If $\arg\min(f,K)$ is a singleton, then $P(f,K)$ is Tykhonov well-posed.*

Proposition 3.9 [6] : *Let K be a closed subset of R^n, $x^* \in Ker\ K$ and let $f \in IAR(K,x^*)$ be a lower semicontinuous. If $\arg\min(f,K)$ is a compact, then $P(f,K)$ is Tykhonov well-posed in the generalized sense.*

4. Vector increasing along rays functions and vector optimization

We consider the vector optimization problem:

$$VP(f,K) \qquad \min_C f(x) \qquad \forall x \in K$$

where f is a function from R^n to R^m and $K \subseteq R^n$. The order on the space R^m is induced by a closed convex pointed cone $C \subseteq R^n$ with $\operatorname{int} C \neq \emptyset$ (nonempty interior).
The solutions of $VP(f,K)$ are usually called point of efficiency, but here we prefer to call them minimizers.

The point $x^* \in K$ is said to be e- minimizer for $VP(f,K)$ when

$$f(x)-f(x^*) \notin -C \setminus \{0\} \qquad \forall x \in K$$

The point $x^* \in K$ is said to be w- minimizer for $VP(f,K)$ when

$$f(x)-f(x^*) \notin -\operatorname{int} C \qquad \forall x \in K$$

The solutions of $VP(f,K)$ can be characterized as solutions of suitable scalar optimization problems. For this we introduce the concept of oriented distance from a point $y \in R^m$ to a subset $A \subseteq R^m$, given by

$$\Delta_A(y) = d_A(y) - d_{R^m}(y)$$

where $d_A(y) = \inf_{a \in A} \|y-a\|$.

We present, now, some basic facts on this concept and the main relations between a vector minimization problem and its scalarized counterparts.

While $d_A(y)=0$ when $y \in$ closure of A and positive elsewhere, $\Delta_A(y)<0$ for $y \in \operatorname{int} A$ (the interior of A), $\Delta_A(y)=0$ for $y \in bd\, A$ (the boundary of A) and positive elsewhere.

It has proved in [12], that when $A = C$ is closed, convex, pointed cone, then we have

$$\Delta_{-C}(y) = \max_{\xi \in C' \cap S} < \xi, y >$$

where $C' = \{x \in R^m :< \xi, c > \geq 0 \ \ \forall c \in C\}$ denotes the positive polar of the cone C and $S = \{\xi \in R^m : \|\xi\| = 1\}$ is the unit sfere in R^m. Further properties of the oriented distance function can be found in [23].

Recently, the function Δ_{-C} has been used to scalarize the vector optimization problem $VP(f,K)$ through the solutions of the scalar problem.

The considered scalar problem is:

$P(\varphi_{x^*}, K) \qquad\qquad \min \varphi_{x^*}(x)$

where $x^* \in K$ and

$$\varphi_{x^*}(x) = \max < \xi, f(x) - f(x^*) > = \Delta_{-C}\left(f(x) - f(x^*)\right)$$

The following theorem states the relations among the solution of problem $P(\varphi_{x^*}, K)$ and those of problem $VP(f, K)$. These relations are investigate in [23] and in [13].

Proposition 4.1 : *i) The point $x^* \in K$ is a strong minimizer of $VP(f, K)$ if and only if x^* is a strong solution of $P(\varphi_{x^*}, K)$; ii) The point $x^* \in K$ is a weak minimizer of $VP(f, K)$ if and only if x^* is a solution of $P(\varphi_{x^*}, K)$.*

Now we may extend to the vector case the notion of IAR function and the IAR property for vector-valued functions. The proposed definition of increasing along rays vector function is not given using a non linear scalarization but using a generalization of the distance notion, known as oriented distance functions from a point to a set. More precisely the function φ_{x^*} allows to introduce the following notion of vector increasing along rays function.

Definition 4.1: *A function $f : K \subseteq R^n \to R^m$ is said vector increasing along rays starting at a point $x^* \in K$, when function $\varphi_{x^*}(x) \in IAR(K, x^*)$.*

To denote that f is a vector increasing along rays starting at x^*, we write $f \in VIAR(K, x^*)$.

The previous definition, defined through the oriented distance function and not through the order induced on R^m by the cone C, reduces to notion of IAR function when: $f : R^n \to R$ .The next propositions, instead, give some basic properties of $VIAR$ functions which should be compared with those in section 3 [9].

Proposition 4.2: *Let* $K \subseteq R^n$ *be a star-shaped and* $x^* \in Ker\, k$ *. If* $f \in VIAR(K, x^*)$ *, then* $\forall t_2 \ge t_1 > 0$ and $\forall x \in K \quad f(x^* + t_2(x - x^*)) - f(x^* + t_1(x - x^*)) \notin -\operatorname{int} C$

Proposition 4.3: *A function* $f \in VIAR(K, x^*)$ *if and only if for every* $x \in K$ *and* $\varepsilon > 0$ *such that* $f(x) \in f(x^*) - C + \varepsilon\beta$ *, it holds* $f(x^* + t(x - x^*)) \in f(x^*) - C + \varepsilon\beta$ *for every* $t \in [0,1]$

Proof: To show that is enough to observe that

$$\{x \in K : f(x) \in f(x^*) - C + \varepsilon\beta\}_{x^*} =$$
$$= \{x \in K : \varphi_{x^*}(x) \le \varepsilon\} = lev_{\le \varepsilon}\varphi_{x^*}$$

and recalling the proposition 3.3.
We denote by $\beta = \{y \in R^m : \|y\| \le 1\}$ the unit ball in R^m .

Proposition 4.4: *If* $x^* \in Ker\ K$ *and* $f \in VIAR(K, x^*)$*, then:*

i) x^* *is a w-minimizer of* f *over K.*

ii) the set $f^{-1}(f(x^*))$ *is star-shaped with* $x^* \in Ker\, f^{-1}(f(x^*))$

Similarly to the scalar case $VIAR$ functions enjoy some well-posedness properties. For more details we refer [17].

5. $MVVI(f', K)$ and $VIAR$ functions

The study of vector optimization problems by means of Minty-type variational inequalities has been first presented in [10] and has been deepened subsequently in [21]. This approach is based on a vector valued variational inequality**.** The extension to the vector case of Minty variational inequalities involves the following problem: *to find* $x^* \in K$ *such that*

$$MVVI(F,K) \qquad < F(x),\, x^* - x >_l \not\geq_c 0 \qquad \forall\, x \in K$$

where the feasible region $K \subseteq R^n$ *is supposed convex and nonempty,* F *is the following function* $F : R^n \to R^m$ *, while* $< \cdot,\cdot >$ *denotes a vector of* l *inner products in* R^n .

This problem is called *Minty vector variational inequality* (for short $MVVI(F,K)$). For $l = 1$ it reduce to the classical Minty variational inequalities.
The vector variational inequalities, introduced in [11] have been studied intensively because they provide a mathematical model for the problem of equilibrium and for investigating vector optimization problems.

We, now, extend the scalar results to the case of vector valued function by means of a non linear scalarization which allows to study a vector function through a family of scalar functions.
Let K a nonempty subset of X. For a given function $F : K \to Y$ it can consider the Minty variational inequality of differentiable type that is defined as the problem of finding a point $x^* \in K$ such that

$$MVVI(f',K) \qquad < f'(x), x^* - x > \leq 0 \qquad \forall\, y \in K$$

where $f'(x,u)$ is the Dini directional derivative of f at x in direction $u \in X$, defined as

$$f'(x,u) = \liminf_{t \to 0^+} \frac{f(x+tu) - f(x)}{t}$$

If $x^* \in Ker\, K$ (that means that K is star shaped) is a solution of $MVVI(f', K)$, then we may assume that f is defined on K, since the directional derivatives in $MVVI$ do not depend on the values of f outside K.

It can be shown that if x^* is a solution of $MVVI(f', K)$, then, differently from the scalar case, f does not necessarily belong to the class $VIAR(K, x^*)$.

To fill the former gap we consider the following result which characterizes $VIAR$ function in terms of a suitable variational inequality. Before recall:

Definition 5.1: *Let f be a function defined an a set $K \subseteq R^n$. We say that f is C-continuous at x^* when for every neighbourhood of $x^*, U \in R^n$, there exists a neighbourhood V of R^m, such that*

$$f(x) \in V + C \qquad \forall x \in U \cap K$$

We say that f f is C-continuous on K when f is C-continuous at any point of K.

Definition 5.2: *Let K be a star-shaped set with $x^* \in Ker\, X$ and let f be a function defined on an open set containing K. The function f is said to be C-radially continuous in K along the rays starting at x^*, if for every $x \in K$, the restriction of f on the interval $R_{x^*,x} \cap K$ is C-continuous.*

If f is C-radially continuous in K, we write for short: $f \in C - RC(K, x^*)$

Proposition 5.1: *Let K be a star-shaped set and $x^* \in Ker\,X$. Assume that f is a function defined on an open set containing K.*

i) *Let $f \in C-RC(K,x^*)$.* If x^* *solves $GMVI(\varphi'_{x^*},K)$, then*

$$f \in VIAR(K,x^*)$$

ii) Conversely, if $f \in VIAR(K,x^*)$, then x^* solves

$$GMVI(\varphi'_{x^*},K)$$

Similarly to the scalar case, the assumption $f \in C-RC(K,x^*)$ appears in only one of the two opposite implications. The next example shows that this assumption cannot be dropped at hall.

Example Let $K = R, x^* = 0$, consider the function $f : R \to R^2$ defined as $f(x) = (g(x), g(x))$, with

$$f(x) = \begin{cases} 1, & if \quad x \neq 2 \\ 3, & if \quad x = 2 \end{cases}$$

Then $f \notin C-RC(K,x^*)$ and it holds $\varphi'_{x^*}(y, x^* - y) \leq 0$ $\forall\, y \in R$ but $f \notin VIAR(K,x^*)$

Corollary

Let $x^* \in \ker K$ and let $f \in C-RC(K,x^*)$. If x^* solves $MVI(\varphi'_{x^*},K)$, then x^* is w-minimizer for f over K.

We close section with some comparison between problem $MVI(\varphi'_{x^*},K)$ and the vector variational inequality problem $MVVI(f',K)$ assuming that f is a function of class C^1 on an open set containing K. In [5] it has been observed that every solution of $MVI(\varphi'_{x^*}, K)$ is also a weak solution of $MVVI(f', K)$. The converse does not necessarily hold as shown by Example 2 in [5]. Anyway, Theorem 9 in [5] ensures that if f is a C-convex function on the convex set K, then any $x^* \in K$ is a weak solution of $MVVI(f', K)$ solves also $MVI(\varphi'_{x^*}, K)$.

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