Many Valued Logics: Interpretations, Representations and Applications

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Abstract

This thesis, as the research activity of the author, is devoted to establish new connections and to strengthen well-established relations between different branches of mathematics, via logic tools. Two main many valued logics, *logic of balance* and *Łukasiewicz logic*, are considered; their associated algebraic structures will be studied with different tools and these techniques will be applied in social choice theory and artificial neural networks. The thesis is structured in three parts.

Part I The logic of balance, for short $Bal(H)$, is introduced. It is showed: the relation with ℓ-Groups, i.e. lattice ordered abelian groups (Chapter 2); a functional representation (Chapter 3); the algebraic geometry of the variety of ℓ-Groups with constants (Chapter 4).

Part II A brief historical introduction of Łukasiewicz logic and its extensions is provided. It is showed: a functional representation via *generalized states* (Chapter 5); a non-linear model for MV-algebras and a detailed study of it, culminating in a categorical theorem (Chapter 6).

Part III Applications to social choice theory and artificial neural network are presented. In particular: preferences will be related to vector lattices and their cones, recalling the relation between polynomials and cones studied in Chapter 4; multilayer perceptrons will be elements of non-linear models introduced in Chapter 6 and networks will take advantages from *polynomial completeness*, which is studied in Chapter 2.

We are going to present: in Sections 1.2 and 1.3 all the considered structures, our approach to them and their (possible) applications; in Section 1.4 a focus on the representation theory for ℓ-Groups and MV-algebras.

Note that: algebraic geometry for ℓ-Groups provides a *modus operandi* which turns out to be useful not only in theoretical field, but also in applications, opening (we hope) new perspectives and intuitions, as we made in this first approach to social theory; non-linear models here presented and their relation to neural networks seem to be very promising, giving both intuitive and formal approach to many concrete problems, for instance degenerative diseases or distorted signals. All these interesting topics will be studied in future works of the author.