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## **Parameter Estimation In Continuous Stochastic Volatility Models**

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# Parameter estimation in continuous stochastic volatility models

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## Abstract

Continuous-time diffusion processes are often used in literature to model dynamics of financial markets. In such kinds of models a relevant role is played by the variance of the process. So assumptions on the functional form of such variance have to be made in order to analyse the distribution of the resulting process and to make inference on the model. In this paper the variance is also modelled by means of a diffusion process. This comes out as continuous time approximation of a GARCH(1, 1) process. Inference on the parameters and properties of the involved estimators are discussed under different choices of the frequency data. Simulations on the model are also performed.

**keywords:** Stochastic volatility, diffusion processes, discrete-time observations

## 1 Introduction

Many econometric studies show that financial time series tends to be highly heteroskedastic. Many of theoretical models have made extensive use of Ito calculus which provides a lot of theoretical instruments to analyse diffusion processes. Moreover, econometricians usually make use of models of dynamic conditional variance, based on discrete time approach of GARCH models first introduced by Engle in 1982 (see [7]). The gap between the two approaches was bridged by Nelson in 1990 ([14]). He developed conditions under which ARCH stochastic difference equations systems converge in distribution to Ito processes as the length of the discrete time goes to zero. Recently, Kallsen and Vesenmayer in [11] have showed that any COGARCH process can be represented as the limit in law of a sequence of GARCH(1, 1) processes. Moreover they argue heuristically that COGARCH and the classical bivariate diffusion limit of Nelson are probably the only continuous-time limits of GARCH. The use of a continuous time approach can be useful

when irregular steps between the data are present or when we have different frequency of the data. Those reasons justify the extensive use of stochastic volatility (SV) models in finance to describe a lot of empirical facts of the stock and the derivative prices.

The estimation of the parameters in such kind of models is still a challenging issue: recently, Figà-Talamanca (see [8]) focused on the Constant Elasticity of Variance stochastic volatility (CEV SV) model and, making use of results in Genon-Catalot (see [10]), she proved that, if the data generating process of a stock price is of CEV SV type in continuous time, then the sample autocovariance of suitable scaled squared returns of a given stock is a consistent and asymptotically normal estimator of the theoretical autocovariance of the mean variance process. Moreover she derived explicitly the asymptotic variance of the estimator under the assumptions that there exists the fourth-moments of the volatility. The aim of the present work is to propose an alternative method to estimate parameters in a SV model and to obtain the optimal properties for our estimators under weaker assumptions. The paper is organized as follows: in the Section 2 we present the model and setup the main definitions, in Section 3 we present the inference on the model and we prove the strong consistency and the asymptotic normality of the proposed estimators. Moreover the asymptotic variance of the estimators is derived by using a moving block bootstrap approach. Section 4 is dedicated to simulation results. Concluding remarks are made in Section 5.

## 2 The model

The so-called stochastic volatility models for describing the dynamics of the price  $S_t$  of a given stock are usually defined through the following bivariate stochastic differential equation:

$$dS_t = \mu dt + \sigma_t dW_{1,t} \tag{1}$$

$$d\sigma_t^2 = b(\theta, \sigma_t^2) dt + a(\theta, \sigma_t^2) dW_{2,t}$$

defined in a complete probability space. Here  $a$  and  $b$  are suitable functions in order to have the existence of a strong solution to (1),  $\mu \in \mathbb{R}$  and  $\theta \in \mathbb{R}^d$  and  $W_1$  and  $W_2$  are two independent Brownian motions. In the GARCH diffusion model the volatility  $\sigma_t^2$  satisfies the following equation:

$$d\sigma_t^2 = (\omega - \theta\sigma_t^2) dt + \alpha\sigma_t^2 dW_{2,t}. \tag{2}$$

In this case, using the centered log-prices  $Y_t$ , we obtain:

$$dY_t = \sigma_t dW_{1,t} \tag{3}$$

$$d\sigma_t^2 = (\omega - \theta\sigma_t^2) dt + \alpha\sigma_t^2 dW_{2,t}.$$

Here,  $\{Y_t\}$  is the observed process and  $\{\sigma_t^2\}$  represents its volatility, so generally it is an unobservable process.

We point out that the model in (3), under some assumptions on the parameters, comes out in a well known paper by Nelson ([14]) as the continuous limit in law of a suitable GARCH model. Moreover, Nelson proved that  $\sigma^2(t)$  is an ergodic diffusion with an inverse gamma invariant probability measure.

If the parameters  $\omega$  and  $\alpha$  in (3) are positive constants, then there exists a strong solution to (3) (see [1], [5]). Moreover, if  $\sigma_0^2$ , i.e. the volatility at initial time  $t_0$ , is a random variable (r.v.) independent on  $W_{2,t}$ , by Ito's formula, we can obtain the explicit expression of the volatility:

$$\sigma_t^2 = \omega F^{-1}(t, W_{2,t}) \int_0^t F(s, W_{2,s}) ds + F^{-1}(t, W_{2,t}) \sigma_0^2 \quad \forall t \geq 0, \quad (4)$$

where  $F(t, W_{2,t}) = \exp\{(\theta + \frac{\alpha^2}{2})t - \alpha W_{2,t}\}$ . For simplicity, in (4) we have assumed  $t_0 = 0$ .

From (4) it is easy to see that the volatility process  $\{\sigma_t^2\}$  is non negative for all  $t \geq 0$ .

After some cumbersome calculations, we obtain that

$$F^{-1}(t, W_{2,t}) \int_0^t F(s, W_{2,s}) ds = \int_0^t \exp\{(\theta + \frac{\alpha^2}{2})s - \alpha W_{2,s}\} ds = \frac{1 - e^{-\theta t}}{\theta}, \quad (5)$$

so we can write the volatility process as:

$$\sigma_t^2 = \sigma_0^2 \Lambda(t) + \frac{\omega}{\theta} (1 - e^{-\theta t}) \quad (6)$$

where  $\Lambda(t) \sim LN(-(\theta + \frac{\alpha^2}{2})t, \alpha^2 t)$ .

### 3 Inference on the model

Let us assume that the data generating the process (3) are given with frequency  $\delta$ , i.e.

$$Y_0, Y_\delta, \dots, Y_{h\delta}, \dots, Y_{n\delta}$$

with corresponding volatilities

$$\sigma_0^2, \sigma_\delta^2, \dots, \sigma_{h\delta}^2, \dots, \sigma_{n\delta}^2.$$

From (6) we can obtain the following recursive relation for the volatility:

$$\sigma_{h\delta}^2 = e^{\{-(\theta + \frac{\alpha^2}{2})\delta + \alpha W_\delta\}} \sigma_{(h-1)\delta}^2 + \frac{\omega}{\theta} (1 - e^{-\theta\delta}) \quad h = 1, 2, 3, \dots, n. \quad (7)$$

To estimate the parameters  $\alpha$ ,  $\theta$ ,  $\omega$ , methods based on classical maximum likelihood or conditional moments do not work (see, for example, [9]). So we propose a method based on the unconditional moments.

**Proposition 1.** *If  $\alpha$  and  $\omega$  are positive constants, the asymptotic moments of the volatility process defined in (2) are:*

$$\lim_{h \rightarrow \infty} \mathbb{E}[\sigma_{h\delta}^2] = \frac{\omega}{\theta}, \quad (8)$$

$$\lim_{h \rightarrow \infty} \mathbb{E}[\sigma_{h\delta}^4] = \frac{\omega^2}{\theta^2} \frac{1 - e^{-2\theta\delta}}{1 - e^{(-2\theta + \alpha^2)\delta}}, \quad (9)$$

$$\lim_{h \rightarrow \infty} \mathbb{E}[\sigma_{h\delta}^2 \sigma_{(h-1)\delta}^2] = e^{-\theta\delta} \lim_{h \rightarrow \infty} \mathbb{E}\sigma_{h\delta}^4 + \frac{\omega^2}{\theta^2} (1 - e^{-\theta\delta}). \quad (10)$$

*Proof.* From the recursive relation for the volatility (7) we obtain:

$$\begin{aligned} \mathbb{E}[\sigma_{h\delta}^2] &= \mathbb{E}[e^{-c\delta + \alpha W_\delta} \sigma_{(h-1)\delta}^2] + \frac{\omega}{\theta} (1 - e^{-\theta\delta}) \\ &= e^{-c\delta + \frac{\alpha^2}{2}\delta} \mathbb{E}[\sigma_{(h-1)\delta}^2] + \frac{\omega}{\theta} (1 - e^{-\theta\delta}). \end{aligned} \quad (11)$$

In (11) it is  $c = \theta + \frac{\alpha^2}{2}$ . For the ergodicity of the process  $\{\sigma_t^2\}$ , we have:

$$\left[1 - e^{(-c + \frac{\alpha^2}{2})\delta}\right] \lim_{h \rightarrow \infty} \mathbb{E}[\sigma_{h\delta}^2] = \frac{\omega}{\theta} (1 - e^{-\theta\delta}) \quad (12)$$

so

$$\lim_{h \rightarrow \infty} \mathbb{E}[\sigma_{h\delta}^2] = \frac{\omega}{\theta}. \quad (13)$$

In the same way, from (7), we have:

$$\begin{aligned} \mathbb{E}[\sigma_{h\delta}^4] &= \mathbb{E}[e^{-2(\theta + \frac{\alpha^2}{2})\delta + 2\alpha W_\delta} \sigma_{(h-1)\delta}^4 + \\ &\quad + \frac{\omega^2}{\theta^2} [1 - e^{-\theta\delta}]^2 + 2e^{-(\theta + \frac{\alpha^2}{2})\delta + \alpha W_\delta} \sigma_{(h-1)\delta}^2] \\ &= e^{-2(\theta + \frac{\alpha^2}{2})\delta + 2\alpha^2\delta} \mathbb{E}\sigma_{(h-1)\delta}^4 + \frac{\omega^2}{\theta^2} [1 - e^{-\theta\delta}]^2 + \frac{2\omega^2}{\theta^2} (1 - e^{-\theta\delta}) e^{-\theta\delta}. \end{aligned} \quad (14)$$

Taking the limit for  $h \rightarrow \infty$ , from (14) we obtain (9).

The (10) can be proved in the following way:

$$\begin{aligned} \mathbb{E}[\sigma_{h\delta}^2 \sigma_{(h-1)\delta}^2] &= \mathbb{E}[e^{-(\theta + \frac{\alpha^2}{2})\delta + \alpha W_\delta} \sigma_{(h-1)\delta}^4 + \frac{\omega}{\theta} (1 - e^{-\theta\delta}) \sigma_{h\delta}^2] \\ &= e^{-\theta\delta} \mathbb{E}\sigma_{(h-1)\delta}^4 + \frac{\omega^2}{\theta^2} (1 - e^{-\theta\delta}). \end{aligned} \quad (15)$$

□

Now, let  $\gamma(k) := \text{cov}(\sigma_{h\delta}^2, \sigma_{(h-k)\delta}^2)$  ( $h \in \mathbb{N}_0$  and  $k = 0, 1, \dots, h$ ) be the autocovariance function of the volatility process. From (7) it is easy to obtain the following recursive relation for  $\gamma(k)$ :

$$\gamma(k) = e^{-\theta\delta} \gamma(k-1) \quad k = 1, 2, \dots, \quad (16)$$

from which  $\gamma(k) = e^{-k\theta\delta}\gamma(0)$ , with  $\gamma(0) = \text{var}(\sigma_{h\delta}^2)$ . So, for the autocorrelation function we obtain:

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} = e^{-k\theta\delta} \quad (17)$$

and it depends only on the parameter  $\theta$ .

In the following proposition we show the relation between the moments of the increment process of the observed process  $\{Y_t\}$  and those one of the volatility process.

**Proposition 2.** *Let  $\{X_t\}$  be the increment process of the observed process  $\{Y_t\}$ :*

$$X_{h\delta} = Y_{h\delta} - Y_{(h-1)\delta} = \sqrt{\sigma_{h\delta}}Z_h, \quad (18)$$

with  $Z_h \stackrel{i.i.d.}{\sim} N(0, 1)$  and independent on  $\sigma_{h\delta}^2$  for each  $h = 1, 2, \dots$ . We have:

$$\begin{aligned} \lim_{h \rightarrow \infty} \mathbb{E}X_{h\delta}^2 &= \delta \lim_{h \rightarrow \infty} \mathbb{E}\sigma_{h\delta}^2 \\ \lim_{h \rightarrow \infty} \mathbb{E}X_{h\delta}^4 &= 3\delta^2 \lim_{h \rightarrow \infty} \mathbb{E}\sigma_{h\delta}^4 \\ \lim_{h \rightarrow \infty} \mathbb{E}[X_{h\delta}^2 X_{(h-k)\delta}^2] &= \delta^2 \lim_{h \rightarrow \infty} \mathbb{E}[\sigma_{h\delta}^2 \sigma_{(h-k)\delta}^2] \end{aligned} \quad (19)$$

*Proof.* It follows from (18) and from the independence of the r.v.'s  $Z_h$  and  $\sigma_{h\delta}^2$  for each  $h = 1, 2, \dots$   $\square$

Let now  $(X_1, X_2, \dots, X_n)$  be a time series of length  $n$  of the increment process  $\{X_t\}$ . Relations (19) suggest to use the r.v.'s:

$$\begin{aligned} M_2 &:= \frac{1}{n\delta} \sum_{i=1}^n X_i^2 \\ M_4 &:= \frac{1}{3n\delta^2} \sum_{i=1}^n X_i^4 \\ E_1 &:= \frac{1}{n\delta^2} \sum_{i=2}^n X_i^2 X_{i-1}^2. \end{aligned} \quad (20)$$

as statistics in the estimation of the parameters  $\omega$ ,  $\theta$  and  $\alpha^2$  in (3).

From (8), (9) and (17), choosing  $k = 1$  and making explicit the parameters  $\theta$ ,  $\omega$  and  $\alpha^2$ , we can conclude that, if there exists the second moment of the volatility, the method based on the moments of the volatility process suggests

the following estimators for  $\theta$ ,  $\omega$  and  $\alpha^2$  in the model (3), respectively:

$$\begin{aligned}\hat{\Theta} &:= f_1(M_2, M_4, E_1) = \frac{1}{\delta} \log \frac{\hat{\gamma}(0)}{\hat{\gamma}(1)}, \\ \hat{\Omega} &:= f_2(M_2, M_4, E_1) = -\frac{M_2}{\delta} \log \hat{\Theta} \\ \hat{\Gamma}^2 &:= f_3(M_2, M_4, E_1) = \frac{1}{\delta} \log \left\{ \frac{1}{\hat{\Theta}^2} \left[ 1 - \frac{M_2^2}{M_4} (1 - \hat{\Theta}^2) \right] \right\}.\end{aligned}\tag{21}$$

Here  $\hat{\gamma}(0)$  and  $\hat{\gamma}(1)$  are the sample variance and covariance of  $\{X_{h\delta}\}$ :

$$\hat{\gamma}(0) = M_4 - M_2^2 \quad \hat{\gamma}(1) = E_1 - M_2^2.\tag{22}$$

### 3.1 Properties of the estimators

In this Section we investigate the properties of the estimators obtained in (21).

**Proposition 3.** *If  $\frac{2\theta}{\alpha^2} > 1$ , the estimators  $\hat{\Omega}, \hat{\Theta}, \hat{\Gamma}^2$  defined in (21) are strongly consistent for  $\omega, \theta$  and  $\alpha^2$ , respectively.*

*Proof.* Let  $\mathbf{V}_n$  be the vector of our statistics, i.e.  $\mathbf{V}_n := (M_2, M_4, E_1)$ . Let us define

$$\mu_2 := \lim_{h \rightarrow \infty} \mathbb{E}X_{h\delta}^2, \quad \mu_4 := \lim_{h \rightarrow \infty} \frac{\mathbb{E}X_{h\delta}^4}{3}, \quad e_1 := \lim_{h \rightarrow \infty} \mathbb{E}[X_{h\delta}^2 X_{(h-1)\delta}^2].$$

For the ergodic theorem (see [3]), if  $E[X_{h\delta}^4] < \infty$ ,

$$\mathbf{V}_n \xrightarrow{a.s.} \mathbf{v} := (\mu_2, \mu_4, e_1).\tag{23}$$

Since  $f_i$  ( $i = 1, 2, 3$ ) defined in (21) are continuous functions of the parameters, we have:

$$f_i(\mathbf{V}_n) \xrightarrow{a.s.} f_i(\mathbf{v}) \quad (i = 1, 2, 3),\tag{24}$$

so the strong consistency holds. Furthermore, from (19) and (9), it's easy to prove that assuming that there exists  $\lim_{h \rightarrow \infty} \mathbb{E}[X_{h\delta}^4]$  is equivalent to assume that the ratio  $\frac{2\theta}{\alpha^2}$  is greater than 1.  $\square$

**Proposition 4.** *If we assume  $\frac{2\theta}{\alpha^2} > 3$ , the estimators  $\hat{\Omega}, \hat{\Theta}$  and  $\hat{\Gamma}^2$  are asymptotically normal:*

$$\sqrt{n}(f_i(\mathbf{V}_n) - f_i(\mathbf{v})) \xrightarrow{d} N(0, \mathbf{a}_i^T \Sigma_{\mathbf{v}} \mathbf{a}_i)\tag{25}$$

with  $\mathbf{a}_i^T = \left( \frac{\partial f_i}{\partial \mu_2}, \frac{\partial f_i}{\partial \mu_4}, \frac{\partial f_i}{\partial e_1} \right)$  ( $i = 1, 2, 3$ ).



*Proof.* The increment process  $\{X_t\}$  is geometrically  $\alpha$ -mixing ([2]). So, if  $E|X_t|^{8+\beta} < \infty$ ,  $\beta > 0$

$$\sqrt{n}(\mathbf{V}_n - \mathbf{v}) \xrightarrow{d} N(\mathbf{0}, \Sigma_{\mathbf{v}}).$$

Moreover, since  $f_i$  ( $i = 1, 2, 3$ ) have continuous partial derivatives and those derivatives are different from zero in the true parameters, we obtain (25). Furthermore, assuming that there exists finite  $E[X_{t\delta}^8]$  corresponds to ask that the ratio  $\frac{2\theta}{\alpha^2}$  is greater than 3.  $\square$

In Proposition 4 we have proved that our estimators are asymptotically normal with zero mean and with covariance matrix  $\Sigma_{\mathbf{v}}$ . In the next section we estimate the terms in the matrix  $\Sigma_{\mathbf{v}}$ .

### 3.2 Estimating the variance of the estimators

We point out that in [8] the author derived the explicit expressions for the terms in  $\Sigma_{\mathbf{v}}$  and suggested a plug-in procedure to estimate each term. Here we prefer to use a procedure that is model free in order to release from changes in the model (3). More precisely, we use a bootstrap technique for dependent data. Such kind of procedures are extensions of the classical bootstrap for independent data first proposed by Efron in [6] in order to preserve the dependence structure of the original data in the bootstrap samples. In this direction two alternative techniques are available. The first one is model based, so it is inconsistent if the model used for resampling is misspecified. Alternatively, nonparametric schemes have been proposed. In this case, blocks of consecutive observations are resampled randomly with replacement from the original time series and assembled by joining the blocks in random order so to obtain a simulated version of the original series ([12] and [15]). This approach, known as blockwise bootstrap or moving block bootstrap (MBB), generally works satisfactory and enjoys the properties of being robust against misspecified models. Moreover, the MBB does not force to select a resampling model and the only parameter to be fixed is the block length. The idea that underlies this scheme is that if blocks are long enough the original dependence will be reasonably preserved in the resampled series. The high applicability and the fact that no specific assumption is made on the structure of the data generating process motivate us to use a MBB approach.

In our case,  $\{X_t\}$  is geometrically  $\alpha$ -mixing and we assumed the existence of the  $\mathbb{E}(X_t)^{8+\beta}$  ( $\beta > 0$ ) (see Proposition 4), so we can use the results in [4]. In particular, choosing the length  $l$  of the blocks such that  $l \rightarrow \infty$  and  $\frac{l}{n} \rightarrow 0$  when  $n \rightarrow \infty$ , the resampled vector  $\mathbf{V}_{\mathbf{n}}^*$  of  $\mathbf{V}_{\mathbf{n}}$ , is such that

$$n \text{ var}^*(\mathbf{V}_{\mathbf{n}}^*) - \Sigma_{\mathbf{v}} \xrightarrow{p} \mathbf{0}$$

so MBB is weakly consistent for the estimation of  $\Sigma_{\mathbf{v}}$ .

## 4 Simulation results

In the setup simulations we choose the real values for the parameters in (3) as follows:

$$\theta = 0.6, \quad \omega = 0.5 \quad \alpha = 0.1.$$

We fix the length between the observations,  $\delta = \frac{1}{4}$  and  $\delta = \frac{1}{12}$ , which corresponds to monthly and three-monthly data respectively. The increment process  $\{X_t\}$  is generated from relation (7) and from:

$$X_{h\delta} = \sqrt{\sigma_{h\delta}}\delta Z_h \quad Z_h \stackrel{i.i.d.}{\sim} N(0, 1) \quad (h = 0, 1, \dots, n).$$

Moreover we choose  $n = \{500, 1000, 2000\}$  time series lengths and for each length we generate  $N = 3000$  Monte-Carlo runs. In Figure 1 results for the statistics  $M_2$ ,  $M_4$  and  $E_1$  are shown for  $\delta = 1/4$  (top box plots) and  $\delta = 1/12$  (bottom box plots). Straight line indicates the real value of  $\mu_2$ ,  $\mu_4$  and  $e_1$  respectively. We can observe that our statistics are consistent as proved in Proposition 3, indeed the widths of the corresponding box plots become smaller and smaller as the length of the time series increases. Moreover also the bias seems to be slight for the three statistics.

In order to evaluate the estimations of the covariance matrix  $\Sigma_{\mathbf{v}}$  through the MBB, we introduce the statistics:

$$T_n^{(1)} = \sqrt{n}M_2 \quad T_n^{(2)} = \sqrt{n}M_4 \quad T_n^{(3)} = \sqrt{n}E_1.$$

The variances of  $T_n^{(1)}$ ,  $T_n^{(2)}$  and  $T_n^{(3)}$  are independent on the time series size. So let  $v^{(j)} = \text{var} T_n^{(j)}$ ,  $j = 1, 2, 3$  be the variance of  $T_n^{(j)}$  calculated on the Monte-Carlo runs. In Tables 1, 2 and 3 the quantities

$$\begin{aligned} MEAN(j) &:= \mathbb{E}_N[\text{var}^*(T_n^{(j)})] & (j = 1, 2, 3) \\ SD(j) &:= \sqrt{\text{var}_N[\text{var}^*(T_n^{(j)})]} & (j = 1, 2, 3) \\ RMSE(j) &:= \sqrt{\mathbb{E}_N[\text{var}^*(T_n^{(j)}) - v^{(j)}]^2} & (j = 1, 2, 3) \end{aligned}$$

are shown for the statistics  $T_n^{(1)}$ ,  $T_n^{(2)}$  and  $T_n^{(3)}$  for the different choices of the length of the time series ( $n = 500, 1000, 2000$ ) and for  $\delta = 1/4$  (left) and for  $\delta = 1/12$  (right).  $MEAN(j)$  ( $j = 1, 2, 3$ ) represents the bootstrap variance of  $T_n^{(j)}$  calculated on  $N$  Monte-Carlo runs;  $SD(j)$  is the standard deviation of the bootstrap variance and  $RMSE(j)$  is its root mean square error. The square bias of the bootstrap variance is measured from the difference  $RMSE - SD$ . So, from Table 1 we can observe that the bias

of  $T_n^{(1)}$  seems to decrease as the length of the time series increases. This is more evident in the case of  $\delta = 1/4$  (left table). In the right table in which  $\delta = 1/12$  we can see that the bias is greater than the case  $\delta = 1/4$ , so the proposed estimators present an higher bias when the distance between the observations  $\delta$  becomes smaller. Indeed, when  $\delta$  goes to zero, we have a situation near the non-stationarity case, as we can see looking at the recursive relation (7). Tables 2 and 3 present a similar situation for the statistics  $T_n^{(2)}$  and  $T_n^{(3)}$ . We point out that from the estimations of  $\Sigma_{\nu}$  and from (25), we can obtain the estimations for the statistics  $\Theta$ ,  $\Omega$  and  $\Gamma^2$  defined in (21).

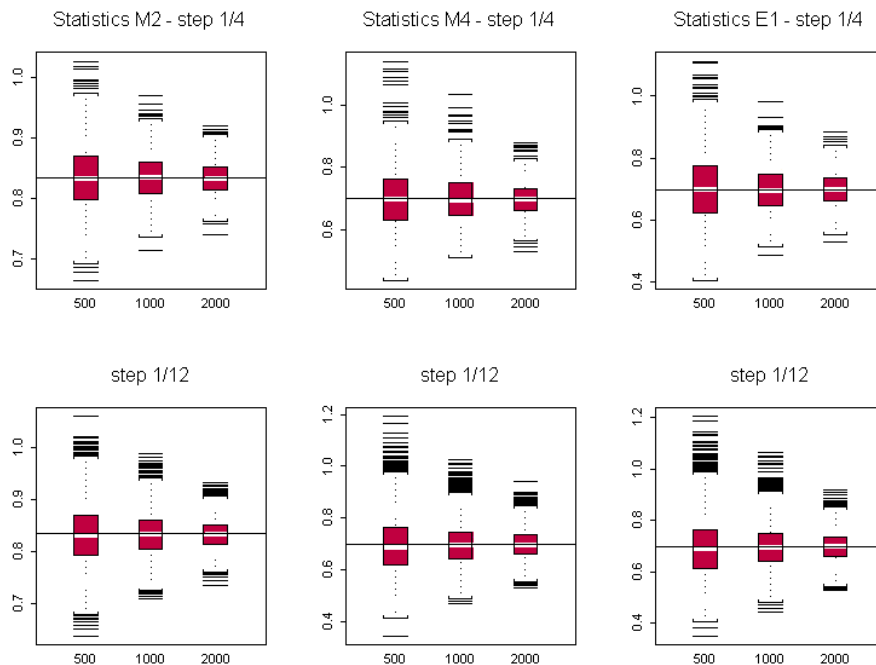


Figure 1: Box-plots for  $M_2$ ,  $M_4$  and  $E_1$  for  $\delta = 1/4$  (top) and  $\delta = 1/12$  (bottom). The straight line represents the real value of the parameter.

## 5 Concluding remark

In the present paper we proposed a method to estimate parameters in the SV model (3) based on the asymptotic unconditional moments of the volatility when the data are discretely sampled. We proved the consistency and the asymptotic normality of the proposed estimators under the hypothesis that

$n$	$\delta = 1/4$			$\delta = 1/12$		
	$Mean(1)$	$SD(1)$	$RMSE(1)$	$Mean(1)$	$SD(1)$	$RMSE(1)$
500	1.395638	0.2992639	0.3265654	1.397150	0.3000958	0.3730771
1000	1.411560	0.2337336	0.2340540	1.400773	0.2336848	0.3315045
2000	1.400609	0.1916976	0.1975044	1.407785	0.1977563	0.2603200

Table 1: Bootstrap variance of  $T_n^{(1)}$  ( $MEAN(1)$ ), its standard deviation ( $SD(1)$ ) and its square root mean square error ( $RMSE(1)$ ) for  $\delta = 1/4$  (left) and for  $\delta = 1/12$  (right).

$n$	$\delta = 1/4$			$\delta = 1/12$		
	$Mean$	$SD$	$RMSE$	$Mean$	$SD$	$RMSE$
500	5.311146	3.682014	3.688477	5.335364	3.481555	3.530922
1000	5.419772	2.662962	2.683963	5.440806	2.875277	2.936198
2000	5.376348	1.873631	1.873332	5.434032	1.949936	1.997512

Table 2: Bootstrap variance of  $T_n^{(2)}$  ( $MEAN(2)$ ), its standard deviation ( $SD(2)$ ) and its square root mean square error ( $RMSE(2)$ ) for  $\delta = 1/4$  (left) and for  $\delta = 1/12$  (right).

$n$	$\delta = 1/4$			$\delta = 1/12$		
	$Mean$	$SD$	$RMSE$	$Mean$	$SD$	$RMSE$
500	6.037674	3.464000	3.480897	6.036609	3.532099	3.582262
1000	5.985166	2.370325	2.426217	6.218848	2.995292	3.049426
2000	6.029416	1.852414	1.852310	6.168984	1.961602	1.974000

Table 3: Bootstrap variance of  $T_n^{(3)}$  ( $MEAN(3)$ ), its standard deviation ( $SD(3)$ ) and its square root mean square error ( $RMSE(3)$ ) for  $\delta = 1/4$  (left) and for  $\delta = 1/12$  (right).

there exist the eight-moments of the observed process  $\{Y_t\}$ . The asymptotic variance of the estimators was also computed using a MBB approach. This approach permits to release from changes in the parameters of the model. The work opens the way to developments in the estimation in the GARCH models exploiting the relations between those models and their continuous limits.

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