Università degli Studi di Salerno
DIPARTIMENTO DI SCIENZE ECONOMICHE E STATISTICHE

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SUBJECTIVE EXPECTATIONS IN ECONOMICS: A STATISTICAL OVERVIEW OF THE MAIN FINDINGS[3]

WORKING PAPER 3.158
Febbraio 2005

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Abstract

In this writing we provide a brief overview on how in different fields such as statistics, econometrics and experimental psychology, the issue of measuring subjective expectations about future uncertain outcomes has been attacked. In many situations realized observed data come from a decision process in which the decision is made by considering future uncertain outcomes, so that the observed data will depend on future outcome via the probabilistic judgement made by the decision maker. For example the decision to buy a car today will depend on our expectations about future incomes, so that observed data about the number of cars sold will depend on peoples' expectations about their future incomes. Expectations are not observable so that we need proper statistical methods of inference to treat these type of problems. Here, we provide a very general overview, and we try to summarize different approaches.

Keywords

Subjective Expectations, Probability Elicitation, Rational Expectations, Prior Specification.

1 Introduction

One of the key assumption in economics theory is the so called “rational expectation hypothesis”. In few words, rational expectation hypothesis assumes that peoples’ expectations about future events are “consistent” and “efficient”; where consistent means they do not make systematic errors, and efficient means that they use all the available information. Moreover, this assumption implicitly assumes that people are “utility maximizer”, this means that people choose their actions maximizing their utility according with some utility representation (e.g. expected utility, max-min expected utility etc.).

From a statistical point of view, this hypothesis is interesting since it represents the theoretical ground for the derivation of a huge number of econometric models. Assuming rational expectations makes the life of Economists easier but on the other hand this force them to make many unrealistic behavioural assumptions, and as a consequence the resulting inference leads to unclear conclusions. Also, it is interesting how econometricians usually avoid to relate the problem of expectations formation to that of prediction, where from my point of view expectations follow prediction.

My plan is to investigate why we need to relax this assumption and how it might be possible to do it without strong restrictions. One of the main proposal in econometrics has been to use survey data on expectations, but there is still a controversy about the consistency of these data with observed behaviour. We think that in order to persuade economists of the importance of eliciting expectations it is very important to test — possibly through experimental sessions — whether peoples' behaviour is consistent with elicited expectations.

Few papers appeared in international economic review use subjective data on expectations elicited via survey questionnaires, but there has been no effort in developing statistical tools to recover expectations. The other issue, that seems very important to me, is to explore the statistical tools we need to elicit people expectations and to understand their learning process and their reaction to new information.

2 *The rational expectation hypothesis as a prior specification hypothesis*

Econometric models predict choice behaviour from data on observed choices, for many years this has been the standard practice. The theoretical tool underlining this approach is the so called "principle of revealed preferences". It was introduced by Samuelson (1938, 1948) who showed that observed choices combined with the assumptions of classical consumer theory implies restrictions on the consumption bundles that a person would choose when she faces her budget constraints. This idea justified the use of observed choice data to infer the underlying decision process. A more practical form of the principle referred as "revealed stochastic preference model" was developed later by McFadden (1974), who tried to answer to the question whether distributions of choices observed from a random sample of individuals in a variety of choice situations, is consistent with rational choice theory which postulates that individuals maximize preferences. McFadden (1974) assumed that a researcher observes a random sample of person with heterogenous preferences each one facing a decision problem, then he showed that data combined with assumptions on population distribution of preferences enable to estimate probabilistic choice model and predict choice behaviour[1].

[1]McFadden (1974) takes into consideration only discrete choice problems with a finite set of outcomes. A complete description of further generalizations to possibly infinite set of

However this approach fails to be reasonable when the decision maker does not know the outcome of her or his decision; in economic theory this situation is referred as “partial information”. For the most part of interesting economics problems, people face partial information, and in practice Economists assume that people have consistent subjective probability distributions on unknown quantities and then they choose maximising their expected utility given their preferences over the unknown outcomes. Again the problem with this approach is that observed data can be consistent with a number of different specification of preferences and subjective probability. In order to get identification Economists assume that decision makers have specific expectations objectively correct, i.e they hold rational expectations. Rational expectations and revealed preferences are what an economist needs to model observed choice data alone.

In order to better understand the consequences of this hypothesis we will introduce some formalism and a simple well known economic models, the so called “permanent income model”.

We can describe a general decision problem as follows. Suppose that X is choice set, i.e. a set of possible actions or quantities that an economic agent will choose. For instance X could be all the possible quantities of a goods a person could buy. Ω is the set of all possible state of nature relevant for the decision, $\omega^* \in \Omega$ is the true state of nature. In the example before Ω is the set of all possible value of future income, and ω^* is the true income a decision maker will face. Each agent have an objective real valued function that values her or his decision $u : X \times \Omega \longrightarrow \mathbb{R}$. The problem of agent is take an optimal decision x^* such that

$$x^* = \operatorname*{argmax}_{x \in X} u(x, \omega^*). \tag{2.1}$$

If ω^* is unknown then problem (2.1) can be solved if and only if there exists a $x^* \in X$ such that for all $(x, \omega) \in X \times \Omega$, $u(x^*, \omega) \geq u(x, \omega)$. Since this is only a special case, Economists need to make some restrictions in terms of behavioural assumptions. Instead of solving problem (2.1), it is assumed

outcomes and to continues choice problem can be found in McFadden (2004) that people solve

$$x^* = \underset{x \in X}{\operatorname{argmax}} \int u(x, \omega) dP, \tag{2.2}$$

where P denotes a subjective probability distribution over Ω. The justification of (2.2) is Savage (1954) axiomatization of subjective expected utility. Since P is subjective then Economist need to restrict the set of all probability distributions over Ω assuming rational expectations, i.e. they assume that decision makers know the objective probability distribution that generates ω^*.

Moreover Economist assume much more than rational expectations. It is also assumed that people use all the available information in order to form rational expectations. If Σ is the set of all available information, it is usually assumed that everyone can access to Σ and thus everyone hold correct probability distribution over ω^* conditional on Σ, in other words the problem that everyone face is to find

$$x^* = \underset{x \in X}{\operatorname{argmax}} \, \mathsf{E}_P[u(x, \omega) | \Sigma], \tag{2.3}$$

where E_P is the expectation taken with the respect to P. Assuming rational expectation along with the hypothesis of stochastic revealed preferences ensure that choice observed data are sufficient to identify a choice model based on maximising expected utility behaviour.

It is obvious that the hypothesis above is just an assumption which states that a rational agent holds a prior beliefs of the unknown P which is consistent with realized data. That is the same as saying that realized past data provide the economic decision maker with an empirical evidence sufficient to specify a coherent empirical prior on the unknown. This is the reason why we can look at the rational expectation hypothesis as a way of imposing an objective empirical[2] prior on the unknown. Though this is a subtle aspect, it is important to notice that this much more that imposing a prior distribution for an unknown quantities. In fact, a prior is never required to be the true

[2]Here by empirical we mean that this prior is derived on the basis of past data.

distribution which moves observables, but in this case it is imposed that P is the true distribution of the unknown so that our rational decision maker will not do any systematic error in predicting future uncertain outcomes.

3 From the theory to the inference: the example of the consumption model

3.1 Theory

Now we want to illustrate how all these concepts are usually embodied in the economics literature, from theory to econometric practices. We are going to present one of the main tool used by economists to analyse and predict future aggregated consumption levels. In the following exposition we will skip many mathematical details because our main object is not to understand techniques; for further details see Lucas and Stockey (1989). In this section we do not discuss the economic contents and the coherence of all those assumptions we are going to make. Many of the following assumptions have no interesting economics content, but we need them in order to achieve mathematical tractability. In the next section, instead, we shall focus on those restrictive assumptions that we need in order to make inference on the model, and we will see how these assumptions are restrictive.

First of all we fix some notation. There is a representative economic agent (a consumer) who live from time $t = 0, 1, \ldots, T$, and in each period he or she faces a budget constraint and has to choose a level of consumption c_t. The time horizon our consumer will face it is assumed to be infinite since she or he incorporates in her or his utility the choice of all the next generations. So that we let $T \to +\infty$. In each period t the consumer has a level of wealth A_t, and perceive an income y_t. She or he can save and invest money at rate r_t, for simplicity we assume that the rate of interest is known for each t and it is constant[3]. We also suppose that $A_0 \geq 0$ and $A_{T+1} \geq 0$. The last two assumptions imply that the consumer is not allowed to have debts at the beginning and at the end of his or her life. The

[3]We could drop this simplifying assumption making the mathematics harder.

consumer can invest his or her wealth at rate r, then in each period t she or he faces the following budget constraint: $A_{t+1} \leq (1+r)(A_t + y_t - c_t)$. Furthermore we assume that there exists a law of motion (or transition equation) for the income y_t, that is, there exists a continuous function $f : \mathbb{R} \longrightarrow \mathbb{R}$, which relates future income to present income through a sequence of i.i.d. random variables $\{\epsilon_t\}_{t=0}^{\infty}$ and past values of income and wealth. We express future income as $y_{t+1} = f(y_t, A_t, \epsilon_t)$, with y_0 and ϵ_0 given. The sequence $\{\epsilon_t\}_{t=0}^{\infty}$ is described by a probability distribution function Q, that is $Q(\epsilon|y,a) = \text{Prob}[e_t \leq \epsilon | y_t = y, A_t = A]$. We need also to suppose that f is measurable with the respect of Q and it satisfies the Feller property[4]. It is important to notice that all the variables involved in this model, except for ϵ, are observable at the beginning of each period t, so ϵ_t is not observable at time t. According with the foundations of economic theory, each consumer is utility-maximizer, i.e. her or his preferences are described by a continuous real valued utility function $u(c_t)$, and the consumer wants to maximize her or his utility given the budget constraint. For simplicity we assume that $u(\cdot)$ is time invariant. Also, we assume that $u(\cdot)$ is strictly concave (meaning strictly decreasing marginal utility), bounded, continuous with continuous first derivatives. In synthesis the problem of each agent for each $t = 0, 1, \ldots, T$ is

$$\max_{\{c_t\}_{t=0}^{T}} \mathsf{E}_Q \left[\sum_{t=0}^{T} \frac{1}{(1+\rho)^t} u(c_t) \middle| \Omega_t \right], \tag{C}$$

$$\text{s.t.} \qquad A_{t+1} \leq (1+r)(A_t + y_t - c_t), \tag{3.1}$$

$$A_0, y_0, \epsilon_0 \quad \text{given}, \tag{3.2}$$

$$y_{t+1} = f(y_t, A_t, \epsilon_t); \tag{3.3}$$

where $(1+\rho)$ is a subjective discount factor for future utility assumed to be constant over time with $\rho \in (0,1)$, and Ω_t is the information set at time t,

[4]This means that for every bounded and continuous real valued function ψ, $\mathsf{E}\left[\psi(y_{t+1}) | y_t = y, A_t = A\right]$ is bounded and continuous in all its arguments. We need this property in order to guarantee the solution of the functional equation (SC) that we are going to see.

it contains all the past and current values about consumption, income and wealth and past values of shocks ϵ. Notice that it is assumed that ρ and Ω_t are set to be equal across individuals for each t. At the end of this economy there is no market to invest in, so it is optimal that $A_{T+1} = 0$, this imply that, going backward up to time 0, the constraint (3.1) can be taken with equality. The problem above is to find a sequence $\{c_t^*\}_{t=0}^{+\infty}$ such that the program (C) is solved; under these assumptions we could easily prove that such solution exists and it is unique. In order to solve this problem we need some more mathematical tool.

Under our assumptions it is possible to show (see Lucas and Stockey, 1989) that our stationary time invariant dynamic programming problem (C) can be equivalently translated into a biperiodal sequential problem using the following Belman equation

$$V(A_t) = \max_{\{A_{t+1}\}_{t=0}^{T}} \left\{ u(c_t(A_{t+1})) + \frac{1}{1+\rho} \mathsf{E}_Q \left[V(A_{t+1}) \middle| \Omega_t \right] \right\}, \quad \text{(SC)}$$

$$\text{s.t.} \quad A_{t+1} = (1+r)(A_t + y_t - c_t), \quad (3.4)$$

$$A_0, y_0, \epsilon_0 \quad \text{given}, \quad (3.5)$$

$$y_{t+1} = f(y_t, A_t, \epsilon_t); \quad (3.6)$$

where $V(\cdot)$ is the value function associated with the program (C). We fixed A_{t+1} as control, A_t as the state, and we got rid of c_t which we can now express as linear function of A_{t+1}. We start programming in time 0, A_0 is given and once the optimal level of A_1 has been chosen, then using equation (3.4) we have c_1^*, so for a general period t. Since the Belmann equation is a contraction, a solution exists and can be calculated using analytic techniques or numerical methods. Note that this is a strictly concave dynamic programming problem, and it can be shown that it has inner solutions. Also the first order condition of the functional equation (SC) are necessary and sufficient for a maximum. Taking the first order conditions for (SC) we have

$$\frac{\partial u(c_t(A_{t+1}))}{\partial c_t} \frac{dc_t(A_{t+1})}{dA_{t+1}} + \frac{1}{1+\rho} \mathsf{E}_Q \left[\frac{\partial V(A_{t+1})}{\partial A_{t+1}} \middle| \Omega_t \right] = 0. \quad (3.7)$$

Let $f'(x) = df(x)/dx$, the previous becomes

$$u'(c_t)\left(-\frac{1}{1+r}\right) + \frac{1}{1+\rho}\mathsf{E}_Q\left[V'(A_{t+1})|\Omega_t\right] = 0; \qquad (3.8)$$

where

$$c_t = A_t + y_t - \frac{A_{t+1}}{1+r}.$$

Note that we will write c_t instead of expression above in order to simplify the notation. By envelope theorem[5] $V'(A_t^*) = u'(c_t^*)$, where the symbol "$*$" means optimal solution to (SC) and as a consequence a solution to equation (3.8). Also by stationarity $V'(A_{t+1}^*) = u'(c_{t+1}^*)$, and thus we can rewrite (3.7) as

$$\frac{u'(c_t^*)}{\mathsf{E}_Q\left[u'(c_{t+1}^*)|\Omega_t\right]} = \frac{1+r}{1+\rho}. \qquad (3.9)$$

The last equation is the so called "Euler equation", it relates optimal current marginal utility with next period expected marginal utility, interest rate and the intertemporal subjective discount rate. Note that at time t, all the information about the past and the present income, wealth, consumption and realized value of innovations ϵ, is in the set Ω_t[6], so that $\mathsf{E}_Q\left[u'(c_t)|\Omega_t\right] = u'(c_t)$. Given an utility function, equation (3.9) becomes a second order difference equation which, together with conditions (3.2) and (3.3), give us a solution which will represent the law of motion of optimal consumption. The economic contents of equation (3.9) is appealing, but is far from our scope.

3.2 Inference

In this section we shall see how the rational expectations hypothesis is important in order to make inference on the model described above without using subjective data.

In this simplified model an economist relates optimal consumption decisions to income and wealth. The only source of uncertainty is the i.i.d.

[5] See any intermediate or advanced book in mathematical programming.

[6] In this modelling, these innovations are identifiable since in each time a consumer has all the equations he or she needs to compute the past error terms.

sequence of innovations which affect the future income. The dynamic of income affects the budget constraint equation (3.4), and as a consequence, the optimal decision about how much to consume in each period. Note that the dynamic of wealth is not important; in fact given A_0, once we know how the income determines the optimal consumption level we get A_1, then the same apply for A_2 and so on, and the whole path of wealth accumulation is recovered. So what really matters for our consumer is the income dynamic. However an econometrician has data about past consumption choices, wealth and income, but he knows anything about the sequence of $\{\epsilon_t\}_{t=0}^{T}$. How shall he or she procede in studying the relationship between consumption choices and income dynamics? Let us see how an econometrician who believes in the rational expectations hypothesis would tackle the problem.

First of all we need some functional form for the utility function. Given choice data on consumption, the revealed stochastic preference theory will allow an econometrician to restrict the family of admissible utility functions. Without enter the theoretical details, we can use some parametric family of utility functions, so that shape will depend on some vector of parameters. Also, we want some utility function which will provide us with a linear marginal utility. We take a quadratic utility function $u(c_t) = ac_t - (b/2)c_t^2$ with marginal utility $u'(c_t) = a - bc_t$. By the linearity of expectation operator, this family of utility functions allows us to write $\mathsf{E}_Q\left[u'(c_t)|\Omega_t\right] = u'(\mathsf{E}_Q[c_t|\Omega_t])$. Applying the Euler equation (3.9) we have

$$\mathsf{E}_Q\left[c_{t+1}^{*}|\Omega_t\right] = \frac{a}{b}\frac{r-\rho}{1+r} + \frac{1+\rho}{1+r}c_t^{*}; \tag{3.10}$$

this equation describes the law of motion of optimal consumption. Up to this point we still do not have the income and shocks in our equations, and also on the left hand side of (3.10) we have a conditional moment.

At this point an economist would introduce the rational expectation hypothesis: a “rational economic agent”[7] uses all the available information, he

[7] Here the wording comes from the standard economic literature, but this does not imply that we accept this simple idea of rationality.

perfectly knows the probabilistic structure of the data generating process and he does not make any systematic forecast error. Translating these words in our example this means that: (i) Ω_t is equal for each consumer and contains everything which is useful to predict y_{t+1}; (ii) our economic agent knows the probabilistic structure of the sequence of innovations $\{\epsilon_t\}_{t=0}^{T}$, i.e. he knows Q and hence the data generating process of future income; (iii) there is no systematic error in predictions. This last statement is formalized introducing an error term $\eta_{t+1} = c^*_{t+1} - \mathsf{E}_Q\left[c^*_{t+1}|\Omega_t\right]$ with the property that $\mathsf{E}\left[\eta_{t+1}|\Omega_t\right] = 0$[8].

Notice that all the randomness of the model comes from the sequence of innovations ϵ_t through the probability distribution function Q, so the error term η depends on ϵ. Let us introduce a real valued function g which is measurable with respect to Q, in the econometric literature is supposed that $\eta_{t+1} = g(\epsilon_{t+1}) = c^*_{t+1} - \mathsf{E}_Q\left[c^*_{t+1}|\Omega_t\right], t = 0, 1, \ldots, T$. The term η_t is the forecast error at time t, and it is a function of the random term of the model. By the previous conclusion, the rational expectation hypothesis implies that $E_Q\left[g(\epsilon_{t+1})|\Omega_t\right] = 0$ for every $t = 0, 1, \ldots, T$.
Introducing this hypothesis allows our econometrician to transform (3.10) into

$$c^*_{t+1} = \frac{a}{b}\frac{r-\rho}{1+r} + \frac{1+\rho}{1+r}c^*_t + \eta_{t+1}. \tag{3.11}$$

We want to simplify our algebra as much as we can, thus without loss of generality we assume that $r = \rho$, and hence (3.11) becomes a random walk

$$c^*_{t+1} = c^*_t + \eta_{t+1}. \tag{3.12}$$

To link optimal consumption to incomes we now need the budget constraint (3.4). We want to base our analysis to a generic period t, however we wrote the budget constraint equation based on $t = 0$. Let us rewrite it as $A_{t+s+1} = (1+r)(A_{t+s} + y_{t+s} - c^*_{t+s})$ with $s = 0, 1, \ldots, T$. Also, we want get rid of A_{t+s+1}. From the budget constraint equation, taking expectations

[8]This is not a property that depends on some reason related to the underlaying probability distribution, this equality is indeed assumed to be valid if rational expextations hold.

and substituting recursively A_{t+s+1} into A_{t+s+2} and so on, we have

$$\sum_{s=0}^{T-t} \frac{\mathsf{E}_Q\left[c^*_{t+s}|\Omega_t\right]}{(1+r)^s} = A_t + \sum_{s=0}^{T-t} \frac{\mathsf{E}_Q\left[y_{t+s}|\Omega_t\right]}{(1+r)^s}. \tag{3.13}$$

Now we substitute (3.12) into the previous and we obtain

$$c^*_t \sum_{s=0}^{T-t} \frac{1}{(1+r)^s} = A_t + \sum_{s=0}^{T-t} \frac{\mathsf{E}_Q\left[y_{t+s}|\Omega_t\right]}{(1+r)^s}. \tag{3.14}$$

Since an interest rate is contained in the interval $(0,1)$

$$\sum_{s=0}^{T-t} \frac{1}{(1+r)^s} = \frac{1+r}{r}\left[1 - \frac{1}{(1+r)^{T-t+1}}\right], \tag{3.15}$$

it follows that (3.14) becomes

$$c^*_t = \frac{r}{1+r}\left[1 - \frac{1}{(1+r)^{T-t+1}}\right]^{-1}\left[A_t + \sum_{s=0}^{T-t} \frac{\mathsf{E}_Q\left[y_{t+s}|\Omega_t\right]}{(1+r)^s}\right]. \tag{3.16}$$

Since we assumed that $T \longrightarrow +\infty$, (3.16) can be written as

$$c^*_t = \frac{r}{1+r}\left[A_t + \sum_{s=0}^{+\infty} \frac{\mathsf{E}_Q\left[y_{t+s}|\Omega_t\right]}{(1+r)^s}\right]. \tag{3.17}$$

The last equation says that the actual optimal consumption depends on the actual wealth plus the sum of all the future expected incomes. Setting

$$y^p_t = \left[A_t + \sum_{s=0}^{+\infty} \frac{\mathsf{E}_Q\left[y_{t+s}|\Omega_t\right]}{(1+r)^s}\right]. \tag{3.18}$$

we have what in economics is called “permanent income” (for details see Friedman, 1957; Hall, 1978). According with this model a consumer bases his or her decisions about consumption on expected level of future incomes and the actual wealth.

To test this model, or to use this model to predict aggregated consumption levels there is still a problem: the econometrician does not dispose of data about expected incomes. The inferential problem of the econometrician will face is the same of the one faced by the consumer, he has to infer the data generating process of incomes. Again, by the rational expectation hypothesis, the solution is that the econometrician has in mind the same data generating process that a representative “rational” agent will have, i.e. econometrician and agents both have the same empirical and objective prior about the stochastic structure that generates incomes. He or she will substitute this model of future income in (3.17) computing all the conditional moments. For example an econometrician could suppose that $y_t \sim$ AR(1) with innovations ϵ_t. With $y_{t+s} = \phi y_{t+s-1} + \epsilon_{t+s}$ we have $\mathrm{E}_Q\left(y_{t+s}|\Omega_t\right) = \phi^s y_t$, thus (3.17) becomes

$$c_t^* = \frac{r}{1+r}A_t + \frac{r}{1+r-\phi}y_t. \tag{3.19}$$

Having data on consumption level chosen and actual income, now the econometrician can estimate and make inference on the model. For instance a model very similar to the one we presented can be find in Campbell and Deaton (1989). The authors estimated the model of the labour income dynamics for the United Stated and they found that it followed an AR(2) process

$$y_t = \mu + (1+\psi)y_{t-1} - \psi y_{t-2} + \epsilon_t,$$

with $\psi > 0$. But this led to a famous inconsistency known as the “excess smoothness paradox”. Using the AR(2) process with $\psi > 0$ they showed that the the theoretical model for the consumption predicts that the variability of $\Delta c_t^* = c_{t+1}^* - c_t^*$ is greater than the variability of $\Delta y_t = y_{t+1} - y_t$, however the empirical evidence suggests the opposite.

3.3 Critiques

In the previous section we have illustrated a basic model of consumer behaviour. This model with its simplicity is not far from what economist usually do in practice. We could drop some simplifications (e.g. we could suppose that the interest rate is not constant over time and that it is different from the subjective discount factor, also we could introduce some more sophisticated utility function, and so on), however these simplifications would not change the main result of the analysis above, the only thing it will change is in a complication of the mathematics. Hence the critiques we shall argue thereafter are not narrowed by these simplifications. It is important to make clear that the kind of arguments we are going to make are general and they do not depend on the peculiar simplifications made in order to simplify the algebra.

We can see the "rationality" assumption as composed by three components: preference-maximizing behaviour, efficient use of information, and perfect ability in recovering data generating processes. We are going to discuss each of them separately.

Preference-maximizing behaviour. The first thing we want to point out is that the general idea of preference-maximizing behaviour, even if it is compelling on the theoretical ground, it is hard to apply in the empirical practice. The decision depend on the subjective utility function together with the subjective idea of the process that generates the data, but we can never identify these two contributes separately. In other words, the decision about the sequence $\{c_t^*\}_{t=0}^T$ comes from a maximization program based on a utility function u and the probability distribution Q about future income levels, now the problem is that the consumption profile we observe form the data could be compatible with different combinations of u and Q. The big issue with this kind of modelling is the lack of observability of hypothetical[9] outcomes, without these, it is truly hard to think about the contemporaneous identification of both u and Q based on past data on c and y.
This argument is more general than the one we want discuss on in this work. It is related with old debate about economics modelling and the related "structural estimation". The idea of the supporter of this view is that we

[9]Notice that econometricians use to refer to "counterfactuals" in many distinct situations, rather than distinguishing between hypothetical knowledge and counterfactual reasoning.

cannot estimate a model based on data without having a “rational theory”[10] about the structure of the economics. Even if we agree that in order to estimate an equation we need some theoretical background which justify it, the position of this “structuralist party” is hard to understand when one tries to introduce some view of rationality. Rationality is a strong and hard concept, and we believe that economists sometimes abuse of it.

Efficient use of information. The other critiques we want to make is about the concept of efficiency. Economists usually assume that people use information efficiently meaning that they condition on all those variables that are needed to predict the desired one. In our example this means that Ω_t contains all those information we need in order to condition our expectation about y and therefor predict optimally next periods' income. This hypothesis is a bit vague, what the structure of Ω_t is? Not only, but also the notion of rationality that economists usually introduce implies that Ω_t is equal for all individuals. In their view if one is rational then he or she knows what the relevant variables needed to predict future income levels. In the previous example we solved the problem assuming that y_t followed an AR(1) process, so this means that in each period a consumer conditioned on past income realization in order to predict next period income.
Following this argument, every econometrician who would test the model above, he should construct the model conditioning on the same information set as an economic agent will do. In fact, an econometrician who has to make empirical analysis starting from data, faces the same inferential problems as the decision maker: he has to make induction from a finite sample. Now if this was the case, every econometrician testing the same model should estimate the same equation using the same variables, but if look at the empirical literature about consumer theory this clearly not the case.

Perfect ability in recovering data generating processes. The assumption that agents are able to infer the true data generating process is too demanding. In the example of consumer theory we saw that an econometrician who bases her or his analysis on choice data alone needs to as-

[10] Again, we would stress that the wording comes from the standard economic literature.

sume that people, by rational expectations hypothesis, have in mind the true probability distribution Q of future income levels. Remaining in the example of consumer theory, empirical economists always disagree on the "true generating process" of income, so that it appears hopelessly optimistic to suppose that expectations are rational, even approximately.
The main problem is that the resulting inference is indeterminate. For instance in Campbell and Deaton (1989) they found that the model presented[11] leads to some anomalous result (e.g. the excess smoothness paradox), now what would the conclusion be? Is the preference-maximizing behaviour wrong? Is the the supposed data generating process for income misleading? Is the peculiar family of utility functions not representative of consumer behaviour? The answers is that we cannot make conclusions, since the number of restricting assumptions we are making is huge, and the effect of this assumptions can be strong.

Even though each of these three components leads to strong restrictions on the economic behaviour, the first one is not intimately connected with the rational expectation hypothesis as the followings. In particular, in our view, the idea that people maximize their expected utility is not realistic, and in fact there exists an huge experimental literature which shows that the expected utility representation of preferences over uncertain outcomes is usually violated. But we do not want to tackle this issue here. The first component, as long as poses strong and controversial restrictions, is much more related to the general decision theory and marginally affects the statistical practice. In this work we shall try to answer to the question whether is possible to drop the rational expectations assumption in order to get less restricted econometric models, and so our research shall focus on those statistical instruments convenient to drop those assumptions related to the second and the third component. As we will see, we think that these issues ar strongly related to peoples' predictive abilities and we will try to find answers to our question starting to think about the predictive tasks one is required to complete when she or he has to think about expectations.

[11] They tested a model with a slightly different setup.

4 Subjective expectations: what has been done

4.1 The need of subjective expectations data

On the theoretical ground economics modelling is always about choices under uncertainty, so that the empirical testing will always deal with estimating equations in which some expectation will appear. We took the example of the consumption model, as our leading example for our analysis, however we want to stress that this problematic is more general.

Choice data are insufficient to estimate models involving expected variables, the only way to fully identify these models is to introduce strong and unrealistic assumptions as we have seen so far, what we want to conclude is that we should try to make any effort in order to combine choice data with data on expectations. For instance, if in the consumption model we could dispose of data about consumers' expectations on future income levels, a structural model about income dynamics and the hypothesis that people have in mind the same model of the economy we have, it would be unneeded; and also we did not need to make the assumption that peoples use information efficiently. All this "restriction saving" would improve the coherence of our modelling, making our inference less indeterminate.

These needs has emerged during the past few years, and few papers on the topic appeared in international economic reviews, this means that the idea of using subjective data, i.e. self-reported data about expectations of future events, still rises doubts amongst economists. They have been skeptical about subjective statements, and usually, they justify their skepticism arguing that "one should believe what people do, not what they say". This is a standard objection which an economist would make. The fact that people could be not able to fully report probability statements is widely investigated in experimental psychology and other related disciplines, however from what we will see later on this is not the case. A good overview history of the debate about the use of subjective data can be found in Dominitz and Manski (1997), and Manski (2004).

However, we think that not many efforts have been done in econometrics in developing the tools needed to elicit expectations, and moreover the actual econometric literature does not pose the issue of expectations in the wider contest of the theory of prediction. We think that the two thinks are intimately related.

With this work we have a plan: we want to assess whether the self-reported probability statements about future scenarios are consistent with observed choices and in this case "what people have in mind should really matter", we want to find optimal tools in order to elicit expectations, also we want to develop a set of statistical tools to predict future behaviour based on both choice data and data about expectations, understanding how people react to new information acquisition.

4.2 Elicitation of subjective expectations

Suppose we want to estimate an equation in which we have, amongst the regressors, next period expected prices' growth, i.e. expected inflation rate about next period. Without making unrealistic assumptions, we need data on expectations. If π_t is the inflation rate that we observe at time t, Ω_t is the information set at time t containing past and current data about inflation, our problem is to collect data on $\mathsf{E}\left[\pi_{t+1}|\Omega_t\right] = \pi_t^e$. We could collect these data about π_t^e in two different ways: (i) at time t we can ask to people to attach probabilities about π_{t+1} and try to recover the probability distribution of $\pi_{t+1}|\Omega_t$ people have in mind, and then we can take the expectation of this π_t^e; (ii) more simply, if $i = 1, 2, \ldots, n$ are subjects interviewed, we could ask to people what is their $\pi_{i,t}^e$, where $\pi_{i,t}^e$ is the expectation reported by individual i at time t about inflation in time $t + 1$. Each one of the two approaches has advantages and limitations, in this section we will examine what is the state of the art of the literature about expectations' elicitation.
It is important to notice that the problem we are analyzing touches a number of grounds. It is strongly related with the psychology involved in the elicitation process, from the statistical point of view it involves a number of issues: the theory of prediction, how to treat this kind of data, how to evaluate the accuracy the expectations recovered, how to predict peoples' reaction to new information, etc.

4.3 Psychological issues: four heuristics

An elicitation methods is the statistical translation of people believes about future events, so that we cannot set up a proper method apart from the psychological research about men's abilities to make probabilistic judgements[12].

Psychological research has been concerned with the problem of how a person quantify the probability of an event, or how he or her judges which event is the more likely to occur. We want to stress that whenever we use the word "probability", we have to be very careful. Probability is a sort of primitive concept in our culture but the content of this concept could be different among individuals, so that when someone ask to us "what is, in your opinion the probability that X occurs?" It's very hard to attach a sort of standard meaning to that answer. One of the direction of psychological research has been devoted to address the question whether people attach to the word probability some common significance.

From experimental psychology literature, it appears that intuitive judgements about probability are based on a limited number of mental operations. The wording used by psychologists to describe this operations is "heuristics". These heuristics are quite effective, but they also lead to errors and systematic bias. We will briefly review some of the most important findings about these .

Heuristic 1: judgement by representativeness.

Suppose we ask what is the probability that an object X belongs to a category Y? Or, what is the probability that Y follows X? The answer requires to evaluate Prob $(X|Y)$ to be assessed, people typically compare the main features of X and Y and assign a probability depending on the degree of similarity between them. A common error made with this kind of judgement is that little or no attention is paid to the unconditional probability of

[12] The most part of this section is inspired to the presentation "Judgement and Forecasting" held by Prof. Nigel Harvey in the seminar series on prediction organized by the Department of Statistical Sciences at University College London (UK)

Y. A complete description of this is given in Kahneman and Tversky (1973). Similar results have been obtained by Hammerton (1975) and Nisbett et al. (1976).

Heuristic 2: judgement by anchoring and adjustment.

A person estimates an unknown quantity by starting from some initial value and then adjusting it up to obtain a final estimate. The starting value is usually called "the anchor". One finding by Slovic (1972) is that is that regardless of the source of the starting value, the adjustment is usually too small, this phenomenon is called anchoring. Kahneman and Tversky (1973) run an experiment in which individuals were asked to estimate various quantities, stated in percentages, they were given randomly chosen starting values and first they were asked whether the value they had been given was too high or too low, and then to adjust it until they reached their best estimate. Subjects whose starting values were high ended up with substantially higher estimates than those who started with low values. In this case the Bayes theorem is violated. When subjects use the heuristic judgement-by-representativeness to quantify probabilities, they tend to ignore their priors. Also other experiments have shown that if subjects are made aware of a prior probabilities and are asked to modify these as new data are available, then the assessed posterior probabilities are too close to the prior probabilities, compared with the revision indicated by Bayes theorem. This effect is referred as conservatism (see Edwards and Phillips, 1964). Several experiments have been made to verify the violation of the Bayes theorem changing the structure of the experiment, but it seems that conservatism is a recurrent effect.

Tversky and Kahneman (1971) demonstrated that, in many circumstances, people expect a sample from a population to represent all the essential characteristics of that population, even if the sample is small. They refer to this phenomena as the "law of small numbers", since it seems that people believe that the law of large numbers applies to small numbers as well. The error is readily attributable to belief in the law of small numbers; people expect all samples to have virtually identical characteristics so that they do not updated their estimated probability.

Heuristic 3: judgement by availability.

A person estimates the frequency or the probability of an event by the ease with which examples come to his or her mind. A classic example: suppose you are asked whether a randomly chosen word from an English text is more likely to start with an "r", or have an "r" as its third letter. In this case it is easier to recall words by their starting letter (e.g. r-ubber, r-ank, r-ed, etc.) than by their third letter (e.g. pa-r-ty, wi-r-e, etc.). In a number of experiment, most people judge that "r" is more likely to be the first letter of a word rather than the third one. If we take an English dictionary and we calculate the frequencies of "r" being the first and the third letter of word we discover that it is more likely to have an "r" as a third letter as found in Kahneman and Tversky (1973). Furthermore familiarity and newsworthy events also impact disproportionately on our memory, so we might overestimate the probability of a plane crash with fatalities, for example, particularly if such a crash has happened recently (see Kahneman and Tversky, 1973).

Heuristic 4: hindsight bias.

Knowledge of what has occurred tends to distort memory and people tend to exaggerate their a priori probability for an event that has occurred. Fischhoff and Beyth (1975) conducted an experiment during President Nixon's visit to China and the USSR in 1972, subjects were asked to assess the probabilities of various possible outcomes of his visit, such as "President Nixon will meet Mao at least once?" "The USA and the USSR will agree to a joint space program?". After the visit the subjects were asked to recall the probabilities they had given before Nixon's visit, than they were asked which events they thought had actually occurred. The result of the experiment was that: (i) subjects generally overestimated their a priori probabilities for the events they thought had occurred (ii) they underestimated the priori probabilities for the events they thought had not occurred.

Here we have described only some of the heuristics well known in the psychologic literature, at least we cited those related with the main issues we aim to analyze in this work. An extensive review of the findings can be found in Hogarth (1987).

4.4 The psychology of summaries elicitations

In this section we want to give a brief overview of the main findings about peoples' ability to elicit summary statistics. In general elicitation of summary statistics from multivariate distribution is more complicated if compared with the univariate case. In our work we will not need to elicit expectations about multiple variables so that this brief review is confined to the univariate case. For a complete review about this topic see Garthwaite et al. (2004).

The literature on this topic is vaste and it comes from the 60's to 80's. Here we are interested to evaluate abilities to summarize statistical quantities like mean, median, variance, quartiles, etc. In general it results that people are quite good in interpreting data giving good approximations of the statistical summaries.

Several studies have examined the ability to recover sample proportions. For instance Shuford (1961) proposed a 20×20 matrix to a number of individuals one at time. Each element of the matrix was a blue square or a red square. A subgroup of individuals was shown the matrix for one second, to another subgroup the matrix was shown for ten seconds. After the matrix was shown, an individual was asked the proportion of blue and red squares. The result of the experiment was that the mean of subjects' estimate differed from the sample proportion by $.05$ People seem to be fairly able to estimate proportions.

Similar experiments have been conducted to analyze individuals' abilities to estimate measure of central tendency. Typically they are quit good in identifying the mean and median. However, in an experiment by Peterson and Miller (1964) it resulted that as well as a the distribution of sample is highly skewed, while the estimated mode and median resulted to be reasonably accurate, the estimated mean was biased.

While people seem to be able to estimate the central tendency of the sample they are exposed to, it results that the estimation of the variability is critical. In a number of experiments people appeared to be in troubles when exposed to the concept of variability. Experiments conducted to test the ability to estimate the variance of a sample have not given good results. Experimental psychologists showed that people are more confident with statements about credible intervals. In many experiments an an individual is shown a sample X divided into a partition of subsets $X_1, \dots, X_n$, then he or she is asked what is the probability of an element of X falling in some subset X_i. It seems that peoples have fairly good ability to elicit credible intervals, this technique together with some distributional assumption have been employed to construct estimates of the variance perceived by people. Using this procedure it seems that also the task of measuring variability in a sample is done with reasonable errors, but the direct estimate of variance is highly biased.

An important research aimed to assess the ability to convert phrases — for example "quite probable", "very likely" — into numeric values, and valuing wether verbal expressions of probability have to be preferred to numerical expressions or vice-versa. Wallsten et al. (1986) showed that people are more comfortable to express their uncertainty in verbal expressions rather than providing numerical measures. Unfortunately is not easy to convert verbal expression in numbers as long as different subjects attach to verbal expression different meanings.

5 *Fitting distributions with elicited probabilities*

As we pointed out before the problem of eliciting distributions of unknown quantities is not new. In bayesian analysis the problem of prior selection led to an huge research on methods for prior elicitation based on experts' questionnaires. There are researcher who believe that rather than developing mechanical mathematical methods to specify a prior for a given unknown quantities of interest, the choice of the prior should be based on the real prior knowledge of experts. In this view a number of methods have been proposed in order to elicit this knowledge.

There are to ways of eliciting a probability distribution for a given unknown quantity: (i) we can suppose that the unknown distribution belongs to some family, and then we can elicit the prior knowledge on this quantity trying to elicit believes on the parameter that fully identify the unknown distribution; in this case we are doing a parametric fitting; (ii) we can suppose that the unknown distribution belongs to some wide class of distribution functions and then we find methods to identify the one that is the more coherent with experts prior knowledge; in this case we talk about non-parametric elicitation methods.

Notice that in this section we will talk about findings emerged form the statistical literature. There has been not so much work in this area in econometrics, and the few methods of fitting we know have no bayesian setting, for this reason we believe that it will be better to describe the findings in the two literatures separately.

5.1 Parametric fitting

Suppose we are interested in an unknown quantity X, suppose we are interested to recover the probability distribution over X that a number of peoples have in mind. We can interview these respondents trying to extract information useful to recover such a probability distribution, given that we are imposing that one exists. We can impose that prior knowledge of interviewed about X can be described by a distribution $f_X(x;\theta)$ belonging to some family $\mathcal{F}_\theta$ which members are distinguished by the hyperparameters θ, and then the elicitation task it is reduced to choosing appropriate values for θ in order to capture the main features of respondents' opinion.

Notice that in this framework we have that the researcher expresses her or his own prior knowledge on the unknown quantity of interest. This is the same as saying that this approach involves a two stages elicitation process: in the first stage the researcher believes that the probability distribution of X belongs to some family $\mathcal{F}_\theta$, so he or she chooses $f_X(x;\theta)$ to describe X, in the second stage the researcher sets up a method to elicit θ based on prior knowledge of a number of people participating to a questionnaire. The subjective knowledge of the researcher is fundamental. Very often, rather

than following some logical argument, he family $\mathcal{F}_\theta$ is typically chosen considering the mathematical tractability of the problem. Although, advances in bayesian computation through MCMC methods make it viable to specify more complex families.
In this framework two elicitation tasks have been explored in the statistical literature: elicitation of priors for Bernoulli process and coefficients of linear regression models.

Suppose we are interested in eliciting a prior distribution on the parameter θ (proportion or probability) of a Bernoulli process. We will explore some elicitation scheme proposed.

One method is the so called "probability density function" method by Winkler (1967). The researcher asks to state the most likely value of θ, and to assesses other points of the p.d.f. for θ. Then a respondent is asked to give other two points half as likely as previous one, and then he or she is asked to specify quantiles for θ defined as points that divide the area under the graph of the p.d.f. in specified proportions. Then the distribution of θ is estimated. Notice that this method assumes that respondents know what is a quantile and a p.d.f and it can be only considered in these cases. A related method is the "quantile method" explored by Winkler (1972). In this method is simply asked to specify median estimate of θ and to give one or more quantiles for it. It is usually assumed that the respondents' opinion can be represented by a beta distribution (conjugate distribution for Bernoulli sampling scheme), and then a beta distribution is specified matching parameters with elicited quantiles. In this method researcher's prior knowledge is that peoples' believes are partially homogenous. For partially homogenous we mean that, although the researcher lets peoples to vary their prior opinion about the parameter of the beta distribution which is supposed to represent uncertainty about θ, on the other hand the researcher is fixing the family of possible distribution for θ to be a unique distribution, the beta distribution.

On a different philosophy the "hypothetical future sample" tries to recover the distribution of θ letting the sample to vary. Following Winkler (1967), suppose that θ is the proportion of students at the University of

Chicago who are male, the researcher asks questions of the form: "Suppose a random sample of N students in the Chicago University campus were taken and $\bar{n}$ of them were male, what do you think is the probability that one additional student, chosen at random, is male?". For each respondent a number of different hypothetical sample are proposed varying $\bar{n}$ each time. Each hypothetical sample yields an estimate of the hyperparameters and some form of averaging is used to summarize them. Again the researcher assumes that respondents underlying distribution for θ is a beta distribution, so that respondents opinions are used to estimate the parameters of the prior beta distribution. This method has the advantage to check for consistency in the answers because a number of questions are asked to the same subject. A similar method is the "equivalent prior sample" method by Winkler (1967), in which the respondent is asked to describe the sample rather than the parameters of interests. The example in Winkler (1967) will clarify the procedure, the researcher asks: "Can you determine two numbers $\bar{n}$ and N such that your knowledge would be roughly equivalent to having observed exactly $\bar{n}$ males in a random sample of N University of Chicago students?" The researcher, as before, asks different questions letting $\bar{n}$ and N vary, and then assuming the prior distribution is a beta distribution the parameters of the prior are estimated with some averaging method.

The general critique to these methods is that they leads to insufficient revision of opinions, and they produce prior distributions that exhibit too small variances. The reasons why these methods produce unrealistically tight priors has been extensively explored, and one explanation is the bias produced by conservatism. Winkler (1967) found that the quantile method tends to yield distributions that are slightly less tight than the probability density method and much less tight than the hypothetical future sample and equivalent prior sample methods. Compared on an experimental basis, evaluating elicitation with scoring rules, seems that the quantile method is preferable (for an extensive review of these results see Holstein (1971) and Schaefer and k. Borcherding (1973)).

The other context of elicitation of prior is the classic multiple linear regression model with normal errors. Suppose the model of interest is $y_i =$

$\mathbf{x}_i'\beta + \epsilon_i$ where y_i is the response, $\mathbf{x}_i \in \mathbb{R}^k$ is a vector of observable, and $\beta \in \mathbb{R}^k$ is the unknown vector of coefficients to be estimated, and $\epsilon_i \sim$ Normal$(0, \sigma^2)$ where σ^2 is unknown. Our problem is to make inference on β so a prior for it has to be elicited. Usually a conjugate prior distribution species that $\sigma^2 = \alpha\gamma(1/c)$ where $c \sim \chi_\gamma$ and that $\beta|\alpha, \gamma \sim$ Normal$(b, \alpha\gamma R/c)$ where b is k-dimensional vector representing prior knowledge on the mean of β, and R is $k \times k$-dimensional matrix that, together with α and γ, expresses prior knowledge on the variance-covariancematrix of β. The researcher have to elicit b, α, γ, R. Starting from Zellner (1972) a number of researches (Kadane et al. (1980),Garthwaite and Dickey (1991) Oman (1985), amongst the others) have analysed this issue. Here the problem is to specify the joint distribution for the k coefficients of interest. It is a quite complicated task and the respondent have to be selected among people with a deep knowledge of multiple liner regression. Even if, in the psychological experiments we have seen previously, people are quite good in recognizing intercepts and slope of a straight lines, here the problem is much more complicated since we require that people can recognize multiple coefficients and their dependency expressed through the variance covariance matrix. The cited papers are only the most important pieces of research on this topic, many other papers investigated the possibility to elicit prior for such a complicated multidimensional hyperparameters, we are not going to describe these methods. The general critique to them is that this methods use assessment tasks that people are not very good at performing, the main problem seems to be the elicitation of the variance-covariance matrix.

5.2 Looking for non-parametric fitting

As we pointed out in the previous section the procedures based on the elicitation of summaries used to fit a parametric distribution are sensible to the choice of the priors family, where this choice only reflects the opinion of the researcher. This in part contradicts the aim of the elicitation process which should try to embodies as much as possible different opinions on the same quantity of interest.

The revived interest in elicitation in the bayesian community in the last five or six years, has led to a few number of nonparametric approaches.

In Oakley and O'Hagan (2002) the respondents' opinions about the random variable of interest X are supposed to be represented by a probability density function $f_X(x)$. The researcher does not have any knowledge of f but he or se has a prior distribution over possible f. Since f is a function, the researcher prior is a distribution over the space of possible values of the whole function. Then, through some procedure that uses respondents opinions, as usual, a particular f is chosen as prior. Unfortunately to make the problem tractable, Oakley and O'Hagan (2002) have to restrict the possible shapes of f letting it to belong to some wide class of probability density functions. Here we do not want to discuss the method in details. Even if the method is supposed to be non parametric, it is not fully non parametric since a sensible restriction have to be made to identify f. On the other hand this methods has the advantage that not a particular family of priors is chosen by the researcher as starting point, rather, the researcher focus on a wider class of distribution functions.

5.3 Evaluation of elicited opinions

Statisticians are aware of the many difficulties of elicitation and after an elicitation process is completed we ask ourself how we can assess whether the elicited distribution is an adequate representation of the respondents knowledge. Before to continue we want to make clear that the way in which econometricians look to the evaluation step is different to that used by statisticians. For this reason we will describe their approaches separately. The problem is that economists think that peoples' mind obeys to some idea of rationality theoretically formalized so that peoples behaviour in forming beliefs about uncertain quantities can be submitted to a judgement that will classify believes in "rational" and "non-rational". We will discuss this issue in more detail at the end of the next section. Usually statisticians or experimental psychologists ask themselves wether a given elicited opinion is coherent in some specified sense.

The starting point is that we are not interested to classify opinions on uncertain quantities as correct or wrong, we only want to verify that respondents gave us their true opinion. Statisticians use to refer to a concept of coherence called “internal consistency”. They argued that a system of probability statements is coherent if the probabilities are all consistent with the laws of probability. One way to check the quality of a statements is to ask for sets of probability assessments that allow tests of coherence, for instance propose an experiment with two mutually exclusive events X, Y, ask for Prob $\{X\}$, Prob $\{Y\}$ and Prob $\{X \text{or} Y\}$, a statement will be internally consistent if Prob $\{X \text{or} Y\}$ $=$ Prob $\{X\}$ $+$ Prob $\{Y\}$. This is a trivial example, but it can give an idea of what is meant for internal consistency. In more complicated real elicitation procedures is not so easy to design a mechanism to verify internal coherence, nice examples of checking for internal consistency can be found in Lindley et al. (1979).

It is worth to mention that in many of this studies reported opinions are evaluated via scores methods. The researcher sets a score methods that give high points to probabilistic statements that fully reflect personal uncertainty. Rather than being an evaluation methods, the main goal of scoring rules is to induce people to fully state their knowledge, in some sense these methods can be seen as incentives mechanisms.

6 Elicitation in econometrics

6.1 Elicitation techniques

As we pointed out before, not very much work on elicitation has been done in econometrics. Before to continue in discussing some of the main findings, we want to stress the differences between the econometrics approach to elicitation, and the general statistical approach.

Although in the past few years there is a renewed interest in elicitation among statisticians, this literature started many years ago. Garthwaite et al. (2004) contains a general review of the most important findings, in this review the central problem is that of eliciting distributions of priors to solve problems of prior specifications or prior selections. Here the issue that we want address is quite different: we want to elicit the uncertainty about a future quantity summarized by subjective probabilities, so the task we are requiring to survey respondents is not only to assign a probability over uncertain quantities, but we are also requiring to make a prediction. In this sense, as stated before, our work somehow touches to distinct task: probabilistic judgement and forecasting. In a nutshell, this is the main difference between the kind of problems analysed in the statistical literature about elicitation (see Garthwaite et al., 2004), and the kind of problems we want to analyse in this work.

While statisticians focused on techniques for elicitation, econometrician have not done so much work in this direction. The most part of the actual relevant econometric literature have important qualitative conclusions, however there not tremendous innovative works addressing the issue of how to do elicitation. Manski (2004) contains a broad review of the findings where the economic implications are discussed. Survey responses to questions about expectation on future events have been employed to study a number of interesting economic phenomena, we will see some of them.

Domintz and Manski (1997) analyzed perception of job insecurity using data from the SEE. SEE stands for "Survey of Economic Expectations", it is a survey designed by Jeff Domintz and Charles F. Manski and administered as a module in WISCON, a continuous national random-digit telephone survey conducted by the University of Wisconsin Survey Center. In the SEE people are asked about a number of future events and are required to formulate probabilistic statement. Here we report questions asked about the job insecurity (See the SEE Code Book, pag. 4–5)[13]:

> Code RV451 – JOB LOSS: "*I would like you to think about your employment prospects over the next 12 months. What do you think is the PERCENT CHANCE that you will lose your job during the next 12 months?"*
>
> Code RV452 – FIND AS GOOD A JOB: "*If you were to lose your job during the next 12 months... What do you think is the PERCENT CHANCE (or CHANCES OUT OF 100) that the job you eventually find and accept would be at least as good as your current job, in terms of wages and benefits?"*

[13] An introduction to SEE with codebook and data can be downloaded from http://www.faculty.econ.northwestern.edu/faculty/manski/

> Code RV453 – leave job voluntarily (All waves): "*What do you think is the PERCENT CHANCE that you will leave your job voluntarily during the next 12 months?*"
>
> Each respondent currently looking for work was asked to provide a sequence of points on her or his subjective cumulative distribution function (CDF) of beliefs about the time to find a job. Respondent i was asked about three thresholds $FLm1_i$, $FLm2_i$, $FLm3_i$, posed in an increasing order. The questions were as follows:
> Code RV455 – DISTRIBUTION FOR TIME TO FIND JOB: "*What is the PERCENT CHANCE (or what are the chances out of 100) that it will take you less than [FLm] to find a job that you will accept?*"

Domintz and Manski (1997) found some interesting results, for example they found that subjective probability of job loss decreases with the level of education. Rather than summarizing results we want to underline that this study showed that people are willing to answer to question in probabilistic form.

An important contribution was made by Dominitz and Manski (1997), where they analysed household income expectations. This research is mostly related with the example of consumption theory we gave before. The data used by Dominitz and Manski (1997) were taken from SEE. Questions were asked by the telephone, points on the cumulative distribution function (CDF) were elicited starting with some warming-up preliminary training session. Respondents were first asked to report the lowest and highest levels of income that they think possible in the year ahead. The responses to these preliminary questions were used to set thresholds for a series of four probabilistic questions. Here we report from SEE code book (pag. 5):

> Respondent i was asked about four thresholds – $RVFL2_i$, $RVFL3_i$, $RVFL4_i$, and $RVFL5_i$ – posed in increasing order. Again, the only exception occurred if a response of 100% chance was given when one of the thresholds is posed as a coherent subjective distribution must give 100% chance to all subsequent thresholds. The questions were as follows:
> Code RV460-RV468 — DISTRIBUTION OF FUTURE INCOME, BEFORE TAXES: "*What do you think is the PERCENT CHANCE (or*

CHANCES OUT OF 100) that your OWN total income, BEFORE TAXES, will be under $[fill RVFL],000?"

The thresholds about which a respondent was queried were determined by the respondents answer to a pair of preliminary questions asking for the lowest and highest possible incomes...

In order to see how thresholds were set see the CEE code book, it is a quite long procedure, here we skip details. In this report for each respondent i, $F_{i,k} = \text{Prob}\left[y < Y_{i,k}\right], \ k = 1,2,3,4$ are observed, where $\{Y_{i,1}, Y_{i,2}, Y_{i,3}, Y_{i,4}\}$ are the income thresholds about which respondent i is queried. To identify the distribution F, authors assumed that F belongs to the log-normal family. Let $F(Y; M, Q)$ be log-normal distribution with median M and interquartile range Q. Let M_i^*, Q_i^* such that they solve the least square problem

$$\inf_{M_i, Q_i} \sum_{k=1}^{4} \left[F_{i,k} - F(Y_{i,k}; M, Q)\right]^2 ,$$

then M_i^*, Q_i^* are used to estimate respondents' i subjective median and interquartile range and a log-normal distribution is fitted. The authors found that Q tends to rise with M more than proportionally and that Q exhibits significant cross-sectional variations. This would contradict the rational expectations hypothesis. Although Dominitz and Manski (1997) introduce a quite innovative technique to elicit expectations for continuous variable, we think that this approach is not fully satisfying since a restrictive assumption on the class of probability distributions for the expected income is made and this hypothesis is not testable at all.
Evidence of heterogeneity of expectations have also been confirmed by Domintz and Manski (2003, 2004). The authors used data from SEE to study expectations about returns on mutual-fund investments.

Subjective income and inflation expectations data have been used by Jappelli and Pistaferri (2000) to test, amongst other things, for excess sensitivity of consumption. They used data from 1989–1993 rotating panel of the Bank of Italy Survey of Household Income and Wealth (SHIW). The panel offers unique measures of subjective income and inflation expectations. They found that consumption growth is positively correlated with the expected variance of income and uncorrelated with predicted income growth and this is a result which supports the "precautionary saving model", in fact this model predicts that people saving depends on the perceived future risk.

Dominitz (2001) considers the use of expectations and realizations data to estimate income expectations conditional on observed attributes. In this work income expectations data and realizations data are used to model expectations. The data used are taken from the Panel Study of Income Dynamics.

An interesting work on expected inflation formation is Mehra (2002)[14]. He considers three survey measures of inflation expectations: the Livingston Survey of Economists (LSE) in which respondents are selected amongst academic economists; the Michigan Survey of U.S. households (MSH) in which respondents are households without any particular specialization; and the Survey of Professional Forecasters (SPF) in which respondents are selected amongst professional economists and analysts. In this work a näive forecast has been considered as a benchmark, which is simply the most recent one-year growth rate of consumer price index (CPI) known to the survey respondents at the time forecasts are made.

The first result is that all survey measures of expectations are nearer the realized data than those produced by the näive predictive rule. While both the LSE and MSH forecasts perform equally over the full period and the period of rising inflation, the MSH forecasts present the largest mean-deviation from actual data over a period of downward-trending inflation. This later result reverses if we consider median-deviations.

Also a Granger-causality tests have been made, and the result is that survey forecasts contain a forward-looking component and can help in predicting actual future inflation, this does not apply to the exceptions of LSE.

Another finding is that forecast errors are correlated with past information, this would contradict the main "axiom" of rational expectation theory which states the people do not make systematic forecast errors.

In this paper also it is interesting that data on expectations collected — on different individuals with different backgrounds knowledge — exhibit some

[14] Here the approach employed is very similar to that of Thomas (1999).

pattern in the deviation of expectations form actual data, and that mean-deviations and median-deviations present significant cross-sectional and longitudinal variations.
Also it is interesting to notice that expectations of respondents in MSH — i.e. household with no particular required knowledge of economics or statistics — fully reflect the path of realized inflation rate in median but they are "biased" in mean during periods of structural changes in the process of prices' growth. This result seems to confirm what has been found in the experimental psychological literature (see Peterson and Miller, 1964).

In this section we reviewed some of the relevant existing literature in econometrics where subjective expectations are used. In this review we have certainly noticed that, although subjective data are used, there has not been a development of tools to recover subjective distributions over uncertainty quantities of interest. The only exception is Dominitz and Manski (1997), but has we already pointed out in this paper a restrictive assumption on the family of distribution to which the expected income belongs is made. Dominitz and Manski (1997) is an important contribution, but more sophisticated techniques are needed.

6.2 Evaluating elicited expectations

Once we have elicited expectations or other forms of probabilistic judgements we need to assess the quality of our analysis. Often we read wording like "accuracy of expectations", "bias of expectations", "correctness of expectations", and other expressions like these. In our view this is an improper wording if we refer to expectations that peoples hold in their mind. Suppose I observe the price for apples today, say it is 1 euros per kilogram, suppose one ask to me "what price do you expect for apples tomorrow?", and suppose my answer is 1.1 Euros per kilogram. Now, suppose that the next day we go on the fruit market and the price of apples is 1.15 Euros. An economist would say that I made a "mistake" which can be measured by the forecast error, which is -5 cents of euro, he would also argue that by rational expectation hypothesis, if we repeat this experiment an huge number of time, this error should be equal to 0 on average.

Why are we talking about an error? There are several reasons why we should not talk about errors, we give for instance an extreme one. Suppose we are willing to make the extremely unrealistic assumption that a data generating process of prices can be identified, and that I have a proper subjective probability distribution over this data generating process. It could be that from the time I made my statement about expected prices to the time in which the new price is formed in the next period, something in the information set, on which I conditioned my stated expectation, has changed. This is almost always the case in economics. We measure economics variables at discrete time interval and we make probabilistic judgements on information acquired at discrete time intervals, but economic phenomena occurs in continuous time, so that the information we would like to observe changes continuously over time.
In this sense even in a world of extremely smart people who are able to recover the data generating process of apples' price, if prices are measured at discrete time intervals, and if their expectation stated at time t about apples' price at time $t+1$ is different from the true apple price we pay at time $t+1$, we cannot say that a forecast error has been made. To say that a forecast error has been made, we should be sure that nothing has changed in the available information between the time when we stated our expectation and the time when the new price is available. We should agree that such an assumption would be unrealistic and impossible to assess.

So we do not agree with the wording which is common amongst economists. We believe that at most we could talk about "coherence" of probabilistic judgements. The concept of coherence that here is inspired to De Finetti. As we pointed out before, we do not believe in that "it is important what people do, not what people think", instead we believe that it is important to assess whether what people think is coherent with what they do. If what people do is coherent with the probabilistic judgement they made, we can use this judgement to predict future behaviours. Suppose we have an urn containing blue and red balls, we know the proportion of the two kind of balls, suppose we attach a lottery to the urn, that is, if a red ball is drawn then nothing is payed, if a blue ball is drawn 5 euros are payed. There are many players, we can decide if we want to participate to the lottery or not, if we decide to participate then we pay 2 Euros we draw the ball and we get 5 Euros if the result is a blu ball, otherwise we can pass to the next player and will never partecipate to the game. Suppose that it is my turn, suppose that before I state whether I want play or pass, someone ask to me which is the probability that I would attach to the event that a blu ball is extracted in the next drawn. Suppose I say “2%” and next I am asked if I want to play or not, and I say that I want to play. In this case I would say that I am not coherent because my probabilistic statement implies that I expect to lose roughly 2 Euros and still I want to play the game. When we talk about coherence this is what we mean.

Now we will review some of the econometric analysis that has been made about the evaluation of elicited probabilistic judgement. Again, we will try to avoid confusing words as much as we can.

One thing that econometricians do is to simply compare individual expectations with realizations data. Dominitz (1998) used a one year follow-up to a preliminary 1993 version of SEE to evaluate the accuracy of respondents expectations about weekly earnings. He assumed that people hold a subjective distribution for earnings which is continuous and that realizations are cross-sectionally independent. He estimated the empirical distribution of expectations and the empirical distribution of realized data, and then he measured the distance between the two distribution by comparing quantiles. Applying this criterion, he found that expectations and realizations did not match entirely. Dominitz (1998) found that expectations were “too optimistic” (said the author), in the sense that central tendency of expectations exceeds central tendency of realizations, and “too confident”, i.e., spread of realizations exceeds spread perceived by respondents.
We should notice that this kind comparisons need panel data, in fact we need to estimate the empirical distribution of expectations at each period (cross-section estimate), and then we need to compare — following some statistical criterion — this distribution with the estimated empirical distribution of realized data in the next period (longitudinal comparison).
Furthermore, the idea behind this procedure is that we should find in the

data what peoples expect, the problem of this philosophy is that realized data can be determined by other factors that are not under the control of respondents, so that realized data will not reflect peoples' action based on their expectations.

Another procedure used in literature has been the comparison of mean expectations and mean realizations. Domintz and Manski (1997) used this approach to evaluate the one-year-ahead expectations from SEE in 1994, they compared expectations with the realizations reported by the new sample of respondents interviewed in 1995, simply comparing means of expectations and realizations. All SEE respondents were asked to elicit expectations of health insurance coverage, crime victimization, and the job-loss questions (see SEE codbook pag. 3-5), the comparison showed that mean expectations and realizations of health insurance match up closely.

The last procedure we explore is the comparison of mean expectations with historical realizations. This methods assume that successive cohorts of persons have the same distribution of realizations for a given variable of interest. Under this assumption the previous method of comparing mean of expectations and realized data, it is applied comparing expectations and realized data of different cohorts. This is needed when the researcher wants to apply the method described before, but she or he does not have measured data on realizations for each period for each individual. Fischhoff et al. (2000) conducted a study on a numbers of life events eliciting expectations in different cohorts, then they compared mean of expectations with historical realized data on different cohorts. They concluded that the result of comparison largely depends on the expectations of teens, over the years the distribution of teens' expectations change sensibly and comparison with historical realized data on different cohorts is misleading.

Our opinion is that comparison is that when we compare expectations data with realizations, we should be careful in our conclusion. For the kind of variables involved in the cited studies, even if peoples' actions are coherent with their expectations, realized data not only depend on peoples' action but they also depend on a number of factors out of peoples' control. Suppose we collected data on expected future earnings and then we want to compare expected earnings with realized earnings. Suppose that after we measured expectations, and before we measure realized earnings, a new unexpected law has passed which changes the discipline of labor contracts. Suppose we compare expectations with realized data, then —using the wording popular among economists— could we conclude that peoples' do not have "accurate expectations"? We think that we cannot conclude that peoples' have "inaccurate expectations", simply we cannot make any comparison.

Another comment is that comparison based on summary of distributions, and especially the mean, is not reasonable. First of all a distribution is not only characterized by central tendency, and moreover we have already seen that from experimental psychological literature it results that probabilistic judgement is biased in mean.

7 Conclusions

In this short chapter we tried to give an overview of the problem of eliciting probability distributions. The starting point has been the need of less unrealistic modelling in econometrics as pointed out in Manski (2004) (amongst the others). In fact, in econometrics very often modelling considers unobservable expectations amongst independent variable. Trying to estimate this models implies that to achieve fully identification of the joint probability of the response, econometricians have to make unclear and unrealistic assumptions. In the last few years there has been an increasing interest by econometrician in eliciting probability distribution in order to estimate expectations. We also described researches on elicitation in psychology and bayesian statistic where these kind of works started many years ago.

Although there has been a tremendous effort by econometricians in this direction of the research, we think that the methods employed are not fully satisfying. Econometricians still think that people have in mind some model which correspond to a probability distribution on uncertain outcomes. Our approach will be more unpretentious than this, in fact, we think that an individual could be even not able to assign a probability distribution on uncertainty outcomes; and even in this case we will not say that those individuals are “not rational”. Assigning probability to uncertain outcomes can be very hard even in experiment involving two or three outcomes. On these basis, we believe any elicitated probability distribution is false, in the sense that it cannot represent to true uncertainty which people have in mind. Our effort will be devoted to set up reasonable methodology and statistical tools in order to represent this uncertainty as close as possible to that perceived by peoples. We see the need of bayesian methods in this field because of the necessity to fully express the starting uncertainty via priors probability and then to revise this uncertainty trough posteriors.

In our future work our basis will be the evidence collected up this moment. Evidence suggests that people are better to express their opinions in terms of quantiles and other summaries, rather than to perform judgements in terms of probabilities. The experimental psychological literature showed that people are not able to perform probabilistic judgement when the situation becomes complex and the number of variables increases. For this reason we do not think that multivariate elicitation is achievable, so that we will try to develop methods that reduce multivariate elicitation to univariate where possible.

At this stage we do not have a clear idea of what we are going to do. What is clear is the object of our research. We will try to set up a method of elicitation in which expectations of a random quantity X will be computed taking the expectation of the predictive distribution over X where this predictive distribution will be computed starting from an elicitated prior and then getting the posterior given realized data on X and data on expectations about X. We think that with this framework we will add knowledge to what have been done. We can express peoples' uncertainty on X trough their elicited prior, the revision process through the posterior will give us knowledge on the updating process of information, also we will combine past data with new data on expectations and we will take into account peoples' heterogeneity allowing prior uncertainty to vary among them.

As we said so far this is a rough draft. Before to apply any methods to real data we need to follow a number of steps. We need to elaborate appropiate methods of priors elicitation suited for our problem, eventually we need to test this method trough an experimental session. Then we want to develop the statistical tools needed, and finally we will apply this techniques to some real world datasets.

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