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**On neutrino mixing in Quantum Field Theory**

ABSTRACT

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## ABSTRACT

Neutrino physics is one of the most important areas in research on the fundamental interactions, both theoretically and experimentally. A central issue in the study of these particles is related to the properties of mixing and flavor oscillations. In particular, a necessary condition for the mixing is that neutrinos have mass, a feature which is not considered by the Standard Model, and which poses the problem of the origin and nature of these masses.

The theoretical basis of neutrino mixing has been studied in many details and in the late '90 a quantum field theory (QFT) formalism for mixed fields has been developed thanks to which it was found that mixing transformations induce a condensed structure in the flavour vacuum with possible phenomenological consequences. In particular, the usual formulas of Pontecorvo oscillation are arising as the relativistic limit of exact formulas in the context of field theory. Within this framework it is inserted our work, finalized to the study of algebraic properties of the mixing transformations generator in QFT and its components.

In quantum mechanics (QM) the mixing transformation looks like a rotation operating on massive neutrino states. We show explicitly that such a rotation is not sufficient for implementing the mixing transformation at level of fields. It is necessary, in fact, also the action of a Bogoliubov transformation which operates a suitable mass shift. Such a property of Bogoliubov transformations has been already known and used since long time, e.g. in renormalization theory or in the dynamical generation of mass. We then analyze the condensate nature of the flavor vacuum and the rôle played by the non commutativity between the rotation and the Bogoliubov transformation. This structure of the vacuum also suggests a thermodynamical interpretation which we investigate, showing peculiarities in the thermal behavior due to

the character of the particle-antiparticle condensate involved in the flavor vacuum. The key point in our analysis is the non commutativity between rotation and Bogoliubov transformations, a feature which turns out to be at the origin of the inequivalence among mass and flavor vacua. From another point of view, the Bogoliubov transformations are shown to naturally arise when studying the neutrino mixing in the context of the Noncommutative Spectral Geometry in Alain Connes' construction and the algebra doubling he introduces. Given the algebraic nature of our arguments, we have good reasons to believe that the results we have obtained are general and also extend to the mixing phenomenon of any particle, even if our analysis is limited to the case of two Dirac neutrinos.

The thesis is structured as follows: after an *Introduction* to the arguments of our work is presented; in *Chapter 1* we briefly summarise the main aspects of the Standard Model and of neutrino mixing. *Chapter 2* is dedicated to neutrino mixing and oscillations, considering both the quantum mechanics approach and the quantum field theory formalism. In *Chapter 3* we analyze the mixing generator, decomposing it into components, and the flavor vacuum structure, studying its thermodynamical properties. A noncommutative structure will arise. *Chapter 4* is focused on non commutativity, giving some known examples of noncommutative systems, and briefly presenting noncommutative geometry and noncommutative spectral geometry (NCSG) elements. In *Chapter 5* we introduce further notions on Alain Connes' construction; we summarize how neutrinos appear within this construction and we relate the algebra doubling, which is a crucial element of the NCSG model, to the Hopf non-commutative algebra and Bogoliubov transformations, which play a key role in the neutrino mixing. In *Chapter 6*, in order to better understand the mixing phenomenon, we study a classical system analogue for it. We then close with our *Conclusions and Outlook*.

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The results obtained and presented in this thesis have been published in the following international journals:

- M.V. Gargiulo, M. Sakellariadou, G.Vitiello,  
*Doubling of the Algebra and Neutrino Mixing within Noncommutative Spectral Geometry*,  
EPJ C (2014) **74** 2695 ;
- M.V. Gargiulo, M. Sakellariadou, G.Vitiello,  
*Noncommutative spectral geometry, Bogoliubov transformations and neutrino oscillations*,  
Journal of Physics: Conference Series (2015) **626** 012014 ;
- M.Blasone, M.V.Gargiulo and G.Vitiello,  
*Disentangling mass and angle dependence in neutrino mixing*,  
Journal of Physics: Conference Series (2015) **626** 012026 ;
- M. Blasone, M.V. Gargiulo, G. Vitiello,  
*On the role of rotations and Bogoliubov transformations in neutrino mixing*,  
Phys. Lett. B (2016) **761** 104;
- M.Blasone, M.V. Gargiulo, G.Vitiello,  
*Semiclassical aspect of neutrino mixing*  
Journal of Physics: Conference Series (2017), in press.

Work in progress

- M.Blasone, M.V. Gargiulo, G.Vitiello,  
*Noncommutative aspects of neutrino mixing*,  
work in progress.