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EFFICIENCY GAINS AND MERGERS

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Abstract

In the theoretical literature, strong arguments have been provided in support of the efficiency defense in antitrust merger policy. One of the most often cited results is due to Williamson (1968) that shows how relatively small reduction in cost could offset the deadweight loss of a large price increase. Furthermore, Salant et al. (1983) demonstrate that (not for monopoly) mergers are unprofitable absent efficiency gains. The general result, drawn in a Cournot framework by Farrell and Shapiro (1990), is that (not too large) mergers that are profitable are always welfare improving. In the present work we challenge the conclusions of this literature in two aspects. First, we show that Williamson's results underestimate the welfare loss due to a price-increasing merger and overestimate the effect of efficiency gains. Using the simple linear Cournot model, we show that efficiency gains needed to compensate for the deadweight loss are much larger than Williamson's. Then, we prove that the conditions for welfare improving mergers defined by Farrell and Shapiro (1990) hold true only when consumers are adversely affected. This seems an argument to disregard their policy prescriptions when antitrust authorities are more "consumers-oriented". In this respect, we provide a necessary and sufficient condition for a consumer surplus improving merger: in a two firm merger, efficiency gains must be larger than the pre-merger average markup.

Keywords: Mergers; Efficiency Gains; Cournot oligopoly

JEL Classification: D43, L11, L22.

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1 Introduction

The purpose of the present work is to analyze the role of efficiency gains in horizontal mergers, to evaluate the effect on welfare and to critically review the main contribution in the literature on this topic.

Mergers represent a major economic issue in the economic literature as well as for antitrust policy. To provide an idea of the numbers and resource involved, over the period 1981-1998, there were nearly 70,000 merger announcements worldwide, with each deal worth at least 1 million U.S. dollars, of which nearly 45,000 were actually implemented. The average deal was valued at 220 million U.S. dollars (base year 1995). Of these, 42% were horizontal mergers, defined as those involving two companies with sales in the same 4-digit industry, 54% were conglomerate mergers, and 4% were vertical mergers.[1]

As pointed out by Schmalensee (1989), mergers have also been an important source of increase in market concentration, particularly outside the US.

The relationship between market concentration and welfare dominated the economic debate since the first wave of mergers at the end of the nineteenth century; the common wisdom was that the higher the concentration, the lower the welfare. This wisdom was challenged some decades ago when new strands in economic literature questioned this relationship.

Some works, such as Salant and Shaffer (1999), showed how higher market concentration can imply higher welfare in a Cournot oligopoly.[2]

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[1] For this data see Gugler et al. (2003).

[2] At a first glance this argument seems counterintuitive but it clearly relies on an important property of Cournot equilibrium: the more efficient firms produce more. Indeed, assume linear costs and an initial equilibrium with n symmetric firms. Suppose that cost are redistributed with a mean preserving increase in the variance: total quantity, that depends only on the average marginal cost, remains unchanged. Now more efficient firms produce more than less efficient firms. Total output is unchanged and so is social gross surplus, but total cost are reduced. Welfare has increased together with concentration.

In other works strong arguments were provided in support of the efficiency defense in antitrust merger policy. One of the results most often cited was provided by Williamson (1968) that showed how relatively small reduction in cost, even in a merger for monopoly, could offset the deadweight loss of a large price increase.

The theoretical literature on horizontal mergers largely relies on the standard Cournot model. A central postulate is that the pre-merger and the post-merger situations are represented as Cournot equilibrium points involving different market structures, with the merged entity being treated as a single player in the post-merger situation. An important result in this framework was provided by Salant et al. (1983). Using the simple linear demand and cost setting, they showed that mergers without cost savings are unprofitable unless they involve more than 80% of the market. Even if Davidson and Deneckere (1984) showed in the Bertrand setting with differentiated products, that every merger is profitable, the result in Cournot competition had some implications.[3] One of these implications was that socially inefficient mergers are unprofitable as long as they are not merger for monopoly. Some subsequent works, such as Perry and Porter (1985), challenged this point of view suggesting that anticompetitive profitable mergers are possible when fixed asset combination is taken into account and when mergers occur in industries with increasing marginal cost in the short run.[4] But then Farrell and Shapiro (1990) provided an important result: they defined conditions under which profitable mergers are welfare improving in a general Cournot setting. Considering synergies, learning and asset combination they are able to show that the reallocation of production to the more efficient firms can lead to welfare improvement. Their benchmark work is based on the characterization of the *external effect* of a merger, that is the sum of the effects on consumers and outsiders – incumbent firms not involved in the merger.

The empirical literature on merger profitability exhibits some degree

[3]This difference in the results is due to complementarity and substitutability of the choice variables and to the fact that in Bertrand competition the merger entity continues to produce all the differentiated product of the pre-merger firms. In the model of Salant et al. (1983), on the contrary, merger resolves in a "lock-up" of all the merging firm but one. Perry and Porter (1985) for this argument.

[4]On merger profitability see also McAfee and Williams (1992). Faulí-Oller (1997) shows that profitability of mergers increases with the concavity of demand function.

of controversy, mostly of a quantitative sort. Gugler et al. (2003), in the largest cross-national study to date, report that nearly 60% of all the horizontal mergers were profitable. On the other hand, two other broad-base studies concluded that the profitability of acquired firms declined after the merger for U.K. firms (Meeks, 1977) and for U.S. firms (Ravenscraft and Scherer, 1988), while in Mueller (1980) divergent results studying OECD countries are reported. Trying to explain this rather mixed picture, Amir et al. (2003) modeled the effect of a merger when non-merging firms are uncertain about the efficiency gains reached by the merging firms. They show how the result depends not only on the magnitude of cost reduction but also on the belief of the outsiders; a merger can be profitable even if efficiency gains do not actually materialize, provided that non-merging firms sufficiently believe that the merger will generate large enough cost reduction.

The contribution of this work is twofold.

First of all we challenge the common wisdom that small cost reductions more than offset the deadweight loss due to a price-increasing merger. Indeed, even though this result is due to the pioneering work of Williamson (1968), it remains very popular in the literature. Many works – such as Bloch (1995); Sapienza (2002); Schmalensee (2004); Shapiro and Willig (1990); Willig et al. (1991), just to provide few examples – refer to Williamson's result as the clearest theoretical evidence of the importance of efficiency gains in merger analysis. In the simple linear Cournot framework, we show that the cost reduction needed to offset the allocative efficiency cost of a price-increasing merger are much larger than Williamson's. To provide an example of the magnitude of this difference, Williamson reports that, with a demand elasticity of 1, a reduction of just 0.5% of the average cost can offset the negative effect on welfare of a 10% price increase. In our setting the efficiency gains must exceed 11%. There are two main reasons for this huge difference. First of all, in the Williamson's analysis efficiency gains are computed *on the whole production of the industry*. This is of course adequate only when a merger for monopoly is considered. Second, he assumes a competitive price in the pre-merger situation, so that the deadweight loss is just a loss in the consumer surplus and not in profits. In the present work we consider the case in which a merger occurs in an oligopoly in which firms compete à la Cournot.

Extending the analysis to a more general Cournot setting the *external effect* defined by Farrell and Shapiro (1990) is fully characterized. We first show that the effect of a merger on consumers has always the opposite sign to the effect on outsiders; so it is possible to show that the condition defined by Farrell and Shapiro for a welfare-improving merger holds only when consumers are adversely affected. The latter seems an argument to disregard their policy prescriptions when antitrust authorities are concerned about consumer protection.

At this regard we provide a clearcut condition for a merger not to reduce consumer surplus under Cournot competition: the efficiency gains must be larger than $(m-1)$ times the average pre-merger markup-to-cost (MC) ratio, where m is the number of merging firms.

This work is organized as follows: Section 2 is devoted to the analysis of the Williamson (1968)'s result and to the comparison with the linear Cournot equilibrium analysis. Section 3 extends the analysis and describes, in the linear Cournot setting, the Farrell and Shapiro (1990)'s approach and the underlying forces that drive the result. It is shown that the efficiency gains needed to offset the welfare loss due to the increased market power of the merger is a proportion of the pre-merger MC ratio that is *decreasing* in the number of firms. On the contrary, it is shown that the efficiency gains needed to have a profitable merger is a proportion of the pre-merger MC ratio that is *increasing* in the number of firms. So a threshold in the market share of the merging firms is found as a necessary and sufficient condition to have a profitable merger that increases total welfare. Section 4 extends the Farrell and Shapiro (1990)'s analysis using their *marginal approach*. We show that the effect of a merger on consumers is always opposite to the effect on outsiders' profits, and that the condition defined by Farrell and Shapiro for a welfare-improving merger holds only when consumers are adversely affected. Moreover a necessary and sufficient condition for a merger not to reduce consumer surplus is provided. Section 5 contains the conclusions. Most of the computations are gathered in the Appendix.

2 Williamson's model compared with an equilibrium analysis

In his pioneering work, Williamson (1968) analyzes the welfare tradeoff of market power and efficiency motivations of mergers, and shows that relatively small cost reductions more than offset the allocative efficiency cost of a price increase. Assuming a market in which two or more firms produce a homogeneous good, he analyzes the overall welfare effect of a merger that increases the price and achieves efficiency gains. In Figure 1 his welfare analysis is depicted. If the demand is approximate to a linear function, the deadweight loss due to the price increase is given by $\frac{1}{2}\Delta P \Delta Q$, while the benefit from cost savings is equal to ΔACQ. After

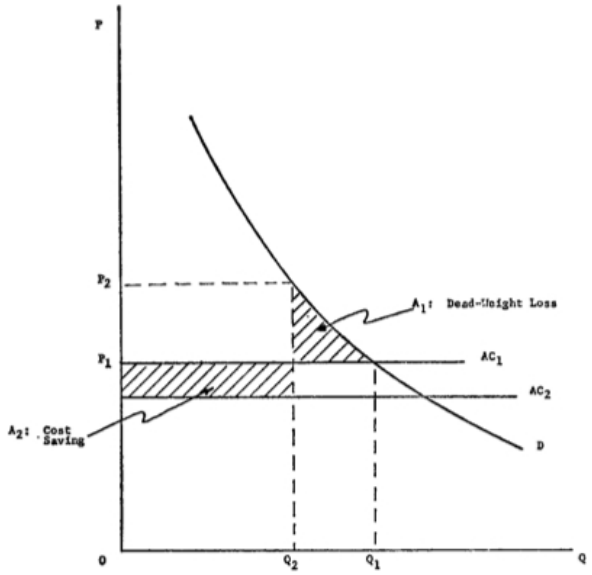

Figure 1: The total welfare effect of a merger, (Williamson, 1968, p.21).

some transformation, Williamson defines the condition under which the total effect is positive; that is,

$$\frac{\Delta AC}{AC} - \frac{k}{2}\eta\left(\frac{\Delta P}{P}\right)^2 \geq 0 \tag{1}$$

where AC is the average cost of production, η is the elasticity of the demand in the initial equilibrium and k is a measure (P/AC) of the

pre-merger market power. Assuming that the initial market power is negligible, so $k = 1$, expression (1) gives rise to the results in Table 1.[5]

$\left(\frac{\Delta P}{P}\right)$	η		
	2	1	0.5
5%	.25%	.12%	.06%
10%	1.00%	.50%	.25%
20%	4.00%	2.00%	1.00%
30%	9.00%	4.50%	2.25%

Table 1: Percentage cost reductions needed to offset some given price increases for selected values of η.

According to his hypothesis, a merger for monopoly, that occurs when $\eta = 1$ and that rises the price of 20%, can be welfare improving if it achieves a reduction in cost of just 2%. Unfortunately, his formulation is erroneous, in the sense that it overestimates the effect of cost reduction.[6] After several attempts, Jackson (1970) provided the (correct) following expression:

$$\frac{\Delta AC}{AC} - \frac{\frac{k}{2}\eta\left(\frac{\Delta P}{P}\right)^2}{1 - \eta\frac{\Delta P}{P}} \geq 0 \tag{2}$$

and numerical examples shown in Table 2.[7]

$\left(\frac{\Delta P}{P}\right)$	η		
	2	1	0.5
5%	.27%	.13%	.06%
10%	1.24%	.55%	.26%
20%	6.66%	2.49%	1.11%
30%	22.49%	6.42%	2.64%

Table 2: Percentage cost reductions needed to offset some given price increases for selected values of η; Jackson (1970)'s version.

[5] This table is taken from Williamson (1968, p. 23).

[6] Indeed, the welfare effect of cost reduction is computed on the pre-merger level of output, while it should be computed at the post-merger level, that is lower.

[7] Jackson (1970, p. 441).

Results are not very different, but formulation (2) is not general, since it works only when the price is competitive at the pre-merger equilibrium; that is, when $k = 1$. Indeed, the welfare loss accounts only for the consumer surplus loss, and not for the loss in profits on the quantity no longer produced.[8] Moreover, and most importantly, the cost reduction in Williamson (1968) and Jackson (1970) is computed *on the whole production of the industry* and not only of the merging firms. Then, his analysis is correct just in case of a merger for monopoly when the starting price is competitive; a situation that is at least uncommon.

Notwithstanding these limitations, Williamson's results are still considered as the clearest evidence of the importance of efficiency gains in merger analysis and his work is one of the most cited paper on this topic.[9]

In what follows Williamson's results are compared with an equilibrium analysis based on the standard Cournot model of oligopolistic competition.

2.1 Equilibrium analysis: the linear Cournot model

Consider an industry described by a linear inverse demand $P = a - bQ$ and, in the pre-merger situation, n symmetric firms with constant marginal cost c that produce a homogeneous good and compete à la Cournot. Assume $a > c > 0$ and $b > 0$.

The equilibrium in the pre-merger situation is described by the following values of total production and price,

$$Q^* = \frac{n\,(a-c)}{b\,(n+1)} \qquad P^* = \frac{a+nc}{n+1}$$

of each firm's production and profit

$$q_i^* = \frac{a-c}{b(n+1)} \qquad \pi_i^* = \frac{(a-c)^2}{b\,(n+1)^2}$$

and of consumer surplus and total welfare

$$CS^* = \frac{1}{2}\frac{n^2\,(a-c)^2}{b\,(n+1)^2} \qquad W^* = \frac{n\,(n+2)\,(a-c)^2}{2b\,(n+1)^2}$$

[8] See DePrano and Nugent (1969) and Jackson (1970) on this point.

[9] See, for example, Bloch (1995); Sapienza (2002); Schmalensee (2004); Shapiro and Willig (1990); Willig et al. (1991).

Assume now that two firms merge experiencing not only an increased market power, but also efficiency gains so that the merged entity has marginal cost $c_m = c - \Delta c$ with $\Delta c \geq 0$.

At the new equilibrium an asymmetry between the merged entity and the outsiders arises. As a general result in Cournot oligopoly, the most efficient firm produces more than the others and the post-merger equilibrium is described by the following values of total production and price,

$$Q^m = \frac{(n-1)(a-c) + \Delta c}{nb} \qquad P^m = \frac{a + (n-1)c - \Delta c}{n}$$

of the merged firm's production and profit,

$$q_m^m = \frac{a - c + (n-1)\Delta c}{nb} \qquad \pi_m^m = \frac{[(a-c) + (n-1)\Delta c]^2}{bn^2}$$

of the *outsiders*' production and profit,[10]

$$q_o^m = \frac{a - c - \Delta c}{nb} \qquad \pi_o^m = \frac{[(a-c) - \Delta c]^2}{bn^2}$$

of consumer surplus

$$CS^m = \frac{1}{2}\frac{[(n-1)(a-c) + \Delta c]^2}{bn^2}$$

and total welfare

$$W^m = \frac{\left(n^2-1\right)(a-c)^2 + 2(n-1)(a-c)\Delta c + \left(2n^2 - 2n - 1\right)\Delta c^2}{2bn^2}$$

Comparing the pre-merger and post-merger equilibrium welfare, the following proposition holds:

Proposition 1 *In a Cournot industry with linear demand, linear cost and n symmetric firms, a two-firm merger is welfare improving if and only if it achieves a reduction in cost such that*

$$\frac{\Delta c}{c} \geq \gamma(n)\frac{P^* - c}{c} \tag{3}$$

[10] As common in the literature we denote as outsiders the firms that are not involved in the merger.

where $\gamma(n)$ *is a decreasing function that depends only on* n *with* $0 < \gamma(n) < 1$ *and* $\frac{P^*-c}{c}$ *is the* markup-to-cost (MC) ratio *at the pre-merger equilibrium. Then, the efficiency gains needed to offset the negative allocative effect of a price increase is an increasing function of the initial MC ratio.*

Proof. See Appendix. ■

This result can be interpreted as a support to the common wisdom that the larger the market power of the firms, the greater the allocative efficiency cost of a merger. Then, the efficiency gains needed to offset this negative allocative effect should increase with the pre-merger market power.

In order to compare this equilibrium analysis with Williamson's results, we can rearrange condition (3), and express the efficiency gains needed to keep the welfare constant as a function of the price increase and of the elasticity at the pre-merger equilibrium.

The results are summarized in Table 2.1.[11]

$\left(\frac{\Delta P}{P}\right)$	η		
	2	1	0.5
5%	4.64%	5.33%	6.37%
10%	[a]	11.86%	15.76%
20%	[a]	[a]	53,24%
30%	[a]	[a]	[b]

Table 3: Percentage cost reduction needed to offset some given price increases for selected values of η; equilibrium analysis in a linear Cournot model.

[a] It is impossible to have this price increase with this value of elasticity at the pre-merger equilibrium.

[b] To offset this price increase the cost reduction should be larger than 100%.

The difference with Williamson's results is striking and has two main

[11] This analysis is carried out treating n as a continuous variable in order to choose given values of the elasticity of the demand and given price increases due to the merger. See Appendix for computational details.

explanations previously highlighted. On the one hand his analysis does not take into account the presence of a markup in the industry, so that the allocative efficiency cost of a price-increasing merger is a loss in consumer's surplus, only. On the other hand the cost reduction in Williamson (1968) is computed on the whole production of the industry.

3 Comparative statics in linear Cournot models

The results in the previous Section highlight how large the efficiency gains should be in order to offset the negative welfare effect of a price-increasing merger, and how optimistic Williamson's analysis is. Treating n as a discrete variable, it is possible to compute the efficiency gains needed to keep the welfare constant and the resulting equilibrium change in price when two firms merge. Table 4 shows this results for selected values of the elasticity and of the number of symmetric firms in the pre-merger equilibrium.

	n[a,b]			
η[c]	2	3	4	5
0.20				
0.33			50.14(*15.62*)	21.28(*10.30*)
0.50		40.88(*17.68*)	16.71(*10.41*)	9.46(*6.87*)
0.67	79.80(*27.53*)	20.44(*13.26*)	10.03(*7.81*)	6.08(*5.15*)
1.00	26.60(*18.35*)	10.22(*8.84*)	5.57(*5.21*)	3.55(*3.43*)
1.50	13.30(*12.23*)	5.84(*5.89*)	3.34(*3.47*)	2.18(*2.29*)
2.00	8.87(*9.18*)	4.09(*4.42*)	2.39(*2.60*)	1.58(*1.72*)
$\gamma(n)$[d]	26.60	20.44	16.71	14.19

Table 4: Percentage cost reduction needed for a 2 firm merger to keep welfare constant for different values of η and n.

[a] number of firms at the pre-merger equilibrium

[b] in brackets the corresponding price increase, in percentage

[c] the elasticity of demand at the pre-merger equilibrium, in absolute value

[d] efficiency gains, as a proportion of the initial MC ratio, needed to keep the welfare constant

Some comments can be made. First of all, the cost reduction needed to keep the welfare constant is inversely related to the elasticity, as in Table 2.1. This is due to the fact that, in the linear Cournot setting, the elasticity of the demand is increasing in the equilibrium price; and the latter, in turn, is increasing in the marginal cost. But the MC ratio at equilibrium is decreasing in cost; so, a larger elasticity at equilibrium is associated with a lower ratio. Given that the efficiency gains needed to offset the allocative efficiency cost of the price increase are directly related to the MC ratio, they are inversely related to the demand elasticity at the pre-merger equilibrium.

A similar argument can explain the inverse relation between efficiency gains and the number of firms. In fact, the more competitive the market, the lower the allocative efficiency cost of a merger, the smaller the efficiency gains needed to keep the welfare constant.

Finally, it should be emphasized that the efficiency gains must always be larger than the price increase (except for very high values of the elasticity) in order to avoid a reduction in welfare. This is the most evident contrast between our results and Williamson (1968)'s.

As highlighted in the Introduction, Farrell and Shapiro (1990) provide conditions under which a profitable merger is welfare enhancing when firms compete à la Cournot. These conditions essentially affirm that a merger, in order to be profitable, should entail reductions in cost. When the market share of the firms involved in the merger is below some threshold, these efficiency gains imply that the merger increases the welfare. To illustrate the forces underlying this result we provide an illustration in the linear framework.

The first step is to analyze merger profitability. Considering a two firm merger, it is profitable if $\pi_m^m \geq 2\pi_i^*$. Salant et al. (1983) show that a merger in a linear Cournot setting with constant marginal cost and no fixed cost is not profitable if it does not achieve any cost reduction. In the following Proposition the magnitude of these efficiency gains is defined.

Proposition 2 *In a Cournot industry with linear demand, linear cost and n symmetric firms, a two firm merger is profitable if and only if it achieves a reduction in cost so that*

$$\frac{\Delta c}{c} \geq \phi(n) \frac{P^* - c}{c} \tag{4}$$

where $\phi(n)$ is an increasing function that depends only on n. Comparing the two thresholds for profitable and welfare improving merger

$$\phi(n) \geq \gamma(n) \qquad \forall n \geq \frac{1}{2}\left(3+\sqrt{2}+\sqrt{7}\right) \simeq 3.53 \tag{5}$$

Proof. See Appendix. ■

Farrell and Shapiro (1990) find a sufficient condition for a profitable merger to be welfare improving: the sum of the market share of the merging firms must be at most half of the market.[12] In the following Corollary, using the result (5) in Proposition 2, we define a necessary and sufficient condition for the simple linear symmetrical Cournot oligopoly.

Corollary 1 *In a Cournot industry with linear demand, linear cost and n symmetric firms, a necessary and sufficient condition for a profitable merger to be always welfare improving is that the sum of the market shares of the merging firms is lower than $\frac{4}{3+\sqrt{2}+\sqrt{7}} \simeq 0.57$.*

An important feature of Corollary 1 is that it holds even when the merger reduces price, while Farrell and Shapiro sufficient condition holds only when the price increases. This is due to the fact that their results are based on the analysis of the "external effect" of the merger, that is the sum of the effects on consumer surplus and outsiders' profits. Using some properties of Cournot competition they are able to sign this external effect under general conditions; but only when the price increases they are able to define the overall effect on welfare. In section 4 these properties of Cournot competition are presented and discussed in a more general setting.

The threshold for a merger to be consumer surplus increasing can be computed. Comparing the prices (or, alternatively, the consumer surplus) defined in Section 2.1 before and after the merger, the following Proposition holds:

Proposition 3 *In a linear Cournot model, a two firm merger is consumer surplus improving if it achieves a cost saving*

$$\frac{\Delta c}{c} \geq \frac{P^* - c}{c}$$

[12] In this case of linear demand and constant marginal cost.

that is, efficiency gains should be at least equal to the pre-merger MC ratio.

Proof. See Appendix.[13] ■

Table 5 compares the different cost reduction thresholds needed to keep the welfare constant, to have a just profitable merger and to keep the consumer surplus at the pre-merger equilibrium.

Some comments can be made. First of all, a merger for monopoly, when $n = 2$, is profitable even if it does not achieves efficiency gains. Negative values means that even worsening efficiency the merger is profitable. Second, as shown in Corollary 1, merger profitability implies welfare improvement for $n > 3$. In fact, looking at the values of $\gamma(n)$ and $\phi(n)$, this result is due to the fact the efficiency gains needed for a profitable merger increases as n increases, while the reduction in cost needed to have a welfare improvement is decreasing in n.

Third, the condition for profitability is worth discussing. Salant et al. (1983) pointed out that, as the number of firms increases, it becomes more difficult for a merger to be profitable. This result does not hold if we define this difficulty in terms of the necessary cost savings. In fact, recalling condition (4), $\frac{\Delta c}{c}$ depends on $\phi(n)$ and on the MC ratio. While $\phi(n)$ is increasing in n, the MC ratio is decreasing in the same variable; so, the efficiency gains needed to have a profitable merger is the result of these two opposite forces. Table 5 shows that it is first increasing than decreasing when $n > 5$.

Forth, the efficiency gains needed to keep the consumer surplus constant is always equal to the pre-merger MC ratio. Then, profitability never implies a reduction in price.

Finally this table allows us to understand the underlying forces that drive the Farrell and Shapiro (1990)'s result. For $n > 3$ (see for example the values for $n = 5$) welfare can increase even when the efficiency gains are not large enough to have profitability of the merger and consumer surplus improvement. This means that the (positive) effect on outsiders' profit is larger than the (negative) effect on consumer surplus.

This in turn means that, in the Cournot setting, when the sum of the merging firms' market shares is less than the half of the market ($n > 4$),

[13] Note that this result holds in the general Cournot setting, as shown in Proposition 6.

η[b]	n[a]					
	2			3		
	welfare[c]	π	CS	welfare[c]	π	CS
0.5	[d]	[d]	[d]	40.88 (*17.68*)	24.26	[e]
1.00	26.60 (*18.35*)	-17.16	100	10.22 (*8.84*)	6.07	50.00
1.50	13.30 (*12.23*)	-8.58	50.00	5.84 (*5.89*)	3.47	28.57
2.00	8.87 (*9.18*)	-5.72	33.33	4.09 (*4.42*)	2.43	20.00
$\gamma(n)$[f]	26.60			20.44		
$\phi(n)$[f]	-17.16			12.13		
η[b]	n[a]					
	5			10		
	welfare[c]	π	CS	welfare[c]	π	CS
0.5	9.46 (*6.87*)	17.85	66.67	2.05 (*1.84*)	8.73	25.00
1.00	3.55 (*3.43*)	6.69	25.00	0.91 (*0.92*)	3.88	11.11
1.50	2.18 (*2.29*)	4.12	15.38	0.58 (*0.61*)	2.49	7.14
2.00	1.58 (*1.72*)	2.98	11.11	0.43 (*0.46*)	1.84	5.26
$\gamma(n)$[f]	14.19			8.18		
$\phi(n)$[f]	26.77			34.91		

Table 5: Percentage cost reduction needed for a 2 firm merger to keep welfare, insiders' profit and consumer surplus constant for different values of η and n.

[a] number of firms at the pre-merger equilibrium
[b] the elasticity of demand at the pre-merger equilibrium
[c] in brackets the corresponding price increase
[d] there is no pre-merger equilibrium at this level of elasticity
[e] cost reduction needed is higher than 100%
[f] as percentage of the initial MC ratio

the consumer surplus effect is dominated by the outsiders' profit effect. In the next Section we will see that this relation is true in the general Cournot setting and is reversed when the market share of the merging firms is above the threshold.

4 The external effect of a merger in a general Cournot setting

Assume that an industry, in which firms produce a homogeneous good and compete choosing quantity levels, can be described by an inverse demand function $P(Q)$ with $P'(Q) < 0$, and each firm's cost structure is described by the function $c_i(q_i)$.

We define the reaction functions of the firms in the standard way

$$r_i(q_i) \triangleq \arg\max_{q_i \geq 0} \pi_i(q_i, q_{-i}) \qquad \forall i = 1, ..., n$$

where π_i and q_i are profit and quantity of firm i, and q_{-i} is the quantity produced by all the other firms. To have the usual downward sloping reaction functions it is sufficient to assume that:[14]

$$\frac{\partial^2 \pi_i(q_i, q_{-i})}{\partial q_i \partial q_{-i}} < 0$$

Expressing this condition in terms of the primitives:

$$P'(Q) + q_i P''(Q) < 0 \qquad \forall i = 1, ..., n.$$

This condition for all the firms is respected if we assume that the demand function is such that

$$P'(x) + xP''(x) < 0. \tag{6}$$

Moreover, to have uniqueness of the equilibrium, assume that

$$P'(Q) - c_i''(q_i) < 0 \qquad \forall i = 1, ..., n. \tag{7}$$

[14]This is the condition to have a submodular game, that is a game with strategic substitutes. For more on submodularity of Cournot games see Amir (1996).

Assumptions (6) and (7) imply that the reaction functions are decreasing contractions; that is, their slopes

$$r_i'(q_{-i}) = -\frac{P'(q_i + q_{-i}) + q_i P''(q_i + q_{-i})}{2P'(q_i + q_{-i}) + q_i P''(q_i + q_{-i}) - c_i''(q_i)} \tag{8}$$

belongs to the interval $(-1, 0)$. This is a sufficient condition to have uniqueness and stability of the equilibrium.

In the equilibrium each firm's maximization program implies that:

$$P(Q) + q_i^* P'(Q) - c_i'(q_i^*) = 0 \qquad \forall q_i^* > 0 \text{ and } i = 1, ..., n. \tag{9}$$

where q_i^* is the best response to the quantity produced in equilibrium by the others.

In what follows we will use a marginal approach to analyze the effect of a discrete exogenous variation of the rivals output. This approach, used also by Farrell and Shapiro (1990), is originally due to Gaudet and Salant (1988).

Starting from the equilibrium, suppose that the quantity produced by some firms has an exogenous variation. This variation can occur because some firms act differently with respect to a Cournot player as well as some firms experience efficiency gains that reduce their production costs: a merger with efficiency gains could be one of the causes of this exogenous variation of the output.

The equilibrium responses of the other firms and the overall effect on market performance are analyzed. Note that the resulting situation is a Cournot equilibrium for the outsiders and the one (those) that causes the change can either be in a Cournot equilibrium or not. So, the following analysis applies to Cournot market, but also to cases in which some firms act differently.[15]

With a little abuse of notation, we can define the optimal response of firm i to an infinitesimal exogenous change of the quantity produced by the other firms in the following way:

$$dq_i^* = r_i'(q_{-i})\, dq_{-i}$$

[15]For example it can be used to analyze mixed oligopoly where there is a public firm competing with private firms and the effect of a privatization.

Adding on both sides $r_i'(q_{-i})\,dq_i$, we can express the variation of the quantity of the firm i in terms of total output variation:

$$\begin{aligned}\left(1+r_i'(q_{-i})\right)dq_i^* &= r_i'(q_{-i})\,(dq_{-i}+dq_i^*)\\ dq_i^* &= \frac{r_i'(q_{-i})}{1+r_i'(q_{-i})}dQ \qquad (10)\end{aligned}$$

Equation (10) identifies the equilibrium response of firm i to a total output change dQ caused by a change in production of some firm. This analysis is clearly a marginal analysis but, under some regularity conditions, it can be extended to discrete variations, as we will see soon.

By assumptions (6) and (7) the variation of each firm i's output is inversely related to the total output variation. Indeed, defining

$$R_i'(q_{-i}) \triangleq \frac{r_i'(q_{-i})}{1+r_i'(q_{-i})} \qquad (11)$$

it has negative sign because

$$R_i'(q_{-i}) = -\frac{P'(Q)+q_iP''(Q)}{P'(Q)-c_i''(q_i)} < 0$$

This result is important to show some properties of the Cournot competition.

Proposition 4 *The effect of an exogenous variation of the output of some firms on the equilibrium profit of the others is opposite to the effect on consumers. In fact, if some firms increase the output exogenously, total output increases, and price and the other firms' profit decreases. If these firms reduce the output, price increases and the other firms' profit increases, too.*

Proof. The fact that an exogenous variation of the output of some firms changes the equilibrium output in the same direction is due to reaction functions being contractions. Indeed, assuming that the output of some firms j changes for exogenous reasons (for example a change in her cost function or a change in the objective), summing up the variation of all the firms but j

$$\sum_{i\neq j} dq_i^* = dq_{-j} = \sum_{i\neq j} R_i'(q_{-i})\,dQ$$

Adding dq_j on both sides

$$\begin{aligned} dQ &= \sum_{i \neq j} R_i'(q_{-i})\, dQ + dq_j \\ \left(1 - \sum_{i \neq j} R_i'(q_{-i})\right) dQ &= dq_j \\ dQ &= \frac{1}{1 - \sum_{i \neq j} R_i'(q_{-i})} dq_j \end{aligned} \tag{12}$$

Since R_i' is always negative for every firm, dQ has always the same sign of dq_j even if it is smaller; so, if firm j increases the output, at the new equilibrium Q is larger and price is lower. The consumers are then better off.

To see the effect on the other firms' equilibrium profit of a change in total output due to a change of the output of some firms j

$$\begin{aligned} \pi_i^* &= P(Q)\, q_i^* - c_i(q_i^*) \\ \frac{d\pi_i^*}{dQ} &= P'(Q)\, q_i^* + P(Q) \frac{dq_i^*}{dQ} - c'(q_i^*) \frac{dq_i^*}{dQ} \\ &= P'(Q)\, q_i^* + \left[P(Q) - c'(q_i)\right] \frac{dq_i^*}{dQ} \end{aligned}$$

From the first order condition (9), $P(Q) - c(q_i^*) = -P'(Q)\, q_i^*$. Substituting it in the previous expression and by equation (10) and definition (11), we have that:

$$d\pi_i^* = P'(Q)\, q_i^* \left(1 - R_i'(q_{-i})\right) dQ \tag{13}$$

By equation (12) $dQ = \frac{1}{1 - \sum_{i \neq j} R_i'(q_{-i})} dq_j$ and so:

$$\frac{d\pi_i^*}{dq_j} = \frac{1 - R_i'(q_{-i})}{1 - \sum_{i \neq j} R_i'(q_{-i})} P'(Q)\, q_i^* < 0$$

Moreover this condition always holds for any change in Q and we can extend the result to discrete variation of quantity, which completes the proof. ■

This result can easily be applied to mergers with or without efficiency gains. Consider now j to be the subset of firms that merge and assume

that the merged entity acts as Cournot player: if it achieves efficiency gains such that it produces more than the sum of pre-merger output of firms j, then total output increases.[16] In this case consumers are better off and outsiders are worse off. On the other hand, if there is no cost saving, or it is not large enough, the merged firm produces less than pre-merger output, then total output decreases. The consumers are worse off while the outsiders are better off. These results are summarized in the following Corollary.

Corollary 2 *Under assumptions (6) and (7), if a merger is harmful for consumers, it increases outsiders' profits; if a merger reduces the price, it reduces the outsiders' profits.*

Farrell and Shapiro (1990) use the sum of the effect on consumer surplus and outsiders' profits to define the *external effect* of an exogenous change in the production of some firms. They find conditions under which this effect is positive *when the total output shrinks.* In what follows, we characterize this external effect for a general variation of the output, both positive and negative. This allows us to extend and fully characterize the result, that is summarized in the following Proposition.

Proposition 5 *Assume that P'', P''', $c_i''(.)$ are nonnegative and $c_i'''(.)$ is nonpositive in the relevant range. Suppose that an exogenous variation of the output of some firms j occurs. Then, the external effect has always opposite sign to the effect on consumer surplus when the output of the insiders is smaller than a certain threshold $\tilde{q}_j$. When the market share of the insiders is larger, the external effect has the same sign as the consumer surplus effect. In the first case the effect on outsiders' profits dominates the effect on consumer surplus. The reverse is true in the latter case.*

Proof. Total welfare is defined as the sum of consumer surplus and

[16] In the Cournot setting if a firm has lower cost produces more. See an interesting application of this result in Salant and Shaffer (1999).

profits, that is

$$\begin{aligned} W &= \int_0^Q \left[P\left(z\right) - P\left(Q\right)\right] dz + \pi_j + \sum_{i \neq j} \pi_i \\ &= \int_0^Q P\left(z\right) dz - QP\left(Q\right) + \pi_j + \sum_{i \neq j} \pi_i \end{aligned}$$

where, as usual, j represent the insiders – firms that experience an exogenous variation of the output – and $i \neq j$ the outsiders.

Now we want to compute the marginal effect on welfare of a marginal *equilibrium change in* Q due to the exogenous change in q_j; that is, the effect on welfare when the outsiders optimally respond to the variation in the output of the subset of firms j.

$$\begin{aligned} \frac{dW}{dQ} &= P\left(Q\right) - P\left(Q\right) + QP'\left(Q\right) + \frac{d\pi_j}{dQ} + \sum_{i \neq j} \frac{d\pi_i^*}{dQ} \\ &= \frac{d\pi_j}{dQ} + QP'\left(Q\right) + \sum_{i \neq j} \frac{d\pi_i^*}{dQ} \end{aligned}$$

Since it is not possible to determine $\frac{d\pi_j}{dQ}$ without knowing the reasons of the change in the output, we can define the external effect as the difference between the effect on welfare and the effect on insider profits. From equation (13), we can substitute $\frac{d\pi_i^*}{dQ}$:

$$\begin{aligned} \frac{dW}{dQ} - \frac{d\pi_j}{dQ} &= QP'\left(Q\right) + \sum_{i \neq j} P'\left(Q\right) q_i^* \left(1 - R_i'\left(q_{-i}\right)\right) \\ \frac{dW}{dQ} - \frac{d\pi_j}{dQ} &= QP'\left(Q\right) + P'\left(Q\right) \sum_{i \neq j} q_i^* - P'\left(Q\right) \sum_{i \neq j} R_i'\left(q_{-i}\right) q_i^* \end{aligned}$$

Since $\sum_{i \neq j} q_i^* = Q - q_j$

$$\begin{aligned} \frac{dW}{dQ} - \frac{d\pi_j}{dQ} &= -q_j P'\left(Q\right) - P'\left(Q\right) \sum_{i \neq j} R_i'\left(q_{-i}\right) q_i^* \\ dW - d\pi_j &= -P'\left(Q\right) \left(q_j + \sum_{i \neq j} R_i'\left(q_{-i}\right) q_i^*\right) dQ \end{aligned} \tag{14}$$

So, the external effect has opposite sign with respect to the equilibrium variation of Q when

$$\left(q_j + \sum_{i \neq j} R_i' \left(q_{-i} \right) q_i^* \right) < 0$$

that is, being $R_i' \left(q_{-i} \right)$ always negative,

$$q_j < \tilde{q}_j \triangleq \sum_{i \neq j} -R_i' \left(q_{-i} \right) q_i^* \tag{15}$$

The external effect has the same sign when the opposite is true.

By Proposition 4, the sign of dQ is the same as the consumer surplus, and outsider's profits move always in the opposite direction. When condition (15) is true, the effect on outsiders' profits dominates the effect on consumers, while consumer surplus effect dominates if condition (15) does not hold.

To extend this result to discrete changes of the insiders' output, conditions on second and third derivatives of inverse demand and cost functions are needed.[17] ■

The most important assumption of the previous Proposition is that cost functions should be convex for all the firms. However, this is not a great limitation since concave costs give usually rise to natural monopolies.[18] To better understand the meaning of this threshold, some examples may be useful. Under linear cost and demand, $R_i' \left(q_{-i} \right) = -1$ so $\tilde{q}_j = \sum_{i \neq j} q_i^*$. This means that half of the market is the threshold. In case of linear demand and quadratic costs it can be shown that $R_i' \left(q_{-i} \right) = -\frac{q_i}{\eta}$ where η is the elasticity of the demand at the pre-merger equilibrium.[19] So the threshold is $\tilde{q}_j = \frac{1}{\eta} \sum_{i \neq j} q_i^{*2}$.

[17] See Farrell and Shapiro (1990, p. 116).

[18] However this is not always true. Even when costs are concave (but not too much) it can exists a unique equilibrium in Cournot oligopoly with all firms producing. This is actually a different definition of natural monopoly and it turns out to depend on the demand function, too. The condition under which this equilibrium exists is condition (7) for which c'' can be negative but bigger than P'. See on this Amir (2005).

[19] See Farrell and Shapiro (1990, p. 118) for this result.

In terms of antitrust policy prescriptions, Farrell and Shapiro affirm that mergers between firms whose market share is lower than the threshold should be allowed because the external effect is positive. Looking at the external effect of a merger is then their guideline for antitrust scrutiny.

However, their result holds only when consumers end up paying higher price. Indeed:

Corollary 3 *Under the conditions of Proposition 5, if a merger between some firms whose market shares $q_j < \tilde{q}_j$ reduces the price, the external effect of the merger is negative as long as the output increase is not too large.*

To see this result,note that by (12) we can define the external effect in terms of q_j. From equation (14)

$$dW - d\pi_j = -P'(Q)\frac{q_j + \sum_{i \neq j} R_i'(q_{-i})\, q_i^*}{1 - \sum_{i \neq j} R_i'(q_{-i})} dq_j$$

So $W - \pi_j$, that can be defined as the *external surplus*, is decreasing when $q_j < \tilde{q}_j$ and increasing when $q_j > \tilde{q}_j$.

Under the conditions of Proposition 5, we graph in Figure 2 this *external surplus* as a function of the level of the output of insiders. We can see that when $q_j < \tilde{q}_j$ and the merger determines an increase in the output, so that total output increases and consumers are better off, the external effect is undetermined since it can end up with a quantity higher than $\tilde{q}_j$. But if q_j is small enough, so that the post-merger output cannot be larger than the threshold, we have the paradoxical effect that the external effect is negative even when welfare increases. To get the intuition, suppose that the merged firm produces just the same quantity as the merging firms in the pre-merger situation. It is possible only if it achieves efficiency gains. So, social gross benefit is unchanged, total production costs are lower and total welfare increases. The larger the efficiency gains, the larger the increase in production, the larger the increase in welfare.

This result of course undermines the validity of the Farrell and Shapiro (1990)'s policy prescription for which the external effect is the key element.

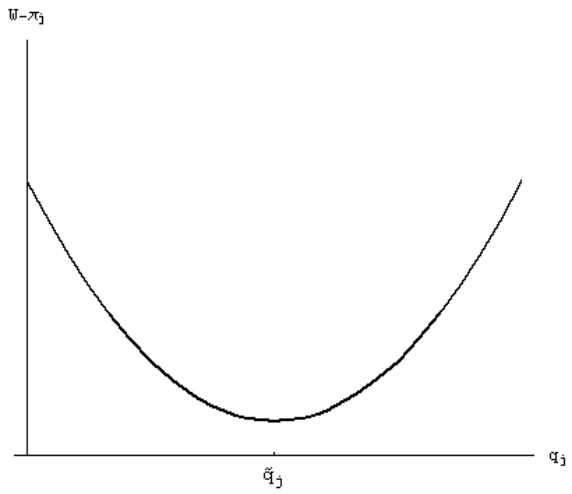

Figure 2: External surplus as a function of insiders' output.

However, using the relation between external surplus and the level of production we can extend the analysis to the case when $q_j > \tilde{q}_j$.

Corollary 4 *Under the conditions of Proposition 5, if a merger between some firms whose market shares $q_j > \tilde{q}_j$ reduces the price, the external effect of the merger is positive.*

The last comment is devoted to the shape of the external surplus. Given that, when q_j increases, price decreases and consumers are better off, the threshold $\tilde{q}_j$ defines the relationship between consumer surplus and outsiders' profits effects. When $q_j > \tilde{q}_j$ the effect on consumers surplus dominates, while outsiders' profit effect dominates if $q_j < \tilde{q}_j$. Then, Farrell and Shapiro policy prescription holds only when consumers are worse off; so, if antitrust authorities are more concerned about consumer protection, their analysis is not of great help.

In the following Proposition we define a very general condition on the efficiency gains that are sufficient to avoid a reduction in consumer surplus.

Assume that $m < n$ firms merge and denote by J the set of these firms, q_J^* the sum of their quantities, and $\bar{c}'$ their average marginal cost at the pre-merger equilibrium.

Proposition 6 *Assuming nondecreasing marginal costs, a merger does not rise the market price if and only if it achieves a reduction in cost, computed at q_J^*, equal to $(m-1)$ times the pre-merger average MC ratio of the merging firms.*

Proof. Since by equation (12) in the proof of Proposition 4 the effect on total output has the same sign of the variation of the quantity of the insiders, all we need is to sign the change in production of the merging firm. By the first order condition of the new firm M,

$$P(Q^M) - P'(Q^M)q_M - c'_M(q_M) = 0$$

where Q^M and q_M are the total output and the production of M in the post-merger equilibrium. Denoting with Q^* total output in the pre-merger situation, firm M will produce more than q_J^* if and only if

$$P(Q^*) - P'(Q^*)q_J^* - c'_M(q_J^*) > 0 \tag{16}$$

Summing up the first order condition of the insiders in the pre-merger equilibrium

$$\begin{aligned} P'(Q^*)q_J^* &= \sum_{i\in J}\left[P(Q^*) - c'_i(q_i^*)\right] \\ &= m\left[P(Q^*) - \bar{c}'\right] \end{aligned}$$

where m is the number of merging firms. Then, substituting in (16)

$$P(Q^*) - m\left[P(Q^*) - \bar{c}'\right] - c'_M(q_J^*) \geq 0$$

or

$$\frac{\bar{c}' - c'_M(q_J^*)}{\bar{c}'} > (m-1)\frac{P(Q^*) - \bar{c}'}{\bar{c}'}$$

that completes the proof. ■

5 Conclusion

Williamson's results are not robust to an equilibrium analysis in a Cournot context. Efficiency gains needed to offset large price increase

due to a merger are very large and, generally, larger than the price increase. His conclusion on the relevance of the efficiency gains in merger scrutiny may be misleading if applied to cases of mergers in oligopoly. In fact, give that cost savings are computed on the *whole production of the industry*, it is only adequate to cases of mergers for monopoly.[20]

Our equilibrium analysis highlights the different and contrasting interests of the parties involved by a change in industry structure: outsiders are worse off when the efficiency gains of the merger are so large that the price decreases and the consumers are better off, and, *vice versa*, consumers are adversely affected by a merger when outsiders' profits increase.

This work is about efficiency defense in the merger scrutiny and takes into account the effect of mergers on the different parties. We have shown how the effect on consumer surplus is essentially dominated by the effect on outsiders' profits of a merger when the merging firms' market share is not so large. The external effect proposed by Farrell and Shapiro as a policy instrument in merger analysis, has, in this cases, is positive only when consumers are worse off. This seems an argument to disregard their policy prescriptions when antitrust authorities are concerned about consumer protection.

In terms of consumer surplus improving merger we provide a general condition under Cournot competition for which a merger is not harmful for consumers. But it requires very high levels of efficiency gains. As a result, efficiency defense becomes a weak argument when antitrust authorities are "consumers-oriented".

A last remark on fixed costs. We have chosen not to consider them explicitly in the analysis since the results are not affected by them. In fact, if they can be considered sunk, then a merger does not achieve any saving on them. If they are assets that the new merged firm can combine in order to reduce marginal cost, then the analysis carried out in the general Cournot framework of Section 4 clearly applies. If the merger achieves savings on fixed costs, the overall welfare effect of the merger increases, but the analysis on external effect, outsiders profits and consumer surplus is not affected at all.

[20] Recall, however, that he assumes also competitive prices in the pre-merger situation.

6 Appendix

Proof of Proposition 1. To define when a two merger increases total welfare, consumer surplus and profit in the pre-merger and post-merger situation are compared. The difference in consumer surplus is:

$$\Delta CS = CS^m - CS^* = \frac{1}{2}\frac{\left[(n-1)(a-c)+\Delta c\right]^2}{bn^2} - \frac{1}{2}\frac{n^2(a-c)^2}{b(n+1)^2}$$

that reduces to the following polynomial expression in terms of Δc and $(a-c)$:

$$\Delta CS = \frac{(n+1)^2\Delta c^2 + 2(n+1)^2(n-1)(a-c)\Delta c - \left(2n^2-1\right)(a-c)^2}{2bn^2(n+1)^2} \tag{17}$$

The difference in profits is given by the difference for the merging firms and for the outsiders. The expression for the former is given by

$$\Delta\pi_m = \pi_m^m - 2\pi_i^* = \frac{\left[(a-c)+(n-1)\Delta c\right]^2}{bn^2} - \frac{2(a-c)^2}{b(n+1)^2}$$

Reducing it in the same polynomial form of (17) we obtain:

$$\Delta\pi_m = \frac{(n-1)^2(n+1)^2\Delta c^2 + 2(n+1)^2(n-1)(a-c)\Delta c}{bn^2(n+1)^2} + \\ - \frac{\left(n^2-2n-1\right)(a-c)^2}{bn^2(n+1)^2} \tag{18}$$

The difference in profits for all the outsiders is:

$$(n-2)\Delta\pi_o = (n-2)\left[\frac{\left[(a-c)-\Delta c\right]^2}{bn^2} - \frac{(a-c)^2}{b(n+1)^2}\right]$$

that reduces to

$$(n-2)\Delta\pi_o = \frac{(n-2)(n+1)^2\Delta c^2 - 2(n-2)(n+1)^2(a-c)\Delta c}{bn^2(n+1)^2} \\ + \frac{(n-2)(2n+1)(a-c)^2}{bn^2(n+1)^2} \tag{19}$$

To have a welfare increasing merger

$$\Delta W = \Delta CS + \Delta \pi_m + (n-2)\,\Delta \pi_o \geq 0$$

Summing up the three polynomial equation and ordering with respect to the relevant variable Δc, the welfare change reduces to

$$\Delta W = \frac{(n+1)^2\left(2n^2-2n-1\right)\Delta c^2 + 2\,(n+1)^3\,(a-c)\,\Delta c}{2bn^2\,(n+1)^2} + \\ - \frac{(2n+1)\,(a-c)^2}{2bn^2\,(n+1)^2} \geq 0 \quad (20)$$

The denominator is always positive so what matters is the numerator that is a second degree polynomial in Δc. The roots are:

$$\Delta c_{1,2} = \frac{-(n+1)^3\,(a-c)}{(n+1)^2\,(2n^2-2n-1)} \mp \\ \mp \frac{\sqrt{(n+1)^6\,(a-c)^2 + (n+1)^2\,(2n^2-2n-1)\,(2n+1)\,(a-c)^2}}{(n+1)^2\,(2n^2-2n-1)}$$

Because the solution of inequality (20) calls for values external to the interval of the roots, the negative root doesn't matter. The solution reduces to

$$\Delta c \geq \frac{a-c}{n+1}\left[\frac{n\sqrt{n^2+8n+4}-(n+1)^2}{2n^2-2n-1}\right] \quad (21)$$

Computing the MC ratio at the pre-merger equilibrium

$$\frac{P^*-c}{c} = \frac{a-c}{c\,(n+1)} \quad (22)$$

and defining

$$\gamma\,(n) \triangleq \frac{n\sqrt{n^2+8n+4}-(n+1)^2}{2n^2-2n-1}$$

the efficiency gains needed to have a welfare improving merger are then

$$\frac{\Delta c}{c} \geq \gamma\,(n)\,\frac{P^*-c}{c}$$

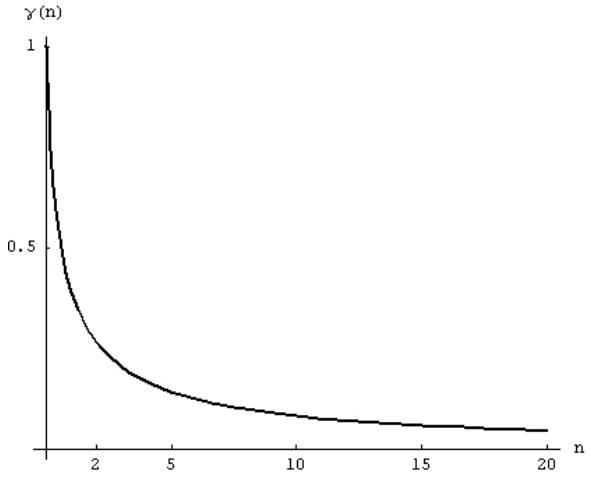

Figure 3: $\gamma(n)$ as a function of the number of firm in the pre-merger situation. It is the cost saving needed to keep the welfare constant as a proportion of pre-merger MC ratio.

that completes the proof. Note that $\gamma(0) = 1$ and that $\gamma(n)$ is decreasing but always positive as shown in Figure 3. ■

Efficiency gains in terms of elasticity, Table 3. Computing (the absolute value of) the elasticity of demand at the pre-merger equilibrium

$$\eta = -\frac{1}{p'}\frac{P^*}{Q^*} = -\frac{1}{b}\frac{a+nc}{n+1}\frac{b(n+1)}{n(a-c)} = \frac{a+nc}{n(a-c)}$$

The MC ratio at the pre-merger equilibrium can be expressed in terms of elasticity:

$$\frac{P^*-c}{c} = \frac{a-c}{c(n+1)} = \frac{1}{n\eta-1} \tag{23}$$

In this sense, the elasticity at the pre-merger equilibrium summarizes the values of the structural parameters of the market. So, the efficiency gains needed to keep the welfare constant can be rewritten in the following way:

$$\frac{\Delta c}{c} = \frac{\gamma(n)}{n\eta-1} \tag{24}$$

When a merger implement the cost savings that keeps the welfare constant, the effect on price is defined by the following expression:

$$\begin{aligned}\frac{\Delta P}{P} = \frac{P^m - P^*}{P^*} &= \left[\frac{a+(n-1)c-\Delta c}{n} - \frac{a+nc}{n+1}\right]\frac{n+1}{a+nc} \\ &= \frac{a-c-(n+1)\Delta c}{n(a+nc)}\end{aligned}$$

Substituting for the threshold value of Δc we obtain the price increase

$$\begin{aligned}\frac{\Delta P}{P} &= \left(\frac{n(1+\eta)}{n\eta-1}c - c - (n+1)\frac{\gamma(n)}{n\eta-1}c\right)\frac{1}{n\left(\frac{n(1+\eta)}{n\eta-1}c+nc\right)} \\ &= \frac{(n+1)(1-\gamma(n))}{n\eta-1}\frac{1}{n\left(\frac{n\eta(n+1)}{n\eta-1}\right)} \\ &= \frac{1-\gamma(n)}{n^2\eta} \qquad (25)\end{aligned}$$

Combining equations (24) and (25) we obtain the efficiency gains needed to keep the welfare constant as a function of the price increase and of the elasticity:

$$\frac{\Delta c}{c} = \frac{1}{n\eta-1}\left(1 - n^2\eta\frac{\Delta P}{P}\right) \qquad (26)$$

Using this relation and considering n as a continuous variable, we obtain the results summarized in Table 3. ■

Proof of Proposition 2. To compute the efficiency gains needed to have a profitable merger, compare the profits of merging firms before and after the merger

$$\begin{aligned}\pi_m^m &\geq 2\pi_i^* \\ \frac{[(a-c)+(n-1)\Delta c]^2}{bn^2} &\geq \frac{2(a-c)^2}{b(n+1)^2}\end{aligned}$$

Being interested only in the positive root of the problem, it is possible to make the square root of both sides and, simplifying the terms

$$\begin{aligned}(n+1)(n-1)\Delta c &\geq \left(\sqrt{2}n-n-1\right)(a-c) \\ \Delta c &\geq \frac{(a-c)}{n+1}\frac{n\left(\sqrt{2}-1\right)-1}{n-1}\end{aligned}$$

Defining

$$\phi(n) \triangleq \frac{n\left(\sqrt{2}-1\right)-1}{n-1}$$

and using the equation (22) for the MC ratio

$$\frac{\Delta c}{c} \geq \phi(n) \frac{P^* - c}{c}$$

To compare $\phi(n)$ and $\gamma(n)$

$$\begin{aligned}
\phi(n) &\geq \gamma(n) \\
\frac{n\left(\sqrt{2}-1\right)-1}{n-1} &\geq \frac{n\sqrt{n^2+8n+4}-(n+1)^2}{2n^2-2n-1} \\
\sqrt{2}n\left(2n^2-2n-1\right)-n\left(n^2-n-1\right) &\geq n(n-1)\sqrt{n^2+8n+4} \\
\sqrt{2}\left(2n^2-2n-1\right)-\left(n^2-n-1\right) &\geq (n-1)\sqrt{n^2+8n+4}
\end{aligned}$$

After some algebraic manipulations, the unique real and positive solution is

$$n \geq \frac{1}{2}\left(3+\sqrt{2}+\sqrt{7}\right) \simeq 3.53$$

In Figure 4 $\phi(n)$ and $\gamma(n)$ are compared. ■

Proof of Proposition 3. The simplest way to compare consumer surplus is to compare the price before and after the merger. A merger increases consumer surplus if

$$\begin{aligned}
P^* &\geq P^m \\
\frac{a+nc}{n+1} &\geq \frac{a+(n-1)c-\Delta c}{n} \\
a-c-(n+1)\Delta c &\leq 0 \\
\frac{\Delta c}{c} &\geq \frac{a-c}{n+1} = \frac{P^*-c}{c}
\end{aligned}$$

where the last equality comes from equation (22). ■

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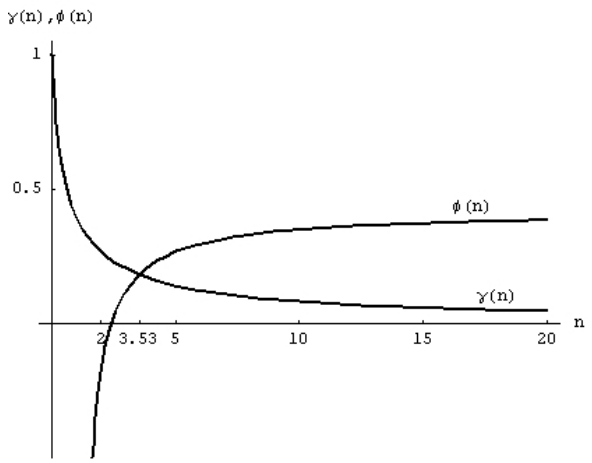

Figure 4: $\phi(n)$ and $\gamma(n)$ as functions of the number of firms in the pre-merger situation. They define the cost savings as a proportion of the pre-merger MC ratio needed to keep the welfare constant and to have merging firms' profits unchanged, respectively.

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