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Empirical Applications of the Interacted Panel VAR Model

Coordinatore

Ch.mo Prof. Sergio Pietro Destefanis

Relatore

Ch.mo Prof. Sergio Pietro Destefanis

Correlatore

Ch.mo Prof. Matteo Fragetta

Candidato

Mario Di Serio

Matr. 8801000005

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Abstract

The Vector Autoregressive (VAR) Models can be considered as a dynamic multivariate extension of the univariate autoregressive models. This family of models has become very popular in macroeconomics analysis after the work of Sims(1980) and they are widely used in time series literature thanks to their flexibility. As a matter of fact, by setting appropriately a VAR model, we can describe efficiently the dynamics of the economy and provide quite accurate forecasts.

During recent years, researchers developed different VAR models with the purpose to represent better the data generating process. Among these, the nonlinear VAR models have gained a central role in macroeconometric analysis in testing the theory, due to their capacity to capture a richer set of dynamics regarding current macroeconomic phenomenons. Depending on the specific model, they can allow, for example, different states (regimes) of the world, to allow the coefficients of the model to vary over time in each time unit, allowing for interactions between variables potentially revealing important information. The first paper included in this thesis is a survey which have the purpose to examine linear and nonlinear VAR models.

The second and third papers present two empirical applications of the Interacted Panel VAR Model, which is a new nonlinear methodology we illustrated over the first paper. Specifically, we analyze in both papers the behavior of government spending multiplier when the interest rate is at the Zero Lower Bound (ZLB). This is a highly topical question since the outbreak of Great Recession, given that many policy makers have wondered whether fiscal stimulus would be able to help the economy to recover from recession. In particular, there exist two different and opposite theoretical predictions. New Keynesian DSGE models show that, when the interest rate is at the ZLB, a raise in government spending has a strong and positive impact on the economy. On the other side, theoretical prediction indicate very low multipliers, showing that an increase in government spending does not stimulate private activity.

Although there exist many theoretical predictions about the size of government spending multiplier at the ZLB, very few empirical evidences are provided. These two paper aim to shed light on the size of the government spending multiplier at the ZLB. Among the nonlinear VAR models, we choose the Interacted (Panel) VAR Model because it offers an important advantage compared to others nonlinear approaches. Thanks to the interaction term, we are able to investigate among the entire sample. This can be done also within a time varying framework, but it implies a larger number of estimates which requires informative priors. In order to be as more agnostic as possible, we also use a Bayesian approach for inference but with uninformative priors.

In the first paper we develop an Interacted VAR Model and conduct our analysis on the United States sample. In order to identify government spending shocks we use the sign restrictions approach, furthermore we use the forecast series of government spending to account for the potential effects of anticipation that can pose serious problems for the identification of government spending shocks. We find that the government spending multiplier ranges between 3.4 and 3.7 at the ZLB, while it ranges from 1.5 to 2.7 away from the ZLB. Then, we develop a Factor-Augmented IVAR (FAIVAR) model with the purpose to address another limited information problem. It confirms our results from a qualitatively point of view. As a matter of fact, the government spending multiplier ranges between 2.0 and 2.1 at the ZLB and between 1.5 and 1.8 away from the ZLB. These results are also in line with some recent studies which predict higher multipliers at the ZLB than in normal times.

In the second paper, we extend our analysis to the Euro Area countries by developing an Interacted Panel VAR Model (IPVAR). Also in this paper, we identify government spending shocks using sign restriction, and use the European Commission forecast of government spending to account for fiscal foresight. We find higher multipliers for times when we are away from the ZLB: the government spending multiplier ranges between 0.33 and 0.88 in the low interest rate state, while it ranges between 1.10 and 1.29 in the high interest rate state. However,we consider a Factor-Augmented IPVAR framework (FAIPVAR), we find that the government spending multiplier at the ZLB is very similar to multipliers computed in normal times, ranging between 1.08 and 1.41 at the ZLB and between 1.26 and 1.39 away from the ZLB. Next, we divide our sample into two groups of countries with high and low levels of debt-to-GDP ratio. The purpose of this exercise is to understand if the size of the government spending multiplier is influenced by the level of debt-to-GDP ratio. Considering, from our point of view, the more reliable specification with factors that contains a richer set of information, we find that if the debt-to-GDP ratio is low, the government spending multiplier is higher than multipliers computed when the debt-to-GDP is high.

Results for both papers are in line with New Keynesian DSGE models predictions, showing that a one unit shock of government spending raises GDP by more than 1%. In case of the US sample, we find that the government spending multipliers are larger when the interest rate is at the ZLB. On the other hand, the EA sample would not seem to support the latter result. Our interpretation is that, the EA findings may be influenced by a subset of countries that experienced high level of debt (especially during the crisis), which we have found to have depressive effect on the multipliers and which might be stronger than the positive effect exerted by the favorable conditions illustrated in some theoretical models at the ZLB. Overall, we argue that a raise in government spending might be a useful additional instrument for policymakers to solve deep recessions, when monetary policy is at the ZLB, although the effect produced by unconventional monetary policies have currently shown to be more difficult to identify.

Contents

Chapter 1

A survey on Linear and Nonlinear Vector Autoregressive Models

Introduction

Starting from the work of Sims (1980), Vector Autoregressive (VAR) Models have proven to be useful instruments to capture the dynamic relationships between variables and to provide quite accurate economic forecast. A VAR model is a system of simultaneous equations where the endogenous variables are regressed on their own lags and lags of the other endogenous variables. Moreover, it allows us to study the impact of innovations in one variable in the system and therefore on all the endogenous variables. This survey has the aim to give an insight into linear and nonlinear VAR time series analysis.

Section 1.1 analyzes the characteristic of linear VAR models. We describe how to approximate correctly the data generating process using a reduced-form VAR model. Here, is crucial to specify correctly the VAR model by properly choosing variables, lag order and eventually modelling breaks. Then we illustrate the estimation procedures with classical least squares and Bayesian methods. We conclude the discussion about reduced form VAR analysis by describing in section 1.1.5 the various strategy to check if our VAR model is subject to misspecifications.

Next, we discuss how to recover the structural VAR innovations starting from the estimated reduced form parameters. Specifically, through section 1.1.6, we illustrates the identification strategies necessary to recover structural parameters. Here, the economic theory play an important role and help us to set the restrictions necessary to recover the structural VAR model. Specifically, we describe three type of restrictions: short-run restrictions, log-run restrictions and sign restrictions.

Section 1.1.6.3 illustrates how to compute impulse responses functions starting from the residuals of the structural form. Moreover, in section 1.1.9, we illustrate also the procedure to recover the generalized impulse response functions, which are obtained without imposing constraints on the reduced form VAR model.

The second part of this survey is focused on the analysis of nonlinear VAR models. As a matter of fact, it can be the case that a standard linear VAR cannot explain well the relationship between the variables of interest. The use of nonlinear VAR models can be suggested by theory or more simply by the observed time series. For example, suppose that we want to compute the fiscal multiplier associated to a government spending shock. The results we would get can depend on whether we are in expansionionary or recessionary phase of the business cycle. In this case, a nonlinear VAR should perform better than standard VAR models. Through section 1.2, we illustrate regime-switching models like Threshold VAR and Smooth Transitions VAR, and the Interacted VAR model.[1]

1.1 Linear Vector Autoregressive Analysis

The Vector Autoregression Model (VAR)[2] is the most common way used to summarize information about comovement among variables. Specifically, it captures the dynamics interdependencies among multiple time series. It is composed by n equation and n variables, where every variable is explained by its own lagged value and (possibly) current and past values of the other n - *1* variables. For example, considering only two variables y_t and z_t:

$$y_t = b_{10} - b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt} \tag{1.1}$$

[1] This survey is not aimed at covering all possible nonlinear VAR specifications. We only present the ones that we retains as the most important.

[2] Introduced by Sims (1980)

$$z_t = b_{20} - b_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt} \tag{1.2}$$

where ε_{yt} and ε_{zt} have standard deviations equal to σ_y and σ_z respectively, and $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$ are white-noise disturbances. Moreover ε_{yt} and ε_{zt} are pure innovations or shocks to y_t and z_t.

The system above constitutes the structural form of a first-order vector autoregression (VAR). As we can see, the two variables affect each other directly through b_{12}, b_{21}, γ_{12}, γ_{21} and indirectly through the error terms.

Reordering and writing 1 and 2 in a matrix form:

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} \tag{1.3}$$

defining $B = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}$, $x_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$, $\mathit{\Gamma}_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}$, $\Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$, $\varepsilon_t = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$ we can rewrite 3 as:

$$Bx_t = \mathit{\Gamma}_0 + \mathit{\Gamma}_1 x_{t-1} + \varepsilon_t \tag{1.4}$$

premultiplying equation 1.4 by B^{-1}:

$$x_t = A_0 + A_1 x_{t-1} + e_t \tag{1.5}$$

where $A_0 = B^{-1}\mathit{\Gamma}_0,\ A_1 = B^{-1}\mathit{\Gamma}_1,\ e_t = B^{-1}\varepsilon_t$.

Equation 1.5 represents the VAR in reduced form, where the error term are:

$$e_{1t} = \frac{\varepsilon_{yt} - b_{12}\varepsilon_{zt}}{1 - b_{12}b_{21}} \tag{1.6}$$

$$e_{2t} = \frac{\varepsilon_{zt} - b_{21}\varepsilon_{yt}}{1 - b_{12}b_{21}} \tag{1.7}$$

They have zero means, constant variances and are individually serially uncorrelated.

Moreover, they incorporates both shocks ε_{yt}, ε_{zt}. The variance-covariance matrix of the e_{1t} and e_{2t} is equal to:

$$\sum = \begin{bmatrix} var(e_{1t}) & cov(e_{1t,}e_{2t}) \\ cov(e_{1t,}e_{2t}) & var(e_{2t}) \end{bmatrix} \tag{1.8}$$

1.1.1 Stability and Stationarity

A stochastic process is called stationary if its mean and variance are time invariant. It means that the time series tend to return to their means and their fluctuations around the mean have constant amplitude. In a VAR context:

$$\begin{aligned} y_t &= a_{10} + a_{11}y_{t-1} + a_{12}z_{t-1} + e_{1t} \\ z_t &= a_{20} + a_{21}y_{t-1} + a_{22}z_{t-1} + e_{2t} \end{aligned} \tag{1.9}$$

we can rewrite the reduced form using lag operators:

$$\begin{aligned} (1 - a_{11}L)y_t &= a_{10} + a_{12}Lz_t + e_{1t} \\ (1 - a_{22}L)z_t &= a_{20} + a_{21}Lz_t + e_{2t} \end{aligned} \tag{1.10}$$

solving for z_t and y_t

$$\begin{aligned} y_t &= \tfrac{a_{10}(1-a_{22})+a_{12}a_{20}+(1-a_{22}L)e_{1t}+a_{12}e_{2t-1}}{(1-a_{11}L)(1-a_{22}L)-a_{12}a_{21}L^2} \\ z_t &= \tfrac{a_{20}(1-a_{11})+a_{21}a_{10}+(1-a_{11}L)e_{2t}+a_{21}e_{1t-1}}{(1-a_{11}L)(1-a_{22}L)-a_{12}a_{21}L^2} \end{aligned} \tag{1.11}$$

to have the stability condition satisfied, the roots of the polynomial $(1 - a_{11}L)(1 - a_{22}L) - a_{12}a_{21}L^2$ must lie outside the unit circle.

1.1.2 Specification

1.1.2.1 Choosing variables and lags

Usually the choice of variables of interest are based on institutional knowledge and/or theoretical models. For example DSGE model can give good advice on which variables to include in a VAR. In theory, we can include all variables we need in our VAR. However, adding variables means loosing degree of freedom making our estimates more imprecise. The same happens for lags: choosing long lag length will erodes degrees of freedom. Specifically, we need to estimate np parameters plus intercept - where n are the number of variables, p the lag length - which have to be less than number of observations T. It is generally inopportune to add to each equation more than $T/3$ parameters, thus we should impose the following constraint:

$$3np < T \tag{1.12}$$

On the other hand, if we choose short lag length, we could incur in misspefication problems due to the fact that we might not fully capture the persistence of the variables considered. Economic theory can help us to choose the right lag length, otherwise we could use statistical criteria.

The likelihood ratio test, compares the residuals of two VAR with same variables but different lag length. Following Sims (1980), we compare the unrestricted system with restricted system using:

$$(T - c)\left(ln\left|\sum_r\right| - ln\left|\sum_u\right|\right) \tag{1.13}$$

where T is the number of observations, c is the number of parameters estimated in each equation of the unrestricted system, $ln\left|\sum_u\right|$ is the natural logarithm of the determinant of the variance-covariance matrix of the unrestricted model, $ln\left|\sum_r\right|$ is the natural logarithm of the determinant of the variance-covariance matrix of the restricted model.

The above statistic is asymptotically distributed as a χ^2 which have degrees of freedom equal to the number of restrictions. Given the significance level, if the value of this statistic is less than the value of χ^2 we cannot reject the null hypothesis and therefore we choose the restricted model.

It is also possible to choose different lag lengths for each variable in our VAR model. In this way, we obtain an unbalanced VAR, named near-VAR.[3] Thus, if the equation of the unrestricted model have a different number of regressors, c represents the maximum number of regressors of the longest equation.

The likelihood ratio test is applicable only if we have nested models. Moreover, since it is based on asymptotic theory, it is not advisable when we have a small sample. The AIC and SBC can be considered as alternatives to likelihood ratio test:

$$AIC = T\, ln\left|\sum\right| + 2N \tag{1.14}$$

$$SBC = T\, ln\left|\sum\right| + N\, ln(T) \tag{1.15}$$

where $\left|\sum\right|$ is the determinant of variance/covariance matrix of residuals and N is the total number of parameters estimated in all equations. In this case we will choose the model which have the minimum value of AIC or SBC.

[3]In this case, it is important to highlight that OLS estimates are not efficient. We should estimates the VAR coefficients using seemingly unrelated regressions (SUR).

1.1.2.2 Granger Causality

The Granger Causality test is a test of causality which determines if the current and past values of one variable are useful to forecast the future value of another variable. In practice, we want to know if the lag and current value of one variable enter into the equation of another variable. Writing out a n-equation VAR:

$$\begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{nt} \end{bmatrix} = \begin{bmatrix} A_{10} \\ A_{20} \\ \vdots \\ A_{n0} \end{bmatrix} + \begin{bmatrix} A_{11}(L) & A_{12}(L) & \vdots & A_{1n}(L) \\ A_{21}(L) & A_{22}(L) & \vdots & A_{2n}(L) \\ \vdots & \vdots & \vdots & \vdots \\ A_{n1}(L) & A_{n2}(L) & \vdots & A_{nn}(L) \end{bmatrix} \begin{bmatrix} x_{1t-1} \\ x_{2t-1} \\ \vdots \\ x_{nt-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ \vdots \\ e_{nt} \end{bmatrix} \tag{1.16}$$

where A_{i0} are the intercept terms, $A_{ij}(L)$ are polynomials in lag operator L which are denoted by $a_{ij}(1)$, $a_{ij}(2)$..., the terms e_{it} are white-noise disturbances. The Granger causality test states that the variable j does not Granger cause variable i if $A_{ij}(L) = 0$.

Considering a model with only two variables with p lags, $\{y_t\}$ does not Granger cause $\{z_t\}$ if $A_{21}(L) = 0$. To test this hypothesis we can use an F-test on the following restriction:

$$a_{21}(1) = a_{21}(2) = a_{21}(3) = \ldots = a_{21(p)} = 0 \tag{1.17}$$

It is important to note that we can use the F-test only if all variables in the VAR are stationary. Alternatively, if for example $\{y_t\}$ is $I(1)$ and $\{z_t\}$ is $I(0)$, we can use a t-test. If instead, all variables are in first differences we can use both f-test and t-test.

1.1.3 Classical Estimation

Considering the reduced-form of a generic VAR(p):

$$\begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{nt} \end{bmatrix} = \begin{bmatrix} A_{10} \\ A_{20} \\ \vdots \\ A_{n0} \end{bmatrix} + \begin{bmatrix} A_{11}(L) & A_{12}(L) & \vdots & A_{1n}(L) \\ A_{21}(L) & A_{22}(L) & \vdots & A_{2n}(L) \\ \vdots & \vdots & \vdots & \vdots \\ A_{n1}(L) & A_{n2}(L) & \vdots & A_{nn}(L) \end{bmatrix} \begin{bmatrix} x_{1t-1} \\ x_{2t-1} \\ \vdots \\ x_{nt-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ \vdots \\ e_{nt} \end{bmatrix} \tag{1.18}$$

where A_{i0} are the intercept terms, $A_{ij}(L)$ are polynomials in lag operator L which are denoted by $a_{ij}(1), a_{ij}(2)...$, the terms e_{it} are white-noise disturbances.

We can write it out in a more compact way:

$$x_t = [A_0, A_1, A_2 \dots, A_p]Z_{t-1} + e_t \tag{1.19}$$

where x_t is a vector of n elements which contains all variables included in the VAR, A_0 is a $n \times 1$ intercepts vector, $A_1 \dots A_p$ are the coefficients, $Z_{t-1} = (1, x_{1t-1}, x_{2t-1}, \dots, \mathrm{x}_{\mathrm{nt}-1})'$ and e_t is a $n \times 1$ error terms vector.

We can efficiently estimate $n + pn^2$ parameters equation by equation using OLS. If the process $\{x_t\}$ is normally distributed and $u_t \sim N(0, \Sigma_u)$ we would obtain the same results adopting the maximum likelihood estimator.

Moreover, given that no restrictions on the parameters are imposed, the estimator is also identical to GLS estimator and it is equal to:
$[\hat{A}_0, \hat{A}_1, \hat{A}_2 \dots, \hat{A}_p] = \left(\sum_{t-1}^{T} x_t Z'_{t-1}\right) \left(\sum_{t-1}^{T} Z_t Z'_{t-1}\right)^{-1}$

In the case of non-stationary variables, Sims (1980) and Sims et al. (1990) advice to not differencing, because it causes the lost of important information about the comovements among the data. Thus, we can still use OLS or

ML estimator, which continue to be asymptotically normal under general conditions.[4]

1.1.4 Bayesian Estimation

An issue of VAR models is that they generally involve many variables and consequently many parameters to estimate. Moreover, in many macroeconomic applications some assumptions made on VAR models become somehow unrealistic. For example, considering VAR coefficients constant over time means that the relationship between variables remains constant along time, which might be a strong assumption.[5] Moreover, the volatility of some macroeconomic variables might be not constant over time, and this can be accomodated by allowing the error covariance matrix to change over time.

Although a researcher would prefer to use time-varying VAR Models because of their realism, he has to deal with problem related to over - parameterization. Bayesian estimation, through the introduction of prior information, shrink parameters and so it reduce over-parameterization problems. Bayesian methods consist of conditioning β to a prior distribution of $\sum_e$ (e.g. $p(\beta|\Sigma_e)$). Combining the likelihood function with the prior distribution, we obtain the posterior distribution of β.

The Bayes' rule is represented by equation:

$$p(\theta|\gamma) = \frac{p(\gamma|\theta)p\left(\theta\right)}{p\left(\gamma\right)} \tag{1.20}$$

where p denotes the probability distribution, θ collect the parameters that we want to estimate, γ collect the available data that we use for the esti-

[4] see Park and Phillips (1988, 1989); Sims et al. (1990); Lütkepohl (2005).

[5] In Section 1.2 we focus on some nonlinear VAR model where parameters are allowed to vary over time.

mation. $p(\theta|\gamma)$ represents the posterior distribution, which is the conditional distribution of parameters θ given the data γ. $p(\gamma|\theta)$ represents the likelihood function, which is equivalent to the conditional distribution of the data γ given the parameters θ. $p\,(\gamma)$ is the marginal likelihood ad it is equal to the integral of the joint distribution of the parameters and data:

$$p\,(\gamma) = \int p(\gamma|\theta)p\,(\theta)\,d\theta \tag{1.21}$$

Since the marginal likelihood is a constant, we can conclude that:

$$p(\theta|\gamma) \propto L(\theta;\gamma)p(\theta) \tag{1.22}$$

The term $p\,(\theta)$ is the prior distribution, which is the marginal distribution of the parameters θ. It represents the belief that the researcher have about the parameters. Imposing an uninformative prior distribution means that we have no prior belief about parameters, on the other hand imposing a strong prior means that we have strong belief about the probability distribution of parameters.

Consider now a basic VAR equation:

$$y_t = A_0 + A_1 y_{t-1} + \ldots + A_p y_{t-p} + e_t \tag{1.23}$$

where A_0 is a $N \times 1$ vector denoting the constant term, $\mathrm{A_1}, \ldots, \mathrm{A_p}$are $N \times N$ matrix of autoregressive coefficients and e_t is the error component which is normally distributed with zero mean and covariance matrix equal to Σ.

Denoting a vector $K \times 1$ (where $K = 1 + pN$), which pick up all data found in the right hand side of equation 1.23, such that $x_t = (1, y_{t-1}, \ldots, y_{t-k})$.

Collecting the intercept term and the autoregressive matrices in a $K \times N$ matrix, such that $A = (A_0, A_1, \ldots, A_p)$. We can rewrite equation 1.23 as:

$$y_t' = x_t' A + e_t' \tag{1.24}$$

Considering the vectors y_t', x_t' and e_t' all periods T, we denote $\Upsilon = (y_1, y_2, \ldots, y_T)'$ with dimension $\mathrm{T} \times \mathrm{N}$, $X = (x_1, x_2, \ldots, x_T)'$ with dimension $\mathrm{T} \times \mathrm{K}$, $U = (e_1, e_2, \ldots, e_T)'$ with dimension $\mathrm{T} \times \mathrm{N}$, we can write equation 1.24 as:

$$\Upsilon = XA + U \tag{1.25}$$

where the error term matrix U is conditionally independently normally distributed and follow a matric-variate normal distribution, which have zero mean and covariance matrix equal to Σ :

$$U \sim \mathcal{MN}_{TN}(0_{TN}, \Sigma, I_T) \tag{1.26}$$

where 0_{TN} is a $T \times N$ matrix, and I_t is an identity matrix of order T which represents the coloumn-specific covariance matrix.

Thus, the conditional distribution of Υ follow a normal-Wishart distribution:

$$\Upsilon | A, \Sigma \sim \mathcal{MN}_{TN}(XA, \mathit{\Sigma}, I_t) \tag{1.27}$$

which results from the combination of equation 1.24 and 1.26.

The likelihood function will be equal to:

$$L(A,\Sigma,\Upsilon) = (2\pi)^{-\frac{TN}{2}} |\Sigma|^{-\frac{T}{2}} \times \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}(\Upsilon - XA)'(\Upsilon - XA)\right]\right\} \tag{1.28}$$

which can we rewritten as:

$$\begin{aligned} L(A,\Sigma,\Upsilon) \propto |\Sigma|^{-\frac{T}{2}} \times \exp\left\{-\tfrac{1}{2}tr\left[\Sigma^{-1}(A-\hat{A})'X'X\left(A-\hat{A}\right)\right]\right\} \\ \times \exp\left\{-\tfrac{1}{2}tr\left[\Sigma^{-1}(\Upsilon - X\hat{A})'\left(\Upsilon - X\hat{A}\right)\right]\right\} \end{aligned} \tag{1.29}$$

where $\hat{A} = (X'X)^{-1}X'\Upsilon$ is the least squares estimator of A.

Thus we can conclude that the likelihood function follow a normal-Wishart distribution, which have the following parameters:

$$\begin{aligned} \Upsilon|A,\Sigma \sim \mathcal{MN}_{KN}(\hat{A},\mathit{\Sigma},(X'X)^{-1}) \\ \Sigma|\Upsilon \sim \mathcal{IW}_N\left(\left(\Upsilon - X\hat{A}\right)'\left(\Upsilon - X\hat{A}\right), T-K-N-1\right) \end{aligned} \tag{1.30}$$

As is clear, priors are very important to determine the posterior distribution: they influence the VAR coefficients, drawing them away from OLS estimates, to the prior mean. There exists a rich bouquet of priors for different purpose. For example, one would set a specific prior: to reflect general properties of macroeconomic time series, for fast computations, to have good forecast performance, to achieve good flexibility in the model, and so on.

Furthermore, to derive posterior and predictive densities, we can distinguish between priors for which we can derive results analytically and priors which require Markov chain Monte Carlo (MCMC) methods, such as Gibbs sampler. In order to illustrate the procedure to recover analytically the posterior distribution, we introduce one of the most used prior which is the natural-conjugate prior distribution. The main characteristic of this prior is that the density of the posterior distribution is the same of prior distribution, on the other hand it is important to highlight that this type of prior does not allow restriction on parameters.

Thus, if we assume a normal-Wishart prior distribution we would have:

$$\begin{aligned} A|\Sigma &\sim \mathcal{MN}_{KN}\left(\underline{A}, \Sigma, \underline{V}\right) \\ \Sigma &\sim \mathcal{IW}_N(\underline{S}, \underline{v}) \end{aligned} \tag{1.31}$$

where $\underline{A}$ is the prior mean of A, $\underline{V}$ is proportional to its column-specific covariance matrix, $\underline{S}$ is the scale matrix of the inverse Wishart prior distribution for Σ , and $\underline{v}$ is degree of freedom parameter. These terms are called hyper-parameters, and are specified by the researcher.

Thus, the joint prior distribution would be:

$$p(A, \Sigma) = p(A|\Sigma)p(\Sigma) \tag{1.32}$$

which is proportional to:

$$\begin{aligned} p(A, \Sigma) \propto |\Sigma|^{-\frac{\underline{v}+N+K+1}{2}} &\times \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}(A-\underline{A})'\underline{V}^{-1}(A-\underline{A})\right]\right\} \\ &\times \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}\underline{S}\right]\right\} \end{aligned} \tag{1.33}$$

To get the posterior distribution we have to substitute the likelihood function and the prior distribution in the kernel of the posterior distribution. Thus, we substitute equation 1.29 and 1.33 into equation equation 1.22:

$$\begin{gathered} p(A,\Sigma|\Upsilon) \propto |\Sigma|^{-\frac{\underline{v}+N+K+1}{2}} \times \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}(A-\hat{A})'X'X\left(A-\hat{A}\right)\right]\right\} \times \\ \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}(A-\underline{A})'\underline{V}^{-1}(A-\underline{A})\right]\right\} \times \\ \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}(\Upsilon-X\hat{A})'\left(\Upsilon-X\hat{A}\right)\right]\right\} \times \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}\underline{S}\right]\right\} \end{gathered} \tag{1.34}$$

Therefore, the conditional posterior distribution of A given Σ and Υ and the marginal posterior distribution of Σ can be written as:

$$p(A,\Sigma|\Upsilon) = p(A|\Upsilon,\Sigma)p(\Sigma|\Upsilon) \tag{1.35}$$

where $p(A|\Upsilon,\Sigma)$ and $p(\Sigma|\Upsilon)$ follow a matric-variate distribution and a inverse Wishart distribution, respectively.

$$\begin{aligned} A|\Upsilon,\Sigma &\sim \mathcal{MN}_{KN}(\overline{A},\Sigma,\overline{V}) \\ \Sigma|\Upsilon &\sim \mathcal{IW}_N(\overline{S},\overline{v}) \end{aligned} \tag{1.36}$$

where

$$\begin{aligned} \overline{V} &= \left(\underline{V}^{-1}+X'X\right)^{-1} \\ \overline{A} &= \overline{V}(\underline{V}^{-1}\underline{A}+X'\Upsilon) \\ \overline{v} &= \underline{v}+T \\ \overline{S} &= \underline{S}+\Upsilon'\Upsilon+\underline{A}'\underline{V}^{-1}\underline{A}-\overline{A}'\overline{V}^{-1}\overline{A} \end{aligned} \tag{1.37}$$

The mean of the posterior distribution of A would be:

$$\overline{A} = \overline{V}\underline{V}^{-1}\underline{A}+\overline{V}X'X\hat{A} = \Omega_1\underline{A}\Omega_2\hat{A} \tag{1.38}$$

where $\Omega_1 = \overline{V}\underline{V}^{-1}$ and $\Omega_2 = \overline{V}X'X$. As we can see, the mean posterior distribution is a linear combination of the prior mean $\underline{A}$ and the OLS estimates of Â. Note that if we impose a uninformative prior distribution, by setting the diagonal elements of $\underline{\mathrm{V}}$ to infinity, we simply obtain the the OLS estimates of $\hat{\mathrm{A}}$. On the other hand, if we impose a strong prior, which assign probability equal to 1 to the prior mean and zero probability to OLS estimates we have that the posterior mean will be equal to $\underline{\mathrm{A}}$.

1.1.5 Model Diagnostic

Through this section we basically focus on the following question: does our VAR Model represent appropriately the data generating process of the system of variables? Here, we illustrate some useful tests to answer this important question: test for autocorrelation, test for nonnormality, conditional hetero-skedasticity test and the test for parameters time invariance.

1.1.5.1 Test for Autocorrelation

As we have seen the choice of lag order is a crucial point to set appropriately our VAR model, specifically we should set a lag order which approximately satisfy one of our basic assumption: resulting reduced form residuals are not serially correlated. In other words we have to check that $E\left[e_t e'_{t-1}\right] = 0$ for $i = \mathit{1}, \mathit{2}, \ldots$.

The Portmanteau Test is a useful tool to test the null that $E\left[e_t e'_{t-1}\right] = 0$ for $i = \mathit{1}, \mathit{2}, \ldots$. versus the alternative hypothesis $E\left[e_t e'_{t-1}\right] \neq 0$ for $i = \mathit{1}, \mathit{2}, \ldots$ The associated test statistic is:

$$Q_h = T \sum_{j=1}^{h} tr\left(\hat{C}'_j \hat{C}'^{-1}_0 \hat{C}_j \hat{C}_0^{-1}\right) \tag{1.39}$$

where $\hat{C}_j = T^{-1} \Sigma_{t=j+1}^{T} \hat{e}_t \hat{e}'_{t-j}$, h is the number of autocovariance terms, p is the lag order and $\hat{e}_t$ are the estimated residuals. When the VAR is stationary, there are no parameter restrictions and T and h are large, the distribution of Q_h under the null hypothesis is approximated to a $\chi^2(N^2(h-p))$. In small sample it can be possible that the approximated χ^2 distribution can be different from the actual distribution. For this reason, Hosking (1980) suggested to use a modified statistic:

$$Q_h = T^2 \sum_{j=1}^{h} \frac{1}{T-j} tr \left(\hat{C}_j' \hat{C}_0'^{-1} \hat{C}_j \hat{C}_0^{-1} \right) \tag{1.40}$$

As suggested in Lütkepohl (2005) if we have imposed parameter restrictions in our VAR, we have to adjust degrees of freedom. Specifically, it would be equal to the difference between the number of autocovariances included in the statistic, i.e. N^2h, and the number of parameter estimated in our VAR.

Moreover, it is important to highlight that h should be much larger than p. Thus, to ensure the powerful of the test, we should test for a large number of autocovariances. If instead, we want to test for a small number of autocovariance, it is advisable to use the LM test proposed by Breusch (1978) and Godfrey (1978). Consider an auxiliary model:

$$\hat{e}_t = v + A_1 y_{t-1} + \ldots + A_p y_{t-p} + D_1 \hat{e}_{t-1} + \ldots + D_h \hat{e}_{t-h} + u_t^* \tag{1.41}$$

where $\hat{e}_t$ are the estimated residuals of a VAR model of order p, $u_t^*$is an auxiliary error term and the values of $\hat{e}_t$ with $t \leq 0$ are replaced with zero.

We want to test the null hypothesis $\mathbb{H}_0$: $D_1 + \ldots + D_h = 0$ against the alternative $\mathbb{H}_1$: $D_i \neq 0$ for at least one $i \epsilon \{1, \ldots, h\}$. The associated statistic is computed as:

$$Q_{LM} = T \left(N - tr \left(\widetilde{\Sigma}_e^{-1} \widetilde{\Sigma}_u \right) \right) \tag{1.42}$$

where $\hat{u}_t^*$ are the residuals of the auxiliary model, $\widetilde{\Sigma}_e = T^{-1}\Sigma_{t=1}^{T}\hat{e}_t\hat{e}_t'$, $\widetilde{\Sigma}_u = T^{-1}\Sigma_{\mathrm{t=1}}^{\mathrm{T}}\hat{\mathrm{u}}_{\mathrm{t}}^*\hat{\mathrm{u}}_{\mathrm{t}}^{*\prime}$. The LM statistic is distributed as a $\chi^2(hN^2)$ under the null hypothesis of no autocorrelation. Brüggemann et al. (2006) show that the asymptotic distribution is valid also when there are integrated variables.

1.1.5.2 Test for Nonnormality

Normality of residuals and observed variables is not a strong requirement for VAR modelling. On the other hand, if we find that the distribution of the observations is not normal we can increase efficiency of our model by considering other estimation procedures. Furthermore, if residuals do not have a normal distribution, it can be considered as a sign of other potential problem in our model, e.g. very large residuals.

As an illustration, we describe the normality test developed by Lomnicki (1961) and Jarque and Bera (1987) , which analyses the third and the fourth moment of the residuals and check if they are compatible with normal distribution. Decomposing$\widetilde{\Sigma}_e$ such that $\widetilde{\Sigma}_e = PP'$ and analyze the skewness and kurtosis of the standardized residuals $\hat{e}_t^s = P^{-1}\,\hat{e}_t$.[6]

Consider the vector $\hat{b}_j = \left(\hat{b}_{1j}, \ldots, \hat{b}_{Nj}\right)'$ where $\hat{b}_{nj} = \frac{1}{T}\Sigma_{t=1}^{T}(\hat{e}_{nt}^s)^j$ for j =3, 4. It can be demonstrated that:

$$\lambda_3 = \frac{\hat{b}_3'\hat{b}_3}{6} \xrightarrow{d} \chi^2(N) \quad (1.43)$$

$$\lambda_4 = \frac{\left(\hat{b}_4 - 3_N\right)'\left(\hat{b}_3 - 3_N\right)}{24} \xrightarrow{d} \chi^2(N) \quad (1.44)$$

[6]To decompose $\widetilde{\Sigma}_e$, we follow Lütkepohl (2005) and use a Cholesky decomposition. This approach is described in details in 1.1.6.1.

where $3_{\mathrm{N}} = (3, \ldots, 3)'$ is a N × 1 vector. We use equation 1.43 and 1.44 to test for the symmetry of the distribution and for excess kurtosis, respectively.

1.1.5.3 Conditional Heteroskedasticity Test

Providing that the unconditional error variances are finite, the presence of conditional heteroskedacity of the errors might invalidate consistency of the VAR slope parameters. On the other hand, it weaken the efficiency of the estimator and violate the assumption of i.i.d. errors. Furthermore it can be possible that the errors dynamics may make the fourth moment infinite and consequently complicate the inference procedures.

In order to test for conditional heteroskedasticity we can use once again an LM test. Consider the auxiliary model:

$$vech\left(\hat{e}_t\hat{e}_t^{'}\right) = \delta_0 + D_1 vech\left(\hat{e}_{t-1}\hat{e}_{t-1}^{'}\right) + \ldots + D_q vech\left(\hat{e}_{t-q}\hat{e}_{t-q}^{'}\right) + u_t \quad (1.45)$$

where *vech* is an operator that converts the columns of the matrix from the main diagonal downwards into a column vector, $\hat{e}_t$ are the estimated residuals of a VAR model of order p and u_t^* is an auxiliary error term.

We want to test the following hypothesis:

$$\mathbb{H}_0\colon D_1 + \ldots + D_h = 0$$
$$\mathbb{H}_1\colon D_i \neq 0 \qquad for\ at\ least\ one\ i\epsilon\{1, \ldots, q\}$$

The associated statistic is:

$$LM_{ARCH(q)} = \frac{1}{2}TN\left(N+1\right)\left(1 - \frac{2}{N(N+1)}tr\left(\hat{\Omega}\hat{\Omega}_0^{-1}\right)\right) \quad (1.46)$$

where $\hat{\Omega}$ and $\hat{\Omega}_0$ are the residuals of the model described in equation 1.45 when $q > 0$ and $q = 0$, respectively. The LM statistic can be approximated to a $\chi^2\left(qN^2\frac{(N+1)^2}{4}\right)$.[7]

1.1.5.4 Test for Parameters Time Invariance

We have already seen stability requirements for VAR modelling in 1.1.1. We now focus on the possibility to have changes in parameter values over time. This is an important stationary requirement in a standard VAR analysis, as a matter of fact changes of parameters over time may implicate violation of stability condition.

One of the most used test to check parameter stability over time, is the Chow test. This is substantially a test for structural change, where the null hypothesis is the time invariance of parameters, while the alternative one is a change in parameters value which occurs at a given point in time.

Suppose that we have approximately individuate a date T_B when the break occurs. Let's divide the full sample T into two subsamples T_1 and T_2, such that $T_1 < T_2$ and $T_2 \leq T - T_B$. We can construct a Likelihood-ratio test which compares the maximum of the likelihood obtained in the constant parameter model with its counterpart obtained when we allow for different parameter values.

The associated test statistic is equal to:

$$LR_{Chow} = 2\left[\sup\left(\sum_{t=1}^{T_1} l_t\right) + \sup\left(\sum_{t=T-T_2+1}^{T} l_t\right) - \sup\left(\sum_{t=1}^{T_1} l_t + \sum_{t=T-T_2+1}^{T} l_t\right)\right] \tag{1.47}$$

[7] In finite sample, we can get approximation of critical value using bootstrap procedures. Furthermore, to offset the loose of power of this test in finite sample, we can test each equation for GARCH as an alternative. For further details see Lütkepohl (2005) .

where l_t is the conditional log-density of i^{th} observation vector and it is equal to $l_t = \log f_t\left(y_t | y_{t-1}, \ldots, y_{-p+1}\right)$.

The statistic described in equation 1.47 is distributed under the null hypothesis as a χ^2 distribution, which have degrees of freedom equal to the difference between the total number of the free coefficients estimated in the two subsample and the number of free coefficients in the full sample.

When we don't know exactly the date of the break, or the break is quite plain we may drop some observations from the two subsamples. This operation has also the effect to improve the power of the test in small samples. We should also pay attention to the fact that in small samples the structural change tests may reject the null even if the null is true. This can be explained by the fact that the tests are unable to distinguish between permanent breaks and large transitory dynamics. For this reason, we need to be careful when we interpret the results of tests like Chow test.

1.1.6 Identification

Suppose now the we have estimated our reduced form VAR model and we want to recover the structural form in equation 1.1 and 1.2. The problem we face is that z_t and $\mathrm{y_t}$ are correlated with the error terms $\varepsilon_{\mathrm{yt}}$ and ε_{zt}, respectively. Since the regressors are clearly correlated with error term, we cannot estimate equation 1.1 and 1.2 directly. On the other hand, we can estimate the reduced form VAR model by using OLS and through some appropriate restrictions get the parameters in structural form.

We need some restrictions because the model written in equation 1.1 and 1.2 is underidentified. To make it clear, consider the reduced-form from equation 1.9. We need to estimate 10 parameters in the structural equation (b_{10}, b_{20}, γ_{11}, γ_{12}, γ_{21}, γ_{22}, b_{12}, b_{21}, σ_y and σ_z), but we can estimate only nine parameters in the reduced-form equation (the coefficients a_{10}, a_{20}, a_{11}, a_{12}, a_{21}, a_{22},the variance $var(e_{1t})$, $var(e_{2t})$ and the covariance $cov(e_{1t}, e_{2t})$).

Consider now the VAR described in equation 1.5:

$$x_t = A_0 + A_1 x_{t-1} + e_t \tag{1.48}$$

we can write the variance-covariance matrix of residuals of our VAR as:

$$\Sigma_e = E\left[e_t e_t'\right] = E\left[B^{-1}\varepsilon\left(B^{-1}\varepsilon\right)'\right] = B^{-1}\Sigma_\varepsilon(B^{-1})' \tag{1.49}$$

since we know that $\Sigma_\varepsilon = I$ we can conclude that

$$\Sigma_e = B^{-1}B^{-1'} \tag{1.50}$$

As it is clear, to reach identification we need to know the matrix B^{-1}. As a matter of fact, knowing that $A_1 = B^{-1}\mathit{\Gamma}_1$ and $e_t = B^{-1}\varepsilon_t$ we can recover the structural form:

$$Bx_t = \mathit{\Gamma}_0 + \mathit{\Gamma}_1 x_{t-1} + \varepsilon_t \tag{1.51}$$

Writing equation 1.50 in matrix form and considering only three variable in our VAR:

$$\begin{bmatrix} \sigma^2_{e1} & \sigma^2_{e1,e2} & \sigma^2_{e1,e3} \\ \sigma^2_{e2,e1} & \sigma^2_{e2} & \sigma^2_{e2,e3} \\ \sigma^2_{e3,e1} & \sigma^2_{e3,e2} & \sigma^2_{e3} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}^{-1} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}^{-1'} \tag{1.52}$$

from equation 1.52 we want to derive a system of equation, where we have 9 unknowns, which are the elements of B^{-1}, and only 6 equations (because of the symmetry of Σ_e). For this reason our system is not identified, and we need to impose some restriction to recover the matrix B^{-1}. There are different method to reach identification of the structural form parameters. We illustrates three type of approaches: Cholesky, Blanchard Quah and Sign restriction.

1.1.6.1 Zero Short-run restrictions

Consider the following VAR of 3 variables:

of estimators

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} y_t \\ x_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \\ b_{30} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \\ \varepsilon_{zt} \end{bmatrix} \tag{1.53}$$

The Cholesky decomposition is based on the assumption that ε_{yt} affects contemporaneously all variables, ε_{xt} affects contemporaneously x_t and z_t, and ε_{zt} affects contemporaneously only z_t. In this way we are saying that e_{1t} is attributed only to shock to y_t. Although this decomposition imposes a strong asymmetry, it provides the minimal set of assumptions necessary to identify structural model.

As it is clear, in this context the order of variables is very important and usually it is driven by economic theory. Anyway, we have to be very careful, because we are supposing that some variables have no contemporaneous effect on the others.

This assumptions make the matrix B^{-1} lower triangular. Thus, we can rewrite equation 1.53 as:

$$\begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} y_t \\ x_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \\ b_{30} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \\ \varepsilon_{zt} \end{bmatrix} \tag{1.54}$$

Comparing this system with equation 1.52, now we have 6 unknowns and 6 equations and so we can revert from reduced form to structural form.

We reach identification through the Cholesky factorization of Σ_e. It is based on the concept that a positive definite matrix X can be decomposed as $X = P'P$. For example:

$$if \qquad X = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \qquad then \qquad P = \begin{bmatrix} \sqrt{a} & \frac{b}{\sqrt{a}} \\ 0 & \sqrt{c - \frac{b^2}{a}} \end{bmatrix}$$

where P is an upper triangular matrix and so P′ is lower triangular.

Applying Cholesky factorization to Σ_e:

$$\Sigma_e = P'P \tag{1.55}$$

where P′ is a lower triangular matrix. Furthermore, we know from equation 1.50, that $\Sigma_e = B^{-1}B^{-1'}$ and that B^{-1} is also lower triangular. Thus we can conclude that $P' = B^{-1}$

1.1.6.2 Zero long-run restrictions

An alternative way to zero short-run restrictions is to impose restrictions on long-run responses of variables to a shock. This restrictions was developed by Blanchard and Quah (1989), and allow the researcher to overcome the possible dispute about the right short-run restrictions to impose, by focusing only on the long-run properties of the shock where there is generally more consensus among economists.

Consider the following VAR model:

$$x_t = Ax_{t-1} + B^{-1}\varepsilon_t \quad (1.56)$$

where $A = \Gamma_1 B^{-1}$. For simplicity, Suppose that a shock occurs at time t, the cumulative long run impact on $\mathrm{x_t}$ would be:

$$x_{t,t+\infty} = B^{-1}\varepsilon_t + AB^{-1}\varepsilon_t + A^2B^{-1}\varepsilon_t + \ldots + A^{\infty}B^{-1}\varepsilon_t \quad (1.57)$$

which can be rewritten as:

$$x_{t,t+\infty} = \sum_{j=0}^{\infty} A^j B^{-1}\varepsilon_t = (I-A)^{-1}B^{-1}\varepsilon_t = D\varepsilon_t \quad (1.58)$$

where the matrix D represents the cumulative effect of the shock ε_t on $\mathrm{x_t}$.

To make it clear, considering a VAR composed by 3 equations:

$$\begin{bmatrix} x_{t,t+\infty} \\ y_{t,t+\infty} \\ z_{t,t+\infty} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} \quad (1.59)$$

Each term d_{ij} represents the cumulative long run impact of the shock on our variables of interest, for example, d_{13} represents the cumulative long-run impact of the shock ε_{zt} on x_t

Going back to equation 1.58, we can write DD′ as:

$$DD' = (I-A)^{-1}B^{-1}B^{-1'}(I-A)^{-1'} \quad (1.60)$$

from equation 1.50 we know that $\Sigma_e = B^{-1}B^{-1'}$, thus:

$$DD' = (I - A)^{-1}\Sigma_e(I - A)^{-1'} \tag{1.61}$$

Remembering that to achieve identification we have to identify the matrix B^{-1}, we denote an upper triangular matrix P such that:

$$P'P = (I - A)^{-1}\Sigma_e(I - A)^{-1'} \tag{1.62}$$

Assuming that D is a lower triangular matrix, we can conclude that $D = P'$ and consequently we can define B^{-1} as:

$$B^{-1} = (I - A)D \tag{1.63}$$

1.1.6.3 Sign Restrictions

The sign restrictions approach was developed by Faust (1998), Canova and De Nicolò (2002), and Uhlig (2005). It is an alternative approach which is not based on exclusion restrictions as approaches described in section 1.5.1 and 1.5.2. Moreover, it can be much more easily derived from economic models in comparison to alternative identification approaches.

Considering the Cholesky decomposition of $\Sigma_e = P'P$, we want to find a random squared orthogonal matrix S such that $S'S = I$ and consequently:

$$\Sigma_e = B^{-1}B^{-1} = P'S'SP = \mathcal{P}'\mathcal{P} \tag{1.64}$$

We can construct orthogonal matrices using Givens rotation matrix. For example, in a bivariate model the Givens matrices have the following form:

$$S(\varphi) = \begin{bmatrix} cos\varphi & -sin\varphi \\ sin\varphi & cos\varphi \end{bmatrix} \tag{1.65}$$

where φ ranges from *0* and 2π, such that for each choice of φ we get an orthogonal matrix $S(\varphi)$.

As a result, the matrix $\mathcal{P}'$ is solution for the identification problem[8]:

$$B^{-1} = \mathcal{P}' \tag{1.66}$$

we can generate multiple candidate solution $\mathcal{P}'$ to the identification problem. Among these, we keep only those which generate a structural impact multiplier which satisfy the theory-driven sign restrictions we imposed. In other words, we find a matrix S which satisfy condition 1.64, we identify structural form and find IRFs associated, than we retain them if they satisfy our set of a priori restrictions. We replicate these procedure N times and report the median impulse response function and its confidence intervals.

1.1.7 Impulse Response Function

Consider the reduced form of two-variable VAR(1) model:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \tag{1.67}$$

we can write the model above as a vector moving average (VMA) representation, which enable us to understand what is the path of the variables reactions to a shock:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \overline{y} \\ \overline{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{,} \begin{bmatrix} e_{1t-i} \\ e_{2t-i} \end{bmatrix} \tag{1.68}$$

recalling equation 1.6 and 1.7, the error terms can be written as:

[8] Note that the matrix $\mathcal{P}'$ is not lower triangular anymore.

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \frac{1}{1-b_{12}b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} \tag{1.69}$$

Combining equation 1.67 and 1.68 we obtain:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \overline{y} \\ \overline{z} \end{bmatrix} + \frac{1}{1-b_{12}b_{21}} \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{,} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt-1} \\ \varepsilon_{zt-1} \end{bmatrix} \tag{1.70}$$

which can be rewritten as:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \overline{y} \\ \overline{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix} \begin{bmatrix} \varepsilon_{yt-1} \\ \varepsilon_{zt-1} \end{bmatrix} \tag{1.71}$$

where $\phi_{jk}(i)$ represents the multiplier associated to the i^{th} period after the shock occurs:

$$\phi_i = \frac{A_1^i}{1-b_{12}b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \tag{1.72}$$

thus, the elements $\phi_{jk}(0)$ are the impact multipliers. If the sequence $\{y_t\}$ is stationary, then as i approaches to infinity, the values of $\phi_{\mathrm{jk}}(\mathrm{i})$ converge to zero. This means that if the series is stationary the effect of the shock cannot be permanent.

For example, the cumulative effect of the shock ε_{zt} on the sequence$\{y_t\}$ would be:

$$\sum_{i=0}^{n} \phi_{12}(i) \tag{1.73}$$

since we know that at i^{th} period the values of $\phi_{\mathrm{jk}}(\mathrm{i})$ is zero, we can conclude that the sum above is finite.

The elements $\phi_{11}(\mathrm{i})$, $\phi_{12}(\mathrm{i})$, $\phi_{21}(\mathrm{i})$, $\phi_{22}(\mathrm{i})$ are the impulse responses functions (IRFs). They enable us to recover the path of the variables following a shock ε_{zt} or ε_{yt}.

Thus, to compute IRFs we need to go back from the reduced to the structural form. On the other hand we know that the structural model is underidentified. Basically, it means that we have no way to go from the reduced form to the structural form unless we set appropriate restriction.[9]

It is also important to highlight that, since the IRFs are constructed using estimated coefficients, they enclose error due to parameter uncertainty. For this reason we need to compute confidence intervals which reflect the imprecision of the estimation process.

There exist different methods to computer error bands, we illustrate below a Monte Carlo study for the following AR(p) process:

$$x_t = a_0 + a_1 x_{t-1} + \ldots + a_p x_{t-p} + \varepsilon_t \tag{1.74}$$

First of all we estimate the coefficients a_i and the residuals ε_{i}. Then, we generate bootstrap confidence intervals by randomly generating a T numbers which represents the $\{\varepsilon_t\}$ sequence. The series generated, say $\varepsilon_{\mathrm{t}}^{\mathrm{s}}$, have the same properties of the true error process and can be used to construct a simulated sequence $\{\mathrm{x}_{\mathrm{t}}^{\mathrm{s}}\}$:

$$x_t^s = \hat{a}_0 + \hat{a}_1 x_{t-1}^s + \ldots + \hat{a}_p x_{t-p}^s + \varepsilon_t^s \tag{1.75}$$

[9]Note that we have addressed in detail the identification problem in section 1.1.6.

Now we estimate x_t^s and store the IRFs associated. Than, we repeat the process several times to construct the confidence interval. For example, if we want the 95% confidence interval, we need to exclude the lowest and the highest 2,5%.

If we want to apply this Monte Carlo study to a VAR, we need to remember that in this model the regression residual are correlated. A strategy to get around the problem in a two-variable VAR(1) - like the one described in equation 1.9 - is to draw e_{1t} while keeping fixed the value of e_{2t} corresponding to the same period.

1.1.8 Variance Decomposition

The forecast error variance decomposition is a good instrument to investigate on the relationship between variables of our VAR.

Consider the VMA representation of the structural VAR expressed in equation 1.68. It can be expressed in a more compact way as:

$$x_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i} \tag{1.76}$$

where the h-step-ahead forecast is:

$$x_{t+h} = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t+h-i} \tag{1.77}$$

the corresponding forecast error is:

$$x_{t+h} - E_t \left[x_{t+h} | \mathbb{I}_t \right] = \sum_{i=0}^{h-1} \phi_i \varepsilon_{t+h-i} \tag{1.78}$$

where x_{t+h} is the actual realization of x at horizon $t+h$, $E_t\left[x_{t+h}|\mathbb{I}_t\right]$ is the forecasted value of x at horizon $t+h$ and $\mathbb{I}_t$ represents the information about x_t and the other deterministic variables (if present) up to period t.

Coming back to the two-variables VAR written in equation 1.9 and considering only the $\{\mathrm{y_t}\}$sequence, we can write the n-step-ahead forecast error as:

$$\begin{gathered} y_{t+h} - E_t\left[y_{t+h}|\mathbb{I}_t\right] = \phi_{11}(0)\varepsilon_{yt+h} + \phi_{11}(1)\varepsilon_{yt+h-1} + \ldots \\ \ldots + \phi_{11}(h-1)\varepsilon_{yt+1} + \phi_{12}(0)\varepsilon_{zt+h} + \phi_{12}(1)\varepsilon_{zt+h-1} + \ldots \\ \ldots + \phi_{12}(h-1)\varepsilon_{zt+1} \end{gathered} \tag{1.79}$$

Denoting the error variance of y_{t+h} as $\sigma_{\mathrm{y}}(\mathrm{h})^2$:

$$\begin{gathered} \sigma_y(h)^2 = \sigma_y^2\left[\phi_{11}(0)^2 + \phi_{11}(1)^2 + \ldots + \phi_{11}(h-1)^2\right] + \ldots \\ \ldots + \sigma_z^2\left[\phi_{12}(0)^2 + \phi_{12}(1)^2 + \ldots + \phi_{12}(h-1)^2\right] \end{gathered} \tag{1.80}$$

Then we can decompose the forecast error variance to investigate about the proportion of $\sigma_{\mathrm{y}}(\mathrm{h})^2$ caused by the shock in $\{\varepsilon_{\mathrm{yt}}\}$and $\{\varepsilon_{\mathrm{zt}}\}$:

$$\frac{\sigma_y^2\left[\phi_{11}(0)^2 + \phi_{11}(1)^2 + \ldots + \phi_{11}(h-1)^2\right]}{\sigma_y(h)^2} \tag{1.81}$$

$$\frac{\sigma_z^2\left[\phi_{12}(0)^2 + \phi_{12}(1)^2 + \ldots + \phi_{12}(h-1)^2\right]}{\sigma_y(h)^2} \tag{1.82}$$

Basing on equation 1.80 and 1.81 we can conclude that: if the forecast error variance of $\{\mathrm{y_t}\}$ is not explained by $\{\varepsilon_{\mathrm{zt}}\}$, it means that $\{\mathrm{y_t}\}$ is exogenous and so it is independent from the sequence $\{\mathrm{z_t}\}$ and the $\{\varepsilon_{\mathrm{yt}}\}$ shocks, if instead the forecast error variance of $\{\mathrm{y_t}\}$ is fully explained by $\{\varepsilon_{\mathrm{zt}}\}$, it means that $\{\mathrm{y_t}\}$

is totally endogenous. Apart from these extreme cases, it is pretty common to see that a variable can explain most of its forecast error variance at short horizons. On the other hand, its explanation power decrease at longer horizon.

It is important to highlight that as already explained in section 1.1.6, to recover the sequence $\{\varepsilon_{\mathrm{yt}}\}$ and$\{\varepsilon_{\mathrm{zt}}\}$we need to restrict the B matrix. Different types of restrictions involve different variance decompositions, which should converge to the same values at longer forecasting horizons.

1.1.9 Generalized Impulse Response Function

The Generalized Impulse Response Function (GIRFs) is an alternative way to analyze the consequences of a shock to the variables of interest. It is used when it is difficult to find a plausible identification strategy, since GIRFs do not require any constraint on the reduced form model. The main difference between IRFs and GIRFs is the interpretation that we can give to the shocks: in the first case we can assign to IRFs an economic meaning, while in the second case the GIRFs offer only an identification from a statistical point of view, without any economic content.

Let's define a generic p dimensional VAR process as:

$$x_t = \kappa D_t + \sum_{i=1}^{p} \mathit{\Gamma}_i x_{t-i} + \varepsilon_t \tag{1.83}$$

where $t = \mathit{1}, \ldots, T$, D_t includes deterministic variables, ε_t is assumed to be i.i.d with zero mean and positive definite covariance matrix $\mathit{\Sigma}_{\varepsilon}$.

The corresponding forecast error is:

$$x_{t+h} - E_t\left[x_{t+h}|\mathbb{I}_t\right] = \sum_{i=0}^{h-1} \phi_j \varepsilon_{t+h-i} \tag{1.84}$$

where x_{t+h} is the actual realization of x at horizon $t+h$, $E_t\left[x_{t+h}|\mathbb{I}_t\right]$ is the forecasted value of x at horizon $t+h$ given $\mathrm{\mathbb{I}_t}$, which is an information set that contains the entire history of $\{\mathrm{x_t}\}$and $\{\mathrm{D_t}\}$. ϕ_j is defined as:

$$\phi_j = \sum_{i=1}^{\min p,j} \mathit{\Gamma}_i \phi_{j-1} \tag{1.85}$$

where $j \geq 1$ the starting value ϕ_0 is equal to I_p and the other values of ϕ_j are derived from the matrix $\mathit{\Gamma}_i$.

As in Koop et al. (1996) we define the GIRFs as:

$$GI_x(h, \delta, \mathrm{\mathbb{I}_{t-1}}) = E\left[x_{t+h}|\delta, \mathbb{I}_{t-1}\right] - E\left[x_{t+h}|\mathbb{I}_{t-1}\right] \tag{1.86}$$

where $\delta = \mathrm{x_{t+h}}|\varepsilon_t$. It means that:

$$GI_x(h, \delta, \mathrm{\mathbb{I}_{t-1}}) = \phi_h \delta \tag{1.87}$$

The vector δ is the parameter which determines the path of GIRFs. For purposes of presentation, we shock only one element of $\{\mathrm{\varepsilon_t}\}$sequence. Thus, defining ε_{jt} as δ_j, we can rewrite GIRFs as:

$$GI_x(h, \delta_j, \mathrm{\mathbb{I}_{t-1}}) = E\left[x_{t+h}|\varepsilon_{jt}, \mathbb{I}_{t-1}\right] - E\left[x_{t+h}|\mathbb{I}_{t-1}\right] \tag{1.88}$$

assuming that the $\{\mathrm{\varepsilon_t}\}$ sequence is Gaussian and defining the standard deviation of ε_{jt} as $\delta_j = \sqrt{\omega_{jj}}$:

$$E\left[\varepsilon_t|\varepsilon_{jt} = \sqrt{\omega_{jj}}\right] = \Sigma e_j \omega_{jj}^{-\frac{1}{2}} \tag{1.89}$$

where e_j correspond to the j^{th} column of I_p. It means that:

$$GI_x\left(h, \sqrt{\omega_{jj}}, \mathbb{I}_{t-1}\right) = \phi_h \Sigma e_j \omega_{jj}^{-\frac{1}{2}} \tag{1.90}$$

In this way, we have found the response of x_{t+h} to a standard deviation shock to ε_{jt}.

1.2 Non-linear Vector Autoregressive Analysis

Although linear VAR model may sometime approximate well the true data generating process, economic theory suggests that in some situations can be useful to use nonlinear models. For example, consider the case in which the economy alternates between recession and expansion regimes. In such situations the parameters of our model might differ among states and the transitions between regimes can be modelled with a a stochastic process. Moreover, it can be the case that the parameters of our model evolve continuously over time, following a particular law of motion. In cases like these, a standard linear VAR Model is basically extremely inaccurate.

Consider the structural form of a standard VAR model:

$$B_0 x_t = \mathit{\Gamma}_0 + \mathit{\Gamma}_1 x_{t-1} + \ldots + \mathit{\Gamma}_p x_{t-p} + \varepsilon_t \tag{1.91}$$

which have the following reduced form:

$$x_t = A_0 + A_1 x_{t-1} + \ldots + A_{t-p} + e_t \tag{1.92}$$

We can transform the reduced-form model to admit nonlinearities:

$$x_t = G_t(x_{t-1}, \ldots, x_{t-p}) + e_t \tag{1.93}$$

where G_t is a nonlinear function which depends on t, and the error term are linear as in the standard VAR Model.

In the following sections we illustrate the following nonlinear VAR models: Threshold VAR (TVAR), Smooth-Transition VAR (ST-VAR) and Interacted VAR (IVAR).

1.2.1 Threshold VAR (TVAR) and Smooth-Transition VAR (ST-VAR) Models

The TVAR and ST-VAR model were proposed by Balke and Fomby (1997) and are particularly useful if we want to capture nonlinearities such as regime switching generating asymmetry to shock responses. As a matter of fact, we can identify different regimes using some threshold and describe each of them using a linear model.

Consider the following reduced form VAR:

$$x_t = A_0 + \sum_{i=1}^{p} A_i x_{t-i} + G(y_t, \theta) \left(A_0^+ + \sum_{i=1}^{p} A_i^+ x_{t-i} \right) + e_t \tag{1.94}$$

Where $G(y_t, \theta)$ is a $N \times N$ function matrix, in which the variable y_t and the parameter θ determines the changes in the model coefficients. Thus, if $G(y_t, \theta) \neq 0$ the model is nonlinear and the parameters A_0^+ and A_i^+influence x_t.

A TVAR model, may have the following form of $G(y_t, \theta)$:

$$G(y_t, \theta) = \mathbb{I}(y_t > c) I_K \tag{1.95}$$

where y_t is a scalar variable, $\mathbb{I}$ is an indicator function and c is a constant. In this way, the parameters change when the threshold variable is higher than the value of c. Moreover, we can set multiple threshold by using different values of c.

The main difference between TVAR and ST-VAR model is that the latter allow for less abrupt changes between regimes. Consider for example, the case in which $G(y_t, \theta)$ is an exponential transition function such that:

$$G(x_t, \theta) = \begin{bmatrix} 1 - \exp\left[-\gamma (y_{it} - c_1)^2\right] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 - \exp\left[-\gamma (y_{Nt} - c_N)^2\right] \end{bmatrix} \quad (1.96)$$

where $y_t = (y_{1t}, \ldots, y_{Nt})'$ are transition variables, $\gamma > 0$ and $\theta = (\gamma, c_1, \ldots, c_N)'$. Until a certain period T_B, the transition variables y_t are equal to c_N and the coefficients of our VAR are A_0, $A_1, \ldots A_p$. After T_B, they gradually deviate from c_N and the coefficients of our model become A_0, A_0^+, $A_1 + A_1^+, \ldots, A_p + A_p^+$. In this setting the value of γ rules the celerity of the transition.

Once we set the model, we can estimate it by using nonlinear least-squares (NLS) or maximum likelihood methods or Bayesian methods.

1.2.2 Interacted VAR Model

The Interacted VAR model we illustrate in this section was built on the Interacted Panel VAR developed by Sá et al. (2014) and Towbin and Weber (2013).We choose to illustrate the model specified by Sá et al. (2014)[10], considering a specific case where we suppress panel dimension.

The main characteristic of this model, is the introduction of interaction terms, which allow us to evaluate the reaction of variables of interest at

[10]In comparison with Towbin and Weber (2013), Sá et al. (2014) decided to construct Bayesian error bands and choose to identify shocks through sign restrictions. We illustrate in details the inference and identification procedures through section 3.2.3

different values of the interaction term. Comparing this framework to regime-switching approach like TVAR and ST-VAR, the IVAR does not require to set a particular threshold. Moreover, the number of state can be potentially equal to the number of observation. On the other hand, a threshold model uses the information of each state under consideration separately and the setting of the thresholds is subjected to discretion of researcher if not endogenously modelled.

The IVAR model has the following structural form:

$$B_t x_t = \kappa + \sum_{i=1}^{p} \Gamma_i x_{t-i} + \kappa^1 y_t + \sum_{i=1}^{p} \Gamma_i^1 y_t x_{t-i} + \varepsilon_t \tag{1.97}$$

where $t = 1, ..., T$ denotes time, $i = 1, ..., p$ denotes the lags, κ is a constant, Γ_i is a matrix of autoregressive coefficients , ε_t is vector of residuals which, by assumption are normally distributed such that $\varepsilon_t \sim N(0, \Sigma_\varepsilon)$. The interaction term, y_t , has the capacity to influence both the level and the dynamic relationship between endogenous variables through κ^1 and Γ_i^1.

The matrix B_t is a $q \times q$ lower triangular matrix with ones on the main diagonal. Each component $B_t(w, q)$ of B_t matrix represents the contemporaneous effect of the qth-ordered variable on the wth-ordered variable. It is constructed as follows:

$$B_t = \begin{cases} B_t(w,q) = 0 & for \quad q > w \\ B_t(w,q) = 1 & for \quad q = w \\ B_t(w,q) = B(w,q) + B^1(w,q) y_t & for \quad q < w, \end{cases} \tag{1.98}$$

where the coefficients B_j and B^1 represents the marginal effect of a change in the variable and interaction term, respectively. Moreover, a recursive structure[11] has been imposed to matrix B_t which means that the covariance matrix of the residuals Σ_ε is diagonal.

1.2.2.1 Inference and identification

We start by estimating the recursive form presented in equation 3.1. Since by construction the covariance matrix Σ_ε is diagonal, we proceed by estimating equation by equation using OLS. Furthermore, we set an uninformative Wishart prior and draw from the posterior distribution the parameters of the recursive form. Once coefficients are evaluated at the prespecified values of interaction terms, by inverting the matrix B_t we obtain the reduced form model:

$$x_t = C + \sum_{i=1}^{p} A_i x_{t-i} + C^1 y_t + \sum_{i=1}^{p} A_i^1 y_t x_{t-i} + e_t \tag{1.99}$$

Obviously, the vector of residuals will continue to be normally distributed with mean zero and covariance matrix equal to Σ_t^e.

As we have already discussed, using an uninformative prior lead to the same results of OLS estimation. Anyway, the authors choose a similar setting with purpose to compute Bayesian error bands.

As a matter of fact, the Bayesian approach allows us to distinguish between parameter uncertainty and identification uncertainty. The first is due to the fact that we have a limited set of data, while the latter is a consequence of the fact that we have limited information about the characteristics of the structural shock. The use of Bayesian estimation is useful to account for parameter uncertainty. On the other hand, to account for identification uncertainty, for a given parameter draw d we find a number of rotation matrices that

[11] It implies that all the variables in the system react contemporaneously to the first ordered variable, but the latter does not react on impact to any other variables.

satisfy a particular set of sign restriction.[12] Then, we save the median of the IRFs generated by rotation matrices and move to the next parameter draw. Furthermore, we avoid the possibility to have explosive IRFs by discarding the explosive draws from the unrestricted posterior.

This Monte Carlo simulation, enables us to minimize these two types of uncertainty. The final IRFs obtained will be the median of the median of the IRFs saved during the simulation.[13]

Conclusions

The purpose of this survey has been to provide a brief overview of linear and nonlinear Vector Autoregressive Models.

First, we showed how to correctly setup a linear VAR model and subsequently how to identify the structural form and recover useful causal information through the impulse response functions and variance decomposition.

Second, we have illustrated some nonlinear VAR model, focusing on the most widely used models from our point of view, like the TVAR, ST-VAR and IVAR. This kind of models have become very popular in recent years, due to their capacity to well represent more complex data generating processes. As we have seen, however, we should use these models with caution. As a matter of fact, in such situations, we have to deal with problems related to over-parameterization, which may arise due to short data set. In this context, Bayesian estimation can help us by shrinking coefficients on the base of prior information. Through section 1.1.4 we have seen briefly how to conduct estimation using Bayesian methods. However, reader interested in more details

[12] For further explanations about sign restriction identification approach, see section 1.1.6.3.

[13] The authors reports also the 16th and 84th percentile of the IRFs distribution which reflects parameter uncertainty.

about this topics are referred to Canova (2007), Lütkepohl (2005) and Koop and Korobilis (2010).

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Chapter 2

The Government Spending Multiplier at the Zero Lower Bound: Evidence from the United States

The Government Spending Multiplier at the Zero Lower Bound: Evidence from the United States

Mario Di Serio[a], Matteo Fragetta[a,b,c], Emanuel Gasteiger[b,d]

[a] *Università degli Studi di Salerno, Department of Economics and Statistics, Via Ponte Don Melillo, 84084 Fisciano (SA), Italy*

[b] *Instituto Universitário de Lisboa (ISCTE-IUL), Business Research Unit (BRU-IUL), Av[a] das Forças Armadas, 1649-026 Lisboa, Portugal*

[c] *Università degli Studi di Salerno, CELPE, Centro di Economia del Lavoro e di Politica Economica*

[d] *Freie Universität Berlin, Department of Economics, Boltzmannstraÿe 20, 14195 Berlin, Germany*

Abstract

We estimate state-dependent government spending multipliers for the United States. We use an Interacted Vector Autoregression (IVAR) model to capture the time-varying monetary policy characteristics including the recent zero interest rate lower bound (ZLB) state. We identify government spending shocks by sign restrictions and use a government spending growth forecast series to account for the effects of anticipated fiscal policy. In our baseline specification we find that government spending multipliers range from 3.4 to 3.7 at the ZLB. Away from the ZLB, multipliers range from 1.5 to 2.7. Next, we address the limited information problem typically inherent in VARs by the help of a Factor-Augmented IVAR (FAIVAR). We find that multipliers are lower in this case, ranging from 2.0 to 2.1 at the ZLB and between 1.5 and 1.8 away from it. Thus, in both specifications we find that multipliers are higher,

when the interest rate is lower. Our results are consistent with recent theories that predict larger multipliers at the ZLB.

2.1 Introduction

How large is the government spending multiplier in normal times and how large is it when monetary policy is constrained by the zero interest rate lower bound (ZLB)? The Great Recession has revived the debate regarding this question among policy circles and in academia as it is of high practical relevance. If fiscal stimulus by means of an increase in government spending raises real GDP by more than one-for-one, i.e., each dollar of the government spending increase raises real GDP by more than one dollar, then such a stimulus is highly desirable from a policymaking perspective.

The recent debate has given particular attention to the fact that since the outbreak of the 2008 financial crisis the Fed's monetary policy was accommodative, or, even constrained by the ZLB. It is worthwhile that the accomodative stance also included unconventional monetary policy.[1] Figure 2.1 illustrates monetary and fiscal policy from 1960Q1 to 2015Q4. The key observation regarding the most recent recession is that the Federal Funds Rate was abruptly cut to near zero and has remained there until 2015Q4. Moreover, there has been a dramatic deficit-financed increase in government expenditures during this period. It is frequently argued that in such an extraordinary situation, an increase in government spending is even more effective than in normal times.

A growing theoretical literature examines this claim. There is an increasing number of New Keynesian DSGE models that generates predictions consistent with this claim. See, for instance, Christiano et al. (2011), Eggertsson (2010),

[1] For instance, the Fed announced three rounds of quantitative easing: in November 2008, in November 2010, and in September 2012.

Woodford (2011), Davig and Leeper (2011), or, Coenen et al. (2012). These models predict a government spending multiplier in the range of 3 to 5. Likewise, there is an emerging literature developing reasonable theories that suggest that the government spending multiplier at the ZLB is one or below, and lower than in times without the ZLB binding. See, for instance, Braun et al. (2013), Mertens and Ravn (2014), Aruoba et al. (2017).

Given the wide range of theoretical predictions for the size of the government spending multiplier at the ZLB, empirical evidence is a crucial need for policymakers and academia.[2]

However, the empirical literature providing state-dependent evidence on the size of the aggregate government spending multiplier at the ZLB is still in its infancy. To date, Ramey and Zubairy (2017) is the single paper in this literature according to our knowledge.[3] Ramey and Zubairy (2017) use the local projection method developed by Jordà (2005) and find that the government spending multiplier at the ZLB can be as large as 1.5 in some specifications. Moreover, there is a related, but distinct empirical literature quantifying state-dependent fiscal multipliers in recessions based on regime-switching VAR models. However, as Figure 2.1 illustrates, recessions and episodes where the ZLB is binding do not necessarily coincide. Thus, there is a need for more evidence on the government spending multiplier at the ZLB.

The objective of this paper is to provide further state-dependent evidence on the size of the government spending multiplier at the ZLB from the United States. We extend the literature by proposing an alternative framework to quantify the state-dependent government spending multiplier. To this end we

[2]Christiano et al. (2011, p.81) argue: *'The simple models discussed above suggest that the multiplier can be large in the zero-bound state. The obvious next step would be to use reduced-form methods, such as identified VARs, to estimate the government-spending multiplier when the zero bound binds.'*

[3]Crafts and Mills (2013) and Ramey (2011b) provide evidence for ZLB episodes suggesting multipliers below unity.

use an Interacted Vector Autoregression model (IVAR) building on the panel IVAR in Towbin and Weber (2013) and Sá et al. (2014). The interaction term allows us to derive impulse response functions (IRFs) to a government spending shock at different percentiles of the interest rate distribution. This methodology enables us to investigate among the entire range of historical interest rates for the sample considered: within the same setup, we are capable of computing multipliers for median and low levels of the interest rate distribution, with no need to restrict the sample.

By using the IVAR framework, we can address several potentially problematic issues of alternative frameworks that are used in the literature on state-dependent multipliers. For instance, compared to regime-switching approaches in general, such as Threshold VAR (TVAR) methods, and, the Ramey and Zubairy (2017) approach in particular, the IVAR model does not require to define a particular threshold. Regime-switching approaches use such a threshold to distinguish observations of normal times from ZLB episodes. However, such a threshold is subject to discretion. In addition, the IVAR uses all the information available for the full sample, while a threshold model uses the information of each state under consideration separately. Moreover, the IVAR does not rely on a particular assumption on an approximation of monetary policy over the sample period, i.e., a Taylor (1993)-rule. For instance, parts of the theoretical literature regard the contemporaneous-data Taylor-rule applied in Ramey and Zubairy (2017) as a problematic approximation of monetary policy, because it is not operational. For instance, real-time data is hardly available even for central banks, see McCallum (1999) for a discussion. Finally, an interest rate value implied by an *ex post* application of a Taylor (1993)-rule below some threshold does not necessarily mean that the economy is at the ZLB. Ramey and Zubairy (2017, pp.23-24) are aware of this point and then eliminate certain episodes on a discretionary basis.

An alternative may be to consider a Smooth Transition VAR (STVAR) framework as used by Caggiano et al. (2015) to estimate the government spending multiplier in recessions. However, there are also concerns regarding an STVAR approach that do not apply to the IVAR model. First, the STVAR, similar to a threshold model, allows only for a finite number of states in practice. Second, as emphasized in Caggiano et al. (2017, p.11), the change in monetary policy in times of crises is frequently abrupt and not smooth. The STVAR framework is not designed to capture such abrupt changes. In sum, compared to threshold-based approaches or the STVAR framework, the IVAR offers clear advantages. The interaction term can capture abrupt policy changes and allows for a large number of states. The number of states can equal the number of available observations.

Another key strength of our empirical strategy is that we identify the government spending shock by using sign restrictions and the series of government spending growth forecasts errors used in Auerbach and Gorodnichenko (2012, 2013). The sign restriction approach allows us to use a minimum of economically meaningful and rather uncontroversial identification restrictions.[4] The forecasts errors enable us to address the concerns related to fiscal foresight in Leeper et al. (2013). The series captures the surprise component in a broad measure of government spending and, as we show, is a relevant and strong instrument for the our post WWII sample. An alternative would be to consider the defense news series used in Ramey and Zubairy (2017). However, this is a rather narrow measure that captures just a particular component of government spending. Furthermore, as Ramey (2011b) reports, defense news

[4]The sing restrictions approach is developed in Canova and De Nicolò (2002), Uhlig (2005). Mountford and Uhlig (2009) apply it to fiscal policy.

appears to be a rather weak instrument, when a post WWII sample does not cover the period of the Korean War.

For our sample from 1966Q4 to 2015Q4, we consider two different specifications. Our baseline specification involves the forecast error series, government spending, GDP and the average tax rate. At the ZLB, government spending multipliers are between 3.42 and 3.66. When monetary policy is not constrained by the ZLB, government spending multipliers are between 1.54 and 2.56. Our second specification addresses the generic limited information problem inherent in VARs as a robustness check. On the one side, introducing more and more variables to the VAR adds more information. However, adding additional variables to the VAR implies a loss of degree of freedom. We handle this trade-off by considering a Factor-Augmented IVAR (FAIVAR). Compared to the baseline specification, we obtain lower multipliers in the FAIVAR. Nevertheless, the bottom line result is the same: multipliers are higher when interest rates are lower. At the ZLB multipliers range from 1.98 to 2.10 while multipliers range from 1.48 to 1.79 away from the ZLB. Thus, our results are qualitatively and quantitatively consistent with the claim that increases in government spending are even more effective at the ZLB.

The paper proceeds as follows: Section 2.2 outlines the IVAR model, our baseline specification and data, our inference and identification approach and how we calculate the multipliers; Section 2.3 discusses the main results; Sections 2.4 addresses misspecification concerns; Section 2.5 concludes.

2.2 Methodology

2.2.1 Empirical Model

We use an Interacted VAR Model based on Towbin and Weber (2013) and Sá et al. (2014).[5] The recursive-form is given by:

$$B_tY_t = \kappa + \sum_{k=1}^{L} \Gamma_k Y_{t-k} + \kappa^1 X_t + \sum_{k=1}^{L} \Gamma_k^1 X_t Y_{t-k} + \varepsilon_t, \tag{2.1}$$

where $t = 1, \ldots, T$ denotes time and $k = 1, \ldots, L$ denotes the lag length. Y_t is a $q \times 1$ vector which contains explanatory variables, κ is the constant term, Γ_K is a $q \times q$ matrix of autoregressive coefficients, $\varepsilon_t \sim \mathrm{N}(0, \Sigma)$ is the vector of residuals.

Moreover, X_t denotes the interaction term, which can influence both the dynamic relationship between endogenous variables and their level, trough Γ_k^1 and κ^1 respectively.

The matrix B_t is a $q \times q$ lower triangular matrix with ones on the main diagonal. Each component $B_t(w, q)$ represents the contemporaneous effect of the qth-ordered variable on the wth-ordered variable. It is constructed as follows:

$$B_t = \begin{cases} B_t(w,q) = 0 & for \quad q > w \\ B_t(w,q) = 1 & for \quad q = w \\ B_t(w,q) = B(w,q) + B^1(w,q)X_t & for \quad q < w, \end{cases} \tag{2.2}$$

[5] The exposition follows Sá et al. (2014) although we do not consider a panel of countries.

where $B(w,q)$ and $B^1(w,q)$ are regression coefficients capturing the marginal effects of a change in the interaction term. The recursive structure imposes that all the variables in the system react contemporaneously to the first ordered variable, but the latter does not react on impact to any other variables. The recursive form of the matrix B_t also implies that the covariance matrix of the residuals, Σ, is diagonal.

2.2.2 Baseline Specification

Our data set consists of U.S. quarterly data and goes from 1966Q4 to 2015Q4.[6] In our baseline specification our vector (2.1) of endogenous variables is:

$$Y_t = [\mathrm{FE}_t, \mathrm{G}_t, \mathrm{GDP}_t, \mathrm{T}_t]'. \tag{2.3}$$

This vector Y_t includes variables that are commonly used in the literature (e.g., Blanchard and Perotti, 2002). G_t represents real government spending and we use government consumption expenditures and gross investment as a proxy. T_t denotes the average tax revenue. We use federal government current receipts as a proxy for this variable. Moreover, GDP_t stands for real gross domestic product.

Finally, FE_t denotes a series of forecast errors of the annualized growth rate of real government spending following Auerbach and Gorodnichenko (2012).[7] By this series we address fiscal foresight. In Appendix 2.B we provide evidence that FE_t has high explanatory power regarding the variation in growth of G_t

[6]The choice of this time period is motivated by the availability of the Greenbook and SPF government spending forecasts.

[7]Appendix 2.A contains further information on the computation of this variable.

and is therefore a relevant instrument to control for fiscal foresight that cannot be considered weak.[8]

Variables G_t and GDP_t, are expressed in real terms and considered in levels. T_t is in nominal terms and divided by nominal GDP. With the exception of the average tax rate, the other variables and FE_t have been normalized with an estimate of real potential GDP. Ramey and Zubairy (2017) show that the usual approach of using log of variables requires an *ex post* conversion to dollar equivalents of the estimated elasticities that can produce serious bias. The problem is even more acute in nonlinear models and in particular in our model, where several multipliers can be potentially computed, since the *ex post* conversion requires a factor which is based on the sample average of the ratio of GDP to government spending. With the kind of normalization just described, government spending multipliers can be computed directly[9]. Further details about all variables that we use, transformation and so on, are provided in Table 2.2.

For the interaction term we use the U.S. Shadow Federal Funds Rate developed by Wu and Xia (2016), i.e., $X_t = \mathrm{sr}_{t-1}$. The interaction term allows us to examine how the time-varying interest rate environment affects the transmission mechanism of the government spending shock among the variables in Y_t. However, when we set a specific value of the interaction term, our empirical model implies that the shadow rate remains the same for the 20 quarters, corresponding to the horizon over which we calculate impulse responses. For this reason we investigate the effects of a government spending shock at different percentiles of the shadow rate, specifically at $1st$, $5th$, $13th$, $25th$, $50th$ and $75th$ percentile of its distribution. We consider the range from

[8] For further explanations about the fiscal foresight critique see Leeper et al. (2013).

[9] Further details about the computation of the government spending multipliers are described below. For more details on the bias caused by the *ex post* conversion of the elasticities see Ramey and Zubairy (2017)

the $1st$ to the $13th$ percentile of the Shadow Rate distribution as the *low* interest state, as the $13th$ percentile coincides with a value of the interest rate equal to 0.25. The latter value is conventionally accepted by the literature as the lower bound for monetary policy in using the Federal Funds rate as instrument. Results for the $25th$ percentile and above are associated with the *high* interest state. It is important to emphasize that we use this categorization of percentiles in order to structure the discussion of results later on. However, this is not a threshold that affects our results.

We use the U.S. Shadow Federal Funds Rate as this rate is a more precise indicator of monetary policy after the Federal Funds Rate reached the ZLB: away from the ZLB this series is equal to the effective federal funds rate, but at ZLB Wu and Xia (2016) use a Gaussian Affine Term Structure Model (GATSM) to generate an effective rate. Figure 2.1 illustrates this point. After the abrupt cut in the Federal Funds Rate during the most recent recession, the Federal Funds Rate has been near zero and shows little variation. However, unconventional monetary policy measures have been implemented and the variation in the Shadow Federal Funds Rate in the same period captures this policy. We use first lag of the shadow rate to address potential endogeneity concerns. Specifying $X_t = \mathrm{sr}_{t-1}$ implies that the monetary policy instrument is not endogenous to Y_t. If we were to specify $X_t = \mathrm{sr}_t$ reverse causality could be a problem.[10]

Finally, notice that we choose a lag length of $L = 1$ in order to preserve the parsimony of the model.[11]

[10]Notice that specifying $X_t = \mathrm{sr}_t$ does not have a significant effect on our results and conclusions below.

[11]The lag length has been chosen on the base of the Hannan-Quinn(HQ) and Schwarz-Bayes(SBC) information criteria.

2.2.3 Inference and Identification

As in Uhlig (2005) and Sá et al. (2014) we use Bayesian estimation by setting an uninformative normal-Wishart prior, and start with the estimation of the recursive model described in equation (2.1). Since we know that the covariance matrix Σ is diagonal by construction we can proceed by estimating the model equation by equation. For each equation we draw the recursive-form parameters jointly from the posterior.[12] We evaluate them at a pre-specified value of the interaction term and compute reduced form parameters by inverting the matrix B_t:

$$Y_t = B_t^{-1}\kappa + B_t^{-1}\sum_{k=1}^{L}\Gamma_k Y_{t-k} + B_t^{-1}\kappa^1 X_t + B_t^{-1}\sum_{k=1}^{L}\Gamma_k^1 X_t Y_{t-k} + B_t^{-1}\varepsilon_t, \quad \text{or,} \tag{2.4}$$

$$Y_t = C + \sum_{k=1}^{L} A_k Y_{t-k} + C^1 X_t + \sum_{k=1}^{L} A_k^1 X_t Y_{t-k} + e_t, \tag{2.5}$$

where the vector of residuals $e_t \sim \mathrm{N}(0, \Sigma_t^e)$ and the Cholesky decomposition of the reduced form covariance matrix is given by $V_t = B_t^{-1}\Sigma^{\frac{1}{2}}$.

Government spending shocks are identified by imposing sign-restrictions.[13] Once we have obtained the Cholesky decomposition of the reduced form covariance V_t, the general idea is to obtain combinations of V_t by using an orthogonal matrix Q such that $V_t^* = QV_t$, where orthogonality of the shocks is preserved. Rubio-Ramírez et al. (2010) propose to draw a W matrix from a N(0,1) and use the QR decomposition (householder transformation), obtaining $W = QR$,

[12] As in Sá et al. (2014); Cogley and Sargent (2005); Primiceri (2005) we avoid the possibility to have explosive IRFs by discarding the explosive draws from the unrestricted posterior.

[13] This approach was developed by Canova and De Nicolò (2002), Faust and Rogers (2003), Uhlig (2005). As in Sá et al. (2014) we use the algorithm developed by Rubio-Ramírez et al. (2010).

where Q is the orthogonal matrix required to impose the sign restrictions that allows to preserve orthogonality of the shocks derived from the Cholesky decomposition, since $QV_tQ' = \mathrm{I}$. In this way, candidate draws for the impulse vector are obtained and the impulse responses are calculated, discarding any V_t^* where the sign restrictions are violated in all its columns. Repeating such operations until a desired number of draws meet the required sign restrictions allow to calculate the median responses over the accepted draws.

The set of sign restrictions imposed to obtain identification of a government spending shock are as follow: GDP and government spending responses are constrained to be positive for at least four quarters, while the forecast error for only one quarter (see also Table 2.1). No restrictions are imposed on the average tax variable.[14]

Our procedure accounts for identification uncertainty: for each stable parameter draw of the posterior we find a set of 100 orthonormal matrices that satisfies the sign restrictions. We then compute the corresponding IRFs saving only the median of the 100 identified models.[15] We then repeat this step for each stable draw of the posterior described above for 20.000 parameter draws considering the median of the medians as our estimate of interest.[16]

2.2.4 Multipliers

We estimate the model in normalized levels, similar to Ramey and Zubairy (2017). Thus, there is no need to normalize IRFs in any way, or, to carry

[14] Identification based on sign restrictions is in principle less sensitive to the estimation of the covariance matrix than identification based on short-run restrictions. However we start with estimation of the structural model, where the order might influence results. This is why we have estimated the model with alternative orderings and do not find significant changes.

[15] Uncertainty about identification is due to the fact that we have limited information about the true structural shock. For further details see Sá et al. (2014) and Cogley and Sargent (2005).

[16] We compute 20.000 stable draws, discarding the first 10.000 as burn-in draws.

out the ex-post conversion that is typically applied in the existing literature (see, e.g., Ramey, 2011b). Our IRFs represent the change in the variable of interest to a surprise change in government spending. For instance, for GDP this means $d\text{GDP}(t)/d\text{FE}(t)$.

We compute three types of multipliers denoted by $\mathcal{M}_i \in \{1, 2, 3\}$. $\mathcal{M}_1$ is based on Ramey (2011b), who makes a discrete approximation of the integral of the median IRFs over time horizon $h = 0, 1, ..., H$ given by

$$\mathcal{M}_1 = \frac{\sum_{h=0}^{H} d\text{GDP}(h)}{\sum_{h=0}^{H} d\text{G}(h)}. \tag{2.6}$$

Multipliers 2 and 3 are computed using numerical integration, through the use of the *Trapezoidal* and *Simpson's* rule, respectively. The goal of these two computations is to give more accurate approximations of the integrals in

$$\mathcal{M}_{2,3} = \frac{\int_0^H d\text{GDP}(h)dh}{\int_0^H d\text{G}(h)dh}. \tag{2.7}$$

Following Auerbach and Gorodnichenko (2012), we choose $H = 20$.

2.3 Main Results

In this section we present the macroeconomic effects of a one unit government spending forecast shock obtained for our baseline specification. Figures 2.2 and 2.3 show the IRFs of endogenous variables for the low and high interest rate state respectively.

First, observe that IRFs for government spending and GDP in both states are persistently different from zero, except for very high interest rates. Moreover, the median IRF of the average tax rate is mostly insignificant in the low interest rate state. Thus, we have identified a predominantly deficit-financed spending increase in both states.

The behavior of government spending is similar among states. Government spending peaks on impact and is persistently different from zero throughout the time horizon.

Thus, what are the effects on GDP? In sum, the IRFs for GDP qualitatively resemble the behavior of government spending in their respective state. GDP peaks on impact and has a persistently positive response in the subsequent quarters. However, in the high interest rate state, the median IRF becomes insignificant at an earlier point in time. Taking the behavior of GDP and government spending together, the IRFs suggest that when the interest rate is at the ZLB, a comparable exogenous increase in government spending is more effective in stimulating GDP.

The implied multipliers are consistent with our observations, see Table 2.4. Multipliers, depending on the definition, are in the range of 3.42 and 3.66 in the low interest rate state and around 1.54 and 2.56 in the high interest rate state. Thus, the multipliers also suggest that government spending increases are more effective in the low interest rate state. Moreover, notice that the multipliers for both states are large compared to the VAR literature in general (see, e.g., Ramey, 2011a) and compared to the findings of Ramey and Zubairy (2017) who report multipliers of at most 1.5 at the ZLB and multipliers below unity away from the ZLB.

In sum, our findings cannot be reconciled with theories that suggest that the government spending multiplier at the ZLB is 1 or below, and lower than in the high interest rate state (see, e.g., Braun et al., 2013; Mertens and Ravn, 2014; Aruoba et al., 2017). In addition, our findings, especially for the high interest rate state, contradict with standard Real Business Cycle models (see, e.g., Baxter and King, 1993) that predict a negative wealth effect and lower multipliers due to crowding out of consumption.[17]

In contrast, our results can be reconciled with New Keynesian DSGE models that predict government spending multipliers at the ZLB in the range of 3 to 5 (see, e.g. Christiano et al., 2011; Eggertsson, 2010; Woodford, 2011; Davig and Leeper, 2011; Coenen et al., 2012). For instance, in models such as Christiano et al. (2011) the negative wealth effect of a government spending stimulus is weakened by assumption. As a consequence, co-movement in consumption and real wages due to counter-cyclical markups is possible.[18] An increase in government spending raises aggregate output, marginal cost, and expected inflation. At the ZLB, the key channel to explain the higher multipliers is related to the real interest rate. As expected inflation increases and the nominal interest rate is zero, the real interest rate must fall. In consequence, private consumption increases, raises aggregate output, marginal cost and expected inflation once more. Thus, the ZLB amplifies the effects of government spending on output. As the output increases require an increase in employment, these models also imply real wage increases.

2.4 Robustness

In this section we address misspecification concerns regarding our baseline specification and the results presented above. Notice that we maintain the identification approach described in Section 2.2.3 throughout the robustness analysis.

[17] An increase in government spending lowers the present value of after-tax income. As a consequence, agents lower consumption and increase labor supply. The latter decreases the real wage and higher employment can raise investment.

[18] Thus, in such models multipliers can be large even without considering the ZLB (see, Galí et al., 2007).

2.4.1 Factor-Augmented Interacted VAR Model

In particular, one may argue that our baseline specification is problematic for two reasons that can be addressed by developing a FAIVAR model. First, the choice of variables in Y_t is subject to discretion. Thus, one may argue that our results are due to the particular choice of variables in Y_t.

Second, given the considerations and results in Fragetta and Gasteiger (2014), one may argue that our Interacted VAR model is affected by a generic limited information problem. As a matter of fact, when economic agents make their decisions, they use all available information at the time. In contrast, an econometrician can only take into account a limited set of information, due to the problem related to degrees of freedom.

A FAIVAR model addresses both lines of critique. On the one hand it allows us to take into account the information from a large informational data set and to maintain a small set of variables in Y_t that is necessary for meaningful identification. Thus, discretion in the specification of Y_t is limited to a minimum. On the other hand, the FAIVAR model allows us to overcome the generic limited information problem.

We implement a two-step estimation procedure. Following Bernanke et al. (2005), we use the method of principal components to extract and summarize information from a large dataset.[19] The Bai and Ng (2002, 2007) IC_{p2} criterion suggests to extract four static factors. Thus, we specify the vector of endogenous variables in the FAIVAR model as

[19] We apply the principal components method by using the same informational dataset as used in Fragetta and Gasteiger (2014). Their informational dataset comprises 61 publicly available time series from the Federal Reserve Bank of St. Louis' FRED® Economic Database. As in their case we transform variables to guarantee stationarity according to Dickey and Fuller (1979) and Kwiatkowski et al. (1992) tests.

$$Y_t = [\mathrm{FE}_t, \mathrm{G}_t, \mathrm{GDP}_t, \mathrm{T}_t, \mathrm{F}_t]'. \tag{2.8}$$

where F_t is the 4×1 vector capturing the first four principal components of the informational dataset.

The IRFs in Figures 2.4 and 2.5 depict the low and high interest rate state respectively. Overall, IRFs of government spending and average taxes show a qualitatively similar pattern as in the baseline specification.

However, IRFs for GDP reveal several differences compared to the baseline specification. First, IRFs for GDP are less persistent. This behavior is particular evident in the low interest rate state. In the latter case, one ca also find hump-shaped IRFs. Nevertheless, the lower persistence should be reflected in lower multipliers.

Consistent with this claim, Table 2.4 shows that multipliers are now lower in both states. Moreover, decline of multipliers in the FAIVAR compared to the IVAR is of higher magnitude in the low interest rate state. Nevertheless, multipliers in the low interest rate state range from 1.98 to 2.10, while multipliers range from 1.48 to 1.79 in the high interest rate state. Thus, as multipliers in the low interest rate state exceed the ones in the high interest rate state, we conclude that our baseline results are robust with regard to the particular specification of Y_t and the generic generic limited information problem of (interacted) VARs.

2.5 Conclusions

This paper sheds light on the question of whether the government spending multiplier at the ZLB is larger than in normal times. To this end, we implement an Interacted VAR model and use sign restrictions to identify government spending growth forecast shocks. This framework allows us to account for fiscal foresight and to estimate state-dependent government spending multipliers at all percentiles of the nominal interest rate distribution.

In contrast to the existing state-dependent estimates, we find convincing evidence that government spending multipliers are larger in low interest rate states than in high interest rate states. For our sample from 1966 to 2015, the multipliers at the ZLB are in the range of 3.4 to 3.7. The ones away from the ZLB are between 1.5 to 2.7. Our findings are robust to several important misspecification concerns.

Thus, we conclude that the government spending multiplier at the ZLB is larger than in normal times and within the range of 3 to 5 as predicted by recent New Keynesian DSGE models.

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Appendix

2.A Data

General Information. Table 2.2 contains an overview on the data that we use. If appropriate, nominal variables are transformed into real variables by dividing by the GDP implicit price deflator. Moreover, real variables in levels, if appropriate, are normalized by dividing by real potential GDP. The forecast error that we use is the forecast error for the annualized growth rate of real government spending. We normalize this variable by subtracting the annualized growth rate of real potential GDP.

Forecast Error. Our measure of the forecast error, FE_t builds on the annualized growth rate of real government purchases forecast for time t at time $t-1$, i.e.,

$$\Delta \mathrm{G}^F_{t|t-1} \equiv \left[\left(\frac{\mathrm{G}^e_{t|t-1}}{\mathrm{G}^e_{t-1|t-1}} \right)^4 - 1 \right] \times 100, \tag{2.9}$$

The data source is the Mean Responses of Real Federal Government Consumption Expenditures & Gross Investment (RFEDGOV) and Real State and Local Government Consumption Expenditures & Gross Investment (RSL-

GOV). $\text{G}^e_{t|t-1}$ is the sum of RFEDGOV3 and RSLGOV3, $\text{G}^e_{t-1|t-1}$ is the sum of RFEDGOV2 and RSLGOV2.

As our objective is to compute a series of surprise increases in government spending, we need to control for real-time data. The forecast error for the growth rate of government spending is defined as

$$\text{FE}_t \equiv \left[\left(\frac{\text{G}^{1\text{st}}_t}{\text{G}^{1\text{st}}_{t-1}} \right)^4 - 1 \right] \times 100 - \Delta \text{G}^F_{t|t-1} \tag{2.10}$$

Thus, for this purpose, we have downloaded first release data on real government consumption expenditures and gross investment: state and local (RGSL) from this websiteand real government consumption and gross investment: federal (RGF) from this website. All in quarterly vintages (Billions of real dollars, seasonally adjusted). $\text{G}^{1\text{st}}_t$ is the sum of RGSL and RGF.

Notice that the SPF data is only available from 1981Q4. Thus, for earlier periods, as in Auerbach and Gorodnichenko (2012), we take advantage of the fact that SPF is also quite similar to Greenbook forecasts prepared for FOMC meetings. Thus, we splice data from SPF and Greenbook forecasts and obtain a series which goes from 1966Q4 to 2015Q4.

2.B Explanatory Power of the Forecast Error

Following Ramey (2011b, pp.25-29) we examine the explanatory power of FE_t. In particular, we run regressions such as

$$\Delta \text{G}_t = \beta_0 \text{FE}_t + \sum_{k=1}^{L} \beta_k \text{FE}_{t-k} + \varepsilon_t, \qquad \Delta \text{G}_t \equiv \left[\left(\frac{\text{G}_t}{\text{G}_{t-1}} \right)^4 - 1 \right] \times 100. \tag{2.11}$$

Such a regression can shed light on the question of whether FE_t (or lags of it) can explain part of the variation of the growth in G_t. A high F-statistic is an indicator that this is the case and that FE_t can be considered a relevant instrument to control for fiscal foresight. The results in the second column of Table 2.3 suggest that FE_t is a relevant instrument and that it cannot be considered a weak instrument as the F-statistics are way above the rule-of-thumb critical value of 10.

Notice that even with two lags, $L = 1$, FE_t has considerable predictive power. This is surprising as, by construction, one would expect that it has only predictive power for $L = 0$. The reason for the latter is that FE_t represents a measure for the unpredictable component of $\Delta\mathrm{G}_t$. Therefore our results for $L > 0$ imply that the unpredictable components in $\Delta\mathrm{G}_t$ have some persistence.

The third column in Table 2.3 reports the marginal F-statistic for a regression of the growth rate of G_t on the explanatory variables used in the baseline specification. However, FE_t is excluded, i.e.,

$$\Delta\mathrm{G}_t = \sum_{k=1}^{L} \beta_{k,\mathrm{G}}\mathrm{G}_{t-k} + \sum_{k=1}^{L} \beta_{k,\mathrm{GDP}}\mathrm{GDP}_{t-k} + \sum_{k=1}^{L} \beta_{k,\mathrm{T}}\mathrm{T}_{t-k} + \varepsilon_t. \tag{2.12}$$

Table 2.3 reports low marginal F-statistics and values for R-squared, which suggests that FE_t is a relevant instrument.

Figures

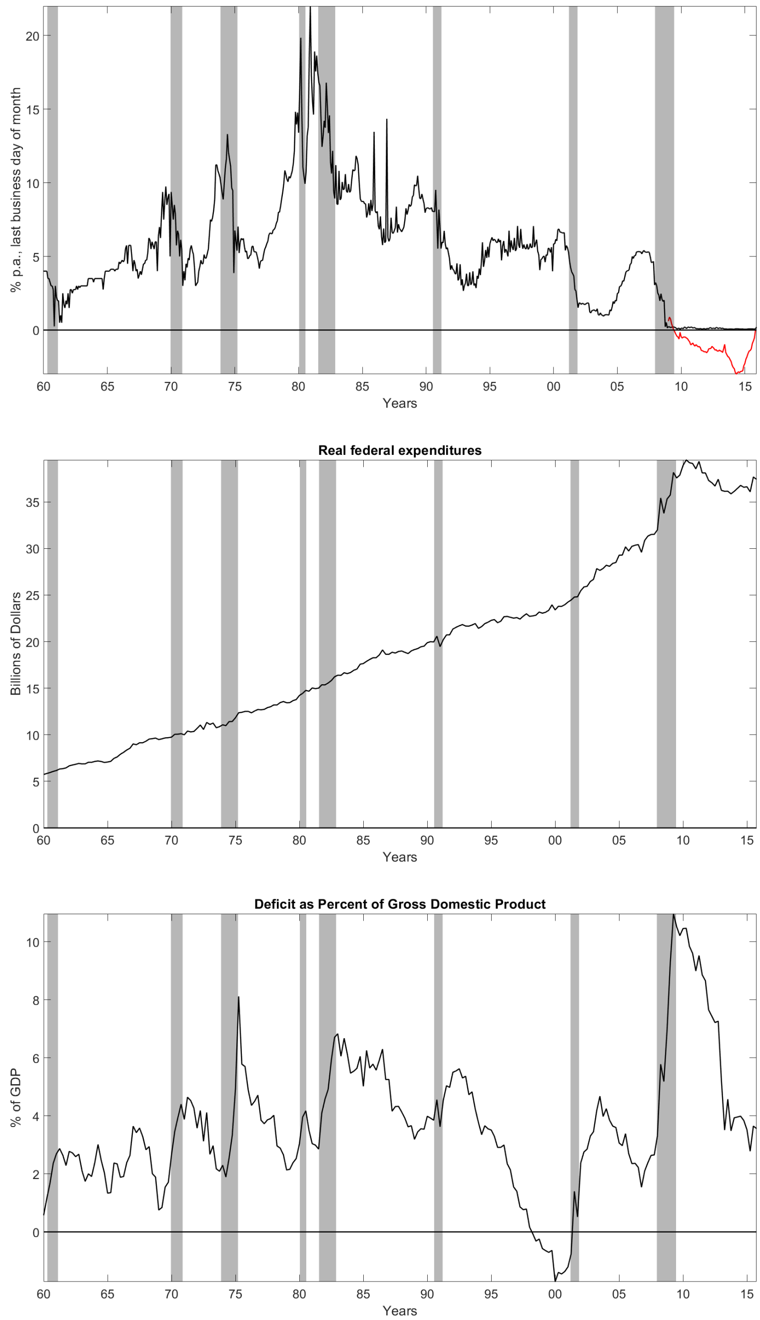

Figure 2.1: Monetary and fiscal policy, 1960Q1 to 2015Q4. The shaded areas indicate recessions according to NBER.

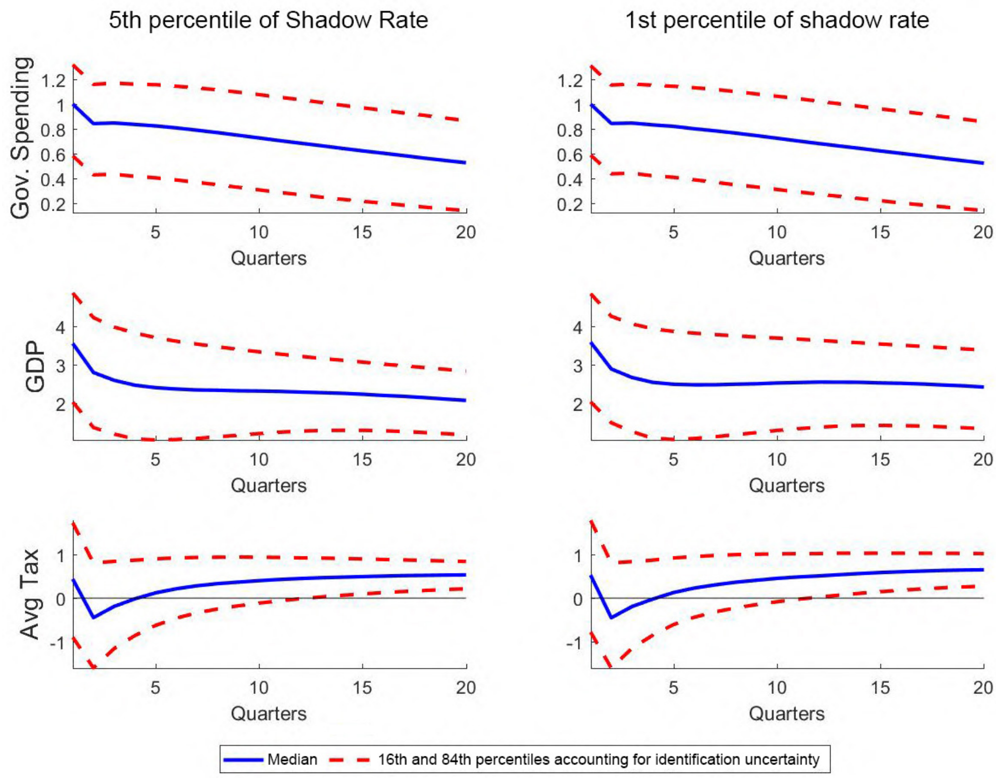

Figure 2.2: IRFs to a one unit government spending growth forecast shock for the baseline specification with $X_t = \mathrm{sr}_{t-1}$ in the *low* interest rate state. The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.

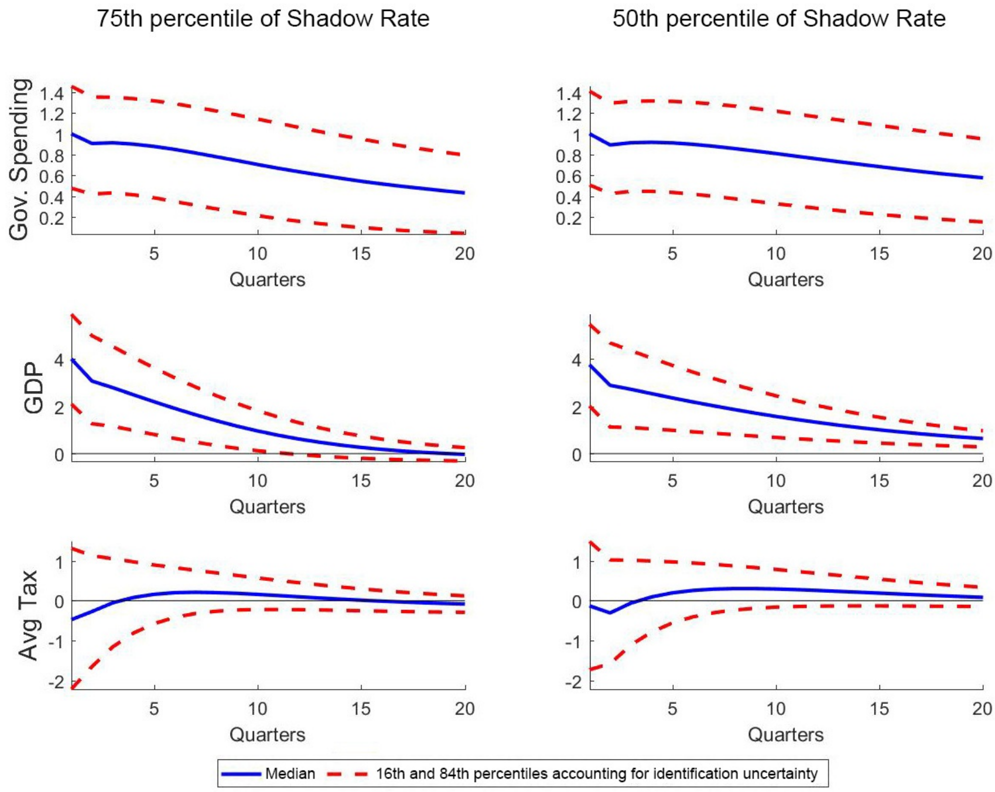

Figure 2.3: IRFs to a one unit government spending growth forecast shock for the baseline specification with $X_t = \text{sr}_{t-1}$ in the *high* interest rate state. The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.

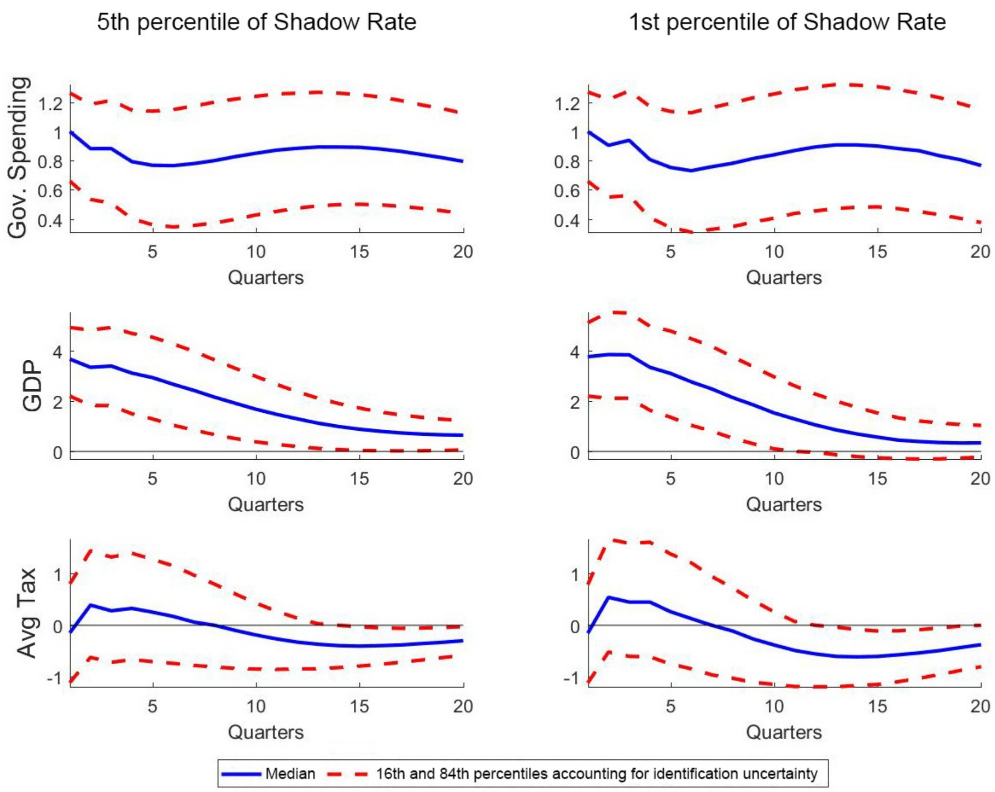

Figure 2.4: IRFs to a one unit government spending growth forecast shock for the specification with $X_t = \mathrm{sr}_{t-1}$ and F_t in the *low* interest rate state. The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.

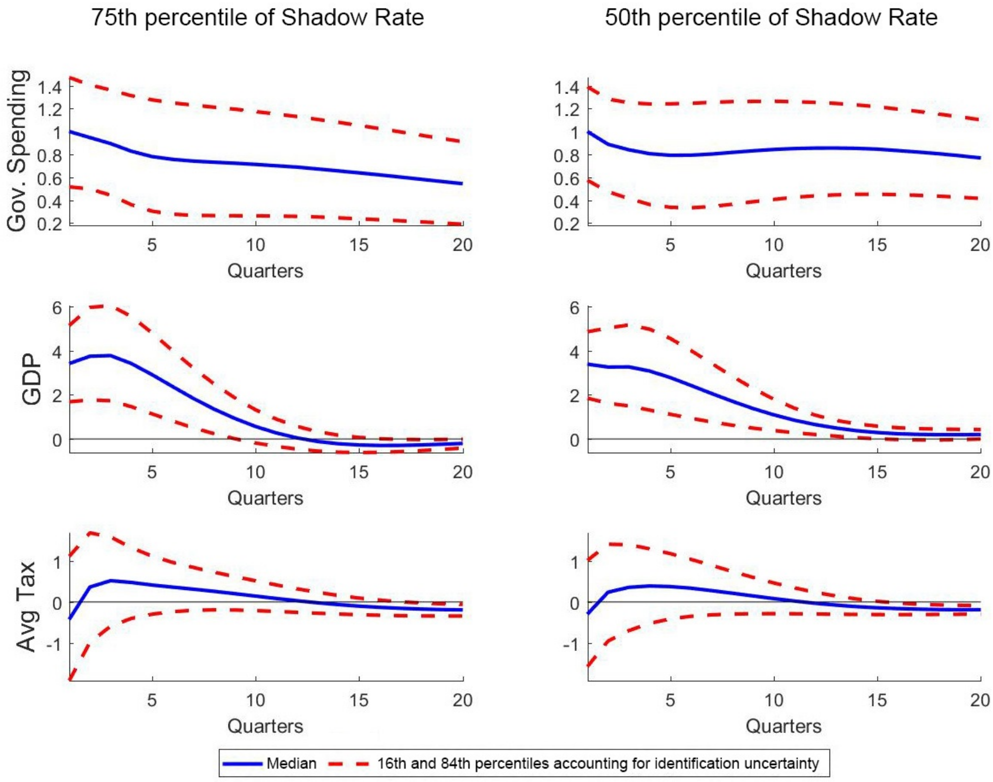

Figure 2.5: IRFs to a one unit government spending growth forecast shock for the specification with $X_t = \mathrm{sr}_{t-1}$ and F_t in the *high* interest rate state. The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.

Tables

Table 2.1: Sign Restrictions for Identifying the Government Spending Shock.

Variable	Sign	Periods
FE_t	+	1
G_t	+	4
GDP_t	+	4
T_t	*	

Table 2.2: Data.

Series	Source	Mnemonic code	Transformation
Forecasts of Real Federal Government Consumption Expenditures & Gross Investment	Federal Reserve Bank of Philadelphia	RFEDGOV	
Forecasts of Real State and Local Government Consumption Expenditures and Gross Investment	Federal Reserve Bank of Philadelphia	RSLGOV	
Greenbook projections of Real Federal Government Consumption and Gross Investment	Federal Reserve Bank of Philadelphia	gRGOVF	
Greenbook projections of Real State and Local Government Consumption and Gross Investment	Federal Reserve Bank of Philadelphia	gRGOVSL	
Real Government Consumption and Gross Investment: Federal	Federal Reserve Bank of Philadelphia	RGF	
Real Government Consumption and Gross Investment: State and Local	Federal Reserve Bank of Philadelphia	RGSL	
Forecast Error of the Annualized Growth Rate of Real Government Purchases	All the above variables are used for the computation, see Appendix 2.A.		Normalized
Nominal Government Consumption Expenditures and Gross Investment	US. Bureau of Economic Analysis	GCE	Real, Normalized
Nominal Federal Government Current Receipts	US Bureau of Economic Analysis	FGRECPT	Real, Average w.r.t. GDP
Real Gross Domestic Product	US Bureau of Economic Analysis	GDPC1	Normalized
Gross Domestic Product: Implicit Price Deflator	US Bureau of Economic Analysis	GDPDEF	
Shadow Federal Funds Rate	Wu and Xia (2016)		
Real Potential Gross Domestic Product	US Congressional Budget Office	GDPPOT	

Table 2.3: Explanatory Power of FE_t.[a]

	R-squared	F-statistic	Marginal F-statistic
L = 0			
1966Q4-2015Q4	0.2474	64.12	
1966Q4-2015Q4	0.0316		2.10
L = 1			
1966Q4-2015Q4	0.2754		
1966Q4-2015Q4	0.0473		3.19

[a]For each lag length L the first line reports results for regression (2.11). The second line reports results for regression (2.12). In the case of $L = 0$, (2.12) uses contemporaneous values.

Table 2.4: Multipliers identified with FE_t.[a]

	Baseline: $X_t = \text{sr}_{t-1}$					
	1st	5th	13th	25th	50th	75th
$\mathcal{M}_1$	3.66	3.34	3.42	2.56	2.10	1.70
$\mathcal{M}_2$	3.64	3.32	3.40	2.53	2.07	1.64
$\mathcal{M}_3$	3.64	3.32	3.40	2.53	2.07	1.64
	Robustness: $X_t = \text{sr}_{t-1}$ and F_t					
	1st	5th	13th	25th	50th	75th
$\mathcal{M}_1$	2.00	2.10	2.06	1.79	1.62	1.51
$\mathcal{M}_2$	1.98	2.08	2.05	1.76	1.60	1.48
$\mathcal{M}_3$	1.98	2.08	2.04	1.76	1.60	1.48

[a]Multipliers $\mathcal{M}_i \in \{1, 2, 3\}$ are calculated as outlined in Section 2.2.4.

Chapter 3

The Government Spending Multiplier at the Zero Lower Bound: Evidence from the Euro Area

The Government Spending Multiplier at the Zero Lower Bound: Evidence from the Euro Area

Adalgiso Amendola[a,c] , Mario Di Serio[a] , Matteo Fragetta[a,b,c]

[a] *Università degli Studi di Salerno, Department of Economics and Statistics, Via Ponte Don Melillo, 84084 Fisciano (SA), Italy*

[b] *Instituto Universitário de Lisboa (ISCTE-IUL), Business Research Unit (BRU-IUL), Av[a] das Forças Armadas, 1649-026 Lisboa, Portugal*

[c] *Università degli Studi di Salerno, CELPE, Centro di Economia del Lavoro e di Politica Economica*

Abstract

We use an Interacted Panel Vector Autoregressive (IPVAR) model, to investigate the effects of a government spending shock when the interest rate is at zero lower bound (ZLB). We also compare the responses of variables of interest at the ZLB with what we get when a government spending shock occurs in normal times (i.e. when the interest rate is larger than 0.25). We identify the government spending shock by sign restrictions and use the European Commission forecasts of government expenditure to account for fiscal foresight. For the baseline specification we find lower multipliers in times in which the ZLB is binding. However, fiscal foresight is not the only problem in fiscal VARs related to limited information problems. Usually, VAR models can only consider a limited number of variables due to degree of freedom problems. Several authors have shown (see Stock and Watson (2005) for a survey) how principal components extracted from a larger number of

variables, can approximate unobserved factors driving most (if not all) of the macroeconomic variables. Therefore, we develop a Factor-Augmented IPVAR model (FAIPVAR) and find that the multipliers are very similar among states, ranging between 1.08 and 1.41 at the ZLB and between 1.26 and 1.39 away from it. We also divide our sample, considering two groups of countries in terms of high and low debt-to-GDP ratios. We find that countries with high levels of debt-to-GDP ratio show relatively lower multipliers. Considering the FAIPVAR model, the government spending multiplier ranges between 2.69 and 3.54 for core countries and between 0.82 and 1.37 for peripheral countries. Therefore, our findings support some recent studies, which suggest that the government spending multiplier is even larger if the debt-to-GDP ratio is low.

3.1 Introduction

The recent world financial crisis and the Great Recession that has followed have renewed interest for the use of discretionary fiscal policies. Starting from 2009, many OECD and developing countries have implemented expansionary fiscal policies with the purpose to soften the effects of the Great Recession. In Europe, the European Commission launched the "European Economic Recovery Plan" (EERP) with the aim to provide coordinated fiscal stimulus to the euro area economies. A natural question arises: has this expansionary fiscal policy succeeded to help Eurozone economies? More specifically, what is the magnitude of the government spending multiplier when monetary policy is constrained at the Zero Lower Bound(ZLB)?[1] Is it larger or smaller than in normal times? There is much uncertainty about these questions: on the one

[1] However, many Euro countries have started to implement austerity measures since 2010. We assume symmetry of responses to a positive or negative fiscal shock. Therefore our estimates are potentially informative about the loss in terms of output implied by austerity measures adopted.

hand, there are researchers who support fiscal stimulus and consequently highlight the Keynesian multiplier effects of a rise in government spending which is even stronger at the ZLB; on the other hand, there are other researchers who criticize fiscal stimulus, arguing that a rise in government spending leads to a very low or even negative fiscal multiplier due to the crowding-out of private consumption and investments.

We join the debate by estimating the government spending multiplier for a set of countries which belong to the Euro Area, in the period that goes from 2000q2 to 2015q4. To this end, we use the Interacted Panel VAR Model (IPVAR) developed by Sá et al. (2014) and Towbin and Weber (2013).[2] Furthermore, in order to account for fiscal foresight, we use the forecasts of government spending made available by the European Commission. The fiscal foresight is due to the fact that most of the fiscal policies are pre-announced and so the economic agents take into account their consequences before they would be actually put in place. More precisely, we add this variable to our specifications with the purpose to purge them from the innovations in the exogenous government spending which are anticipated by agents. Using a sign restrictions approach we identify two shocks: the first identifies the forecast of government spending made at time $t-1$; the second, which is our shock of interest, will be orthogonal to the first and therefore it does not contain expectations made at time $t-1$.

In this baseline specification we find that the government spending multiplier ranges between 0.33 and 0.88 at the ZLB, while away from it, a higher multiplier is found ranging between 1.10 and 1.29. However, although we use a fiscal VAR shared by a large part of the literature (see Blanchard and

[2]For further details about the features of the IPVAR model see Sá et al. (2014). Di Serio et al. (2017) also compare the Interacted VAR model with other nonlinear model used in this kind of literature.

Perotti (2002) for their baseline specification, for example) results might be driven by misspecification concerns in terms of important information that we do not include in our model, but that might be potentially considered by economic agents in determining their choices. Several authors have shown (see Stock and Watson (2005) for a survey) how principal components extracted from a large number of variables, can approximate unobserved factors driving most (if not all) of the macroeconomic variables. We therefore consider a Factor-Augmented IPVAR specification to address such concerns. The results show generally similar multipliers among states. The government spending multiplier ranges between 1.08 and 1.41 in the low interest rate state, and between 1.26 and 1.39 in the high interest rate state. These results show no significant difference between multipliers found in the low and high interest rate states. Overall, we find multipliers that are larger than one, in line with New-Keynesian theoretical predictions.

Then, we proceed our analysis by investigating whether the debt-to-GDP ratio can influence the size of the government spending multiplier. To this end, we consider two different sets of countries. The first is constituted by countries that have had high levels of public debt (higher than 90%) during the 2009-2015 (Belgium, Ireland, Italy, Portugal and Spain).[3] The second one includes countries that during the same period have a debt-to-GDP ratio lower than 90% (Austria, Finland, France, Germany and Netherlands). We find that the government spending multiplier is generally higher for countries which have a lower debt-to-GDP ratio. Considering the specification augmented with factors, the government spending multiplier for countries with low debt-to-GDP ratio ranges between 3.11 and 3.54, while it ranges between 0.82 and

[3]To be more precise, for Spain the public debt is lower and around 85%. However, it has been one of the countries most hit by the sovereign debt crisis. From a low public debt level before the crisis, the latter has caused a rapid deterioration of its public finances.

1.18 for peripheral countries, at the ZLB. The main purpose of this additional exercise is to show qualitative differences between the government spending multipliers at different level of the debt-to-GDP ratio, since they are all subject to the same monetary policy. From our point of view, the findings we get from the full sample specifications are more accurate because they consider countries that all together represent 95.6% of the EA-19 Total GDP.

Our paper is related to a growing theoretical literature which analyzes the size of the government spending multipliers when the interest rate is at the ZLB. Among the others, Christiano et al. (2011), Eggertsson (2010), Woodford (2011), Davig and Leeper (2011) and Coenen et al. (2012) develop New Keynesian DSGE Models which predict higher multipliers at ZLB.[4] On the other hand, Braun et al. (2013), Mertens and Ravn (2014) and Aruoba et al. (2017) argue that the government spending multiplier at ZLB is very small and also lower than in normal times.

Despite the uncertainty about the sign and magnitude of the government spending multiplier, very few studies concerning the effects of government spending at the ZLB have been devoted to the Euro Area. Kilponen et al. (2015) compute the fiscal multiplier using a set of structural macroeconomic models adopted by the European System of Central Banks (ESBC). They find that if temporary fiscal shock happens simultaneously in the Euro Area (as in our empirical strategy), the government spending multiplier has a stronger impact at the ZLB than in normal times. On the other hand, if this shock hits only the economy of one country the relative government spending multiplier is very low and similar to the multiplier computed in normal times. Coenen et al. (2012) evaluate the effectiveness of the implemented fiscal policies during the Great Recession. Specifically, they use the European Central Bank's New

[4]According to their studies, the government spending multiplier at ZLB is in the range of 3 to 5.

Area-Wide Model (NAWM) and find that discretionary exogenous policies of 1% lead to an increase of 1.6% of real GDP. On the opposite side, Cwik and Wieland (2011) find no higher effects of the government spending shock, unless the ZLB state was anticipated and holds for at least two years. Among the models used, the only European Central Bank's Area-Wide Model provides evidences in favor of a government spending multiplier which is higher at the ZLB.

With regard to the literature analyzing the relationship between fiscal multipliers and different level of debt-to-GDP ratio, Sutherland (1997), for example, shows how government spending shocks have expansionary effects when the debt-to-GDP ratio is low, becoming contractionary at high level of debt-to-GDP ratio. Perotti (1999) develops a model which analyzes the effects of both tax and expenditure shocks, finding that the reaction of consumers to a government spending shock can be very different, depending on the initial level of public debt-to-GDP ratio. At a high level of debt-to-GDP ratio, expectations of future increase in taxation generate higher negative wealth effects on fiscal multipliers. On the empirical side, Ilzetzki et al. (2013) follow the Blanchard and Perotti (2002) methodology, estimating a VAR which includes 44 countries from 1960q1 to 2007q4. They find that the size and the sign of government spending multiplier depend on country-specific characteristics. In particular, they find that if debt-to-GDP ratio exceeds 60% of GDP, the fiscal multiplier is not statistically different from zero on impact and negative in the long run (i.e the fiscal multipliers is negative). Kirchner et al. (2010) analyze the effects of government spending shock and the transmission mechanism within the euro area from 1980 to 2008. They find that an increase in debt-to-GDP ratio causes the short run effect to be negative. Nickel and Tudyka (2014) develop an Interacted Panel VAR for

17 European countries from 1970 to 2010, analyzing fiscal multipliers which depend in their model nonlinearly from the debt-to-GDP ratio. Their findings are in line with previous works: the effects of a government spending shock are positive when debt-to-GDP ratio is low, while become negative when this ratio is high.

The paper proceeds as follows: Section 3.2 describes the methodology we use, data and how we calculate the multipliers; Section 3.3 discusses the results of our baseline specification; Sections 3.4 describes result for baseline specification augmented with factors; Section 3.5 shows results for high and low levels of debt-to-GDP ratio; Section 3.6 concludes.

3.2 Methodology

3.2.1 Empirical Model

Our model is built on the Interacted Panel VAR model developed by Sá et al. (2014) and Towbin and Weber (2013).The introduction of interaction terms, allow us to evaluate non-linearities and the reaction of variables of interest at different values of the interest rate.

The model we specify has the following structural form:

$$B_{i,t}y_{i,t} = \sum_{j=1}^{N}\kappa_j D_{j,i} + \sum_{j=1}^{N}\sum_{k=1}^{L}\Gamma_{j,k}D_{j,i}y_{i,t-k} + \kappa^1 x_{i,t} + \sum_{k=1}^{L}\Gamma_k^1 x_{i,t}y_{i,t-k} + \varepsilon_{i,t} \quad (3.1)$$

where $t = 1,...,T$ denotes time, $i = 1,...,N$ denotes the country, $k = 1,...,L$ denotes the lags, κ_j is country-specific intercepts, $\Gamma_{j,k}$ is a matrix of autoregressive coefficients , $D_{j,i}$ is an indicator for each country[5], $\varepsilon_{i,t}$ is the

[5]It is equal to 1 if $i = j$, and 0 otherwise.

vector of residuals which, by assumption, are uncorrelated across countries and normally distributed such that $\varepsilon_{i,t} \sim N(0, \Sigma_\varepsilon)$. The interaction term, $x_{i,t}$, has the capacity to influence both the level and the dynamic relationship between endogenous variables through κ^1 and Γ_k^1.

The matrix $B_{i,t}$ is a $q \times q$ lower triangular matrix with ones on the main diagonal. Each component $B_{i,t}(w,q)$ of $B_{i,t}$ matrix represents the contemporaneous effect of the qth-ordered variable on the wth-ordered variable. It is constructed as follows:

$$B_{i,t} = \begin{cases} B_{i,t}(w,q) = 0 & for \quad q > w \\ B_{i,t}(w,q) = 1 & for \quad q = w \\ B_{i,t}(w,q) = B_j(w,q)D_{j,i} + B^1(w,q)x_{i,t} & for \quad q < w \end{cases} \tag{3.2}$$

where the coefficients B_j and B^1 represent the marginal effect of a change in the variable and interaction term, respectively. Moreover, a recursive structure has been imposed to matrix $B_{i,t}$ which means that the covariance matrix of the residuals Σ_ε is diagonal.

Imposing the shadow rate (i.e.sr) as an interaction term, the coefficient matrices for a country i will be equal to:

$$\Gamma_{sr,k} = \sum_{i=1}^{N} \frac{\Gamma_{i,k}}{N} + \Gamma_k^1 sr \tag{3.3}$$

$$\kappa_{sr} = \sum_{i=1}^{N} \frac{\kappa_i}{N} + \kappa^1 sr \tag{3.4}$$

By using this setting, our results will be averaged across countries[6] and we can compute IRFs for a specific value of interaction term.

3.2.2 Baseline Specification

Our dataset consists of quarterly data for 10 countries that have adopted the Euro from 1999Q1, for which the sample analyzed ranges between 2000Q2 and 2015Q4.[7] Appendix A provides further details about the composition of our dataset and the filter used.

Concerning our baseline specification, we choose variables which are commonly used in this literature:

$$y_{i,t} = [EUF_{i,t}, G_{i,t}, GDP_{i,t}, T_{i,t}]' \tag{3.5}$$

where $G_{i,t}, GDP_{i,t}$ and $T_{i,t}$ represent real government spending, real gross domestic product and average tax revenue, respectively. All the variables are in real term and considered in levels, the average tax revenue is computed by dividing the Total Nominal General Government Revenue series by the nominal GDP. The $EUF_{i,t}$ series represents the forecast of government spending published by the European Commission every six months. In this way, we have the opportunity to purge our VAR from the change in government spending which is anticipated by agents (i.e. fiscal foresight).[8]

[6] We follow the same methodology used by Sá et al. (2014). As explained by Canova and Ciccarelli (2009), the mean group estimator is particularly efficient if dynamic heterogeneity is present. Therefore, also in our case, it should be preferred to a pooled estimator.

[7] Beginning of our sample is due to the computation of the Real Potential GDP.

[8] The fiscal foresight is the phenomenon for which private agents, due to legislative and implementation lags, can anticipate future movements in government spending previously announced, so that not accounting for them in the identification of government spending shocks might give rise to exogeneity problems. See also Leeper et al. (2013) for a theoretical illustration.

We simplify the procedure related to the government spending multipliers computation by dividing all endogenous variables except average taxes by real potential GDP of the corresponding country. In this way we do not use the log of variables and therefore avoid potential bias related to ex post conversion to dollar equivalents of the estimated elasticities.[9]

We use as interaction term the European Central Bank Shadow Rate developed by Wu and Xia (2017). It allows us to be more accurate in terms of inference during the ZLB period. As a matter of fact, after the ZLB is reached, Wu and Xia (2017) develop a shadow-rate term structure model (SRTSM) to describe the economic environment with negative interest rates. Since this rate is available from 2004Q3 onwards, we splice the European Central Bank Shadow Rate with the Main Refinancing Operations (MRO) rate.[10]Thanks to the interaction term we are able to investigate how the government spending shock affects our variables of interest at the different levels of the interest rate. On the other hand, when we set a specific value of the interaction term, our empirical model implies that the shadow rate remains the same for the 20 quarters, corresponding to the horizon over which we calculate impulse responses. For this reason we investigate the effects of a government spending shock at different percentiles of the shadow rate, specifically at 5th, 15th and 31.7th percentiles on one hand and 50th percentile of its distribution on the other hand. We consider the range between the 5th and the 31.7th percentile as the low interest rate state. In particular, the latter percentile corresponds to a value of the shadow rate equal to 0.25, which is conventionally considered

[9]Ex-post conversion require sample averages which migth bias the computation of fiscal multipliers. This problem is even more acute in nonlinear models, such as the one we are adopting here. For further details related to these issues see Ramey and Zubairy (2017).

[10]We choose the MRO rate because it is the most similar rate to the European Central Bank Shadow Rate developed by Wu and Xia (2017).

by the literature as the lower bound for monetary policy. Furthermore, we consider the 50th percentile as the normal time interest rate state.

In order to avoid potential endogeneity problems that might bias our estimates, due to reversed causality issues, we use the first lag of the shadow rate, i.e. sr_{t-1}. We also choose a lag length of $L = 1$. We have chosen the lag length on the base of the Hannan-Quinn(HQ) and Schwarz-Bayes(SBC) information criteria.

3.2.3 Inference and identification

As in Sá et al. (2014), we start by estimating the structural recursive form presented in equation 3.1. More precisely:

1. We proceed by estimating the structural model equation by equation using OLS. We adopt a Bayesian strategy for inference utilizing an uninformative independent Normal–Wishart prior, which use a Montecarlo simulation to recover the posterior distribution of the structural parameters.

2. A draw of the posterior is made and evaluated at prespecified values of the interaction terms.

3. We derive the corresponding reduced form, by pre-multiplying equation 1 for the inverse of $B_{i,t}$

4. We use a sign restriction strategy[11] to identify an unexpected government spending shock. More specifically, we follow the same procedure of Sá et al. (2014), by using the algorithm developed by Rubio-Ramírez et al. (2010). Defining V_x^d as the Cholesky decomposition of the reduced

[11] For more details see Canova and De Nicolò (2002), Faust and Rogers (2003) and Uhlig (2005), among the others.

form variance-covariance matrix Σ_x^d obtained in step 3, we draw an orthonormal matrix Q such that $Q'Q = I$, from which follows $B^d = V_x^d Q$ and $\Sigma_x^d = B^{d\prime} B^d = V_x^{d\prime} Q' Q V_x^d$ where d indicates a stable draw from the posterior distributions.[12] To achieve identification, the impulse responses implied by B^d have to satisfy the following two sets of restrictions: a government spending shock, which raises GDP_{it} and G_{it} for at least four quarters. Following Auerbach and Gorodnichenko (2012), in order to control for anticipation effects, we also identify a forecast government spending shock, imposing an increase for at least one quarter on the response of $EUF_{i,t}$, G_{it} and GDP_{it} (see also table 3.1). Orthogonality of the two shocks should ensure exogeneity of the government spending shock.

5. For every 100 draws of the Q matrix which meet our sign restrictions we save its median value.

6. We repeat step 2 to 5 making 5000 draws from the posterior distributions and use the median over the 5000 medians obtained as our central estimate of interest.[13]

3.2.4 Multipliers

Since we estimate our model in normalized levels, we avoid any concerns related to the ex-post conversation. On this way, we can compute multipliers following the approach of Ramey and Zubairy (2017). Specifically, we compute three types of multipliers. The first is a discrete approximation of the integral of

[12] As in Sá et al. (2014); Cogley and Sargent (2005); Primiceri (2005), we discard any explosive draws from the unrestricted posterior.

[13] Note that we consider the first 10000 parameter draws as burn-in draws.

the median IRFs over time horizon $h = 0, 1, \ldots, 20$ and it is based on Ramey (2011b):

$$\mathcal{M}_1 = \frac{\sum_{h=0}^{H} d\text{GDP}(h)}{\sum_{h=0}^{H} d\text{G}(h)}. \tag{3.6}$$

Multipliers 2 and 3 are numerical integration computed using Trapezoidal and Simpson's rule, respectively.

$$\mathcal{M}_{2,3} = \frac{\int_0^H d\text{GDP}(h)dh}{\int_0^H d\text{G}(h)dh}. \tag{3.7}$$

3.3 Results for Baseline Specification

The impulse response functions for our baseline specification are showed in Figure 3.1. The four columns show the reactions of our variables of interest to an unexpected government spending shock when the shadow rate is at 5th, 15th, 31.7th and 50th percentile of its distribution, respectively. Overall, the responses of our variables of interest are not very persistent. In both states, government spending reacts strongly on impact, it reaches its peak after two quarters, and subsequently reverts quite rapidly to its long run level. The GDP is also very similar among states, even though its reaction seems to be stronger in the high interest state. However in both states, the responses of GDP become insignificant after a few quarters. Average taxes are very different among states: following a government spending shock, they do not rise very much in the high interest rate, while there is substantially no response in the low interest rate state. Overall, their behavior suggests that the government spending shock is mainly deficit financed.

The government spending multiplier (table 3.2) is quite small when the interest rate is at the ZLB. Specifically, it ranges between 0.33 and 0.88 at the ZLB, and from 1.10 and 1.29 away from it.

3.4 Results for Factor-Augmented Specification

Since our results can be influenced by the choice we made about variables and the number of variables is constrained in order to preserve parsimony of the model, defined by the literature as generic limited information problem which can give rise to nonfundamentalness of the shocks (for further details see Forni et al. (2009), Forni and Gambetti (2011))[14], we develop a FAIPVAR model. As a matter of fact, by augmenting our model with principal components as proxies for the unobserved factors affecting most of the macroeconomic variables, we incorporate in our model a large informational dataset and contemporaneously preserve the parsimony of the model.[15]

As in Di Serio et al. (2017), we implement a two-step estimation procedure similar to Bernanke et al. (2005). First, we use the method of principal components to extract summarized information from a large informational dataset.[16] Then, we add the three factors extracted to $\mathrm{Y_t}$.[17] Thus, our FAIVAR model has the following vector of endogenous variables:

[14]Note that by adding the variable EUF_{it} to our specifications, we have already accounted for another kind of limited information problem, which is the fiscal foresight.

[15]For further details about these two issues see Di Serio et al. (2017) and Fragetta and Gasteiger (2014).

[16]For this purpose, we downloaded (if available) from Thomson Reuters Datastream Economics database, the corresponding variables listed in Fragetta and Gasteiger (2014) for all ten countries considered in our analysis. In this way, our informational dataset includes 418 series.

[17]To establish the number of factors to extract, we use the Bai and Ng (2007) IC_{p2}information criterion.

$$Y_{i,t} = [EUF_{i,t}, G_{i,t}, GDP_{i,t}, T_{i,t}, F_t]' \tag{3.8}$$

where $\mathrm{F_t}$ is a 1×3 vector which is common to all countries, but that have a different impact for each country and allows to capture potential spillover effects between countries.

The resulting IRFs are showed in figure 3.2. The first important difference we observe is related to the behavior of the government spending response. As we can see, it is stronger and more persistent in the low interest rate state. Moreover, at the ZLB, GDP increases a lot on impact, then reverts smoothly to its long run level. On the other hand, in the high interest rate state, GDP response becomes insignificant after a few quarters.

The government spending multiplier (table 3.2) is almost equal among states: it ranges between 1.08 and 1.41 in the low interest rate state and between 1.26 and 1.39 in the high interest rate state.

Although these findings do not provide evidences of relevant differences among states, they are in line with theoretical studies which support New-Keynesian government spending effects. Among the others, Coenen et al. (2012) use the European Central Bank's New Area-Wide Model (NAWM) and find multipliers greater than one, as in our case. On the other hand, these findings are in contradiction with Cwik and Wieland (2011), Burriel et al. (2010) and Forni et al. (2009) who find government spending multipliers for the Euro Area below unit.

In addition, comparing these findings to results we get from the baseline specification, we can conclude that the limited information problem, related to the difference in terms of information set usually considered by the econometrician and the one considered by the economic agents, have a significant impact on the results. As a matter of fact, these results prove that our baseline specification underestimates the government spending multipliers, especially when the interest rate is at the ZLB.

3.5 Sub-samples Analysis

The results of section 3.4, show very similar multipliers for both high and low interest rate state. Although these findings broadly support New-Keynesian predictions, they are somehow in contradiction with the theoretical works of Christiano et al. (2011), Eggertsson (2010), Woodford (2011), Davig and Leeper (2011) and Coenen et al. (2012), who find higher multipliers at the ZLB, ranging from 3 to 5. It may be the case that our results are influenced by countries which have high level of debt-to-GDP ratio, which may lower the average value of government spending multipliers, especially at the ZLB.

In this section, we investigate if the reactions of variables of interest may vary across countries conditioned on their level of debt-to-GDP ratio. For this purpose, we create two subsets of countries. The first subset includes countries that have a debt-to-GDP ratio higher than 90% from 2009 on. Basically this subset includes peripheral countries, with the exclusion of Greece which joined the Euro Area only in 2001 and for which data have been continuously revised and with the inclusion of Belgium. The second group is composed by countries that, during the same period, have a debt-to-GDP ratio lower than 90%. Thus, we name the first subsample, which includes Belgium, Ireland, Italy, Portugal and Spain, "*Peripheral Countries*" and the other subsample as "*Core Countries*".

In the next subsections we show results we obtain for the two specifications described in section 3.2.2 and section 3.4.

3.5.1 Results for Baseline Specification

Figure 3.3 and 3.5 show IRFs for peripheral and core countries, respectively. As we can see, there is a huge difference in responses of our variables of interest. Considering peripheral countries, we can note that the response of government spending exhibits more or less the same pattern for both states. Government spending reaches its peak after very few quarters for both states, and it seems a little bit more persistent in the high interest rate state. GDP response mimics the behavior of government spending but it reverts more slowly to zero at the ZLB. Government spending multiplier results are slightly higher at the ZLB, ranging between 1.38 and 2.05, while it ranges between 1.37 and 1.46 away from it. It is also important to point out that the response of average tax is insignificant at the ZLB, while it is quite strong and significant away from it.

Core countries show huge responses of variables of interest to a government spending shock. The responses of government spending and GDP show very similar patterns between the two different states, even if the IRFs away from the ZLB seem to be more persistent for both variables. They are also very large compared to peripheral countries results: at the ZLB, the government spending multiplier ranges between 3.01 and 3.90, while away from the ZLB it ranges from 4.18 to 4.21. Also in this case, the response of average taxes is insignificant at the ZLB, and not so huge in the high interest rate state.

3.5.2 Results for Factor-Augmented Specification

Figure 3.4 show IRFs for peripheral countries. As we can see, Government Spending reacts on impact in the same way for both states. However, its response is more persistent in the low interest rate state. On the other hand, the responses of GDP, is slightly larger in the high interest rate state, although, the GDP response becomes insignificant after a few quarters for both states. The corresponding multipliers are somehow in contradiction with the results of section 3.5.1. As a matter of fact, table 3.3 shows multipliers between 0.82 and 1.18 at the ZLB, and between 1.29 and 1.37 away from the ZLB.

As shown in figure 3.6, results for core countries show generally a huge response of variables of interest to an increase in government spending. The responses of government spending have substantially the same intensity among states. However, the behavior of GDP is very different among states. It mimics the response of government spending at the ZLB, while it exhibits a hump-shaped pattern away from the ZLB. Average taxes rises hugely on impact, especially at the ZLB.

Once again, the government spending multipliers (Table 3.4) are in contradiction with the baseline results: it ranges between 3.11 and 3.54 in the low interest rate state, and between 2.69 and 2.87 in the high interest rate state.

3.5.3 Further Considerations

We can make three considerations from the results we get in section 3.5.1 and 3.5.2. First and foremost, our results support the theoretical works of Sutherland (1997) and Perotti (1999), which predict that a government spending shock is even more effective if the average value of debt-to-GDP ratio is low. Considering the factor augmented specifications, the government spending multiplier for core countries ranges between 2.87 and 3.54, while it ranges between 0.82 and 1.37 for countries with high debt levels (regardless if it was caused by the crisis or not). Our results are also qualitatively similar to the empirical works of Ilzetzki et al. (2013), Kirchner et al. (2010) and Nickel and Tudyka (2014).

Second, the results of section 3.5.2 show lower multipliers for peripheral countries at the ZLB. This might be due to the stronger negative effect that the higher debt-to-GDP ratio have on this subset of economies, with respect to positive potential effect at the ZLB predicted by part of the literature. In fact, we find for core countries a higher multipliers at the ZLB. Therefore, this might explain the magnitude of the government spending multipliers obtained for the full sample.

Third, we point out that with this additional exercise we are aiming to find qualitative differences between the government spending multipliers at different levels of the debt-to-GDP ratio, since in this analysis we are missing an important common factor: monetary policy. Therefore we tend to consider the results shown in sections 3.3 and 3.4 as more reliable.[18]

3.6 Conclusions

This paper tried to infer on what are the consequences of a rise (decrease) in government spending for countries belonging to the Euro Area. In order to identify an unexpected government spending shock, we use an Interacted Panel VAR model utilizing a sign restrictions identifying approach. We consider ten countries belonging to the Euro Area (which represents 95.6% of the EA-19 Total GDP) developing two different specifications: one with variables commonly used in the literature, and a more robust specification with a larger dataset, which allows us to avoid an important limited information problem. In both specifications we use the European Commission forecasts of government expenditure to account for fiscal foresight.

[18]Computing average GDP which take into account both the cross sectional and time dimension for the full sample, we find that all together represent the 95.6% of the EA-19 Total GDP.

The first part of our analysis focused on the size of the government spending multipliers for different levels of the interest rate. Specifically, we tried to answer the following question: is the government spending multiplier at the ZLB larger than in normal times? The baseline specification suggests that the answer is no. We find very low multipliers at the ZLB ranging between 0.33 and 0.88, while they are above unit away from the ZLB, ranging between 1.10 and 1.29. However, these results might be biased due to the few variables considered. For this reason, we have also considered a factor augmented Interacted Panel VAR, where it is possible to take into account a larger amount of information. Considering results obtained using the FAIPVAR model, we find very similar multipliers among states: they ranges between 1.08 and 1.41 at the ZLB and between 1.26 and 1.39 away from it. Overall, we can conclude that these findings are in line with New-Keynesian theoretical studies which argue that a raise in government spending leads to an effects on GDP greater than one. However, these results seems to not support the theoretical predictions of Christiano et al. (2011), Eggertsson (2010), Woodford (2011), Davig and Leeper (2011) and Coenen et al. (2012), who find higher multipliers at the ZLB. Our interpretation is that our findings may be influenced by a subset of countries that experienced high level of debt, which we show to have a depressive effect on the multipliers. Our work, which is not meant to be completely exhaustive, has shown how important is to take into account potential nonlinearity and different structural characteristics when computing fiscal multipliers. Other structural characteristics such as the heterogeneity of labor markets in setting wages, or different taxation in countries belonging to the Euro area might potentially reveal positive or negative effects on the fiscal multipliers that the policy makers might take into account.

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Appendix

3.A Data

Our dataset is composed of quarterly data and goes from 2000q2 to 2015q4. We consider ten out of eleven countries which joined the Eurozone when it came into existence: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal and Spain. According to Auerbach and Gorodnichenko (2013), we exclude Luxembourg because it is a small economy which exhibits large and volatile changes in government spending series.

Our variables of interest are Gross Domestic Product, Total General Government Revenue and Final Consumption Expenditure of Government. All the variables of interest are downloaded from the Eurostat database available on Thomson Reuters Datastream Economics database. We transform Gross Domestic Product and Final consumption expenditure of Government in real terms using GDP implicit price deflator. Then we normalize them by diving by real potential GDP. We also divide Total General Government Revenue by Gross Domestic Product to generate the average taxes series. The details of the Real Potential GDP computation are described in appendix 3.A.1.

We use in our specifications the forecast of the annualized growth of Government Consumption Expenditure made available by the European Commission.

Then we normalize these series by subtracting to it the annualized growth rate of real potential GDP.

3.A.1 Computation of Real Potential GDP

In order to compute the Real Potential GDP series we use the Hamilton (2017) filter to recover the cyclical component of Real GDP and successively subtract to the latter the resulting series. As discussed by Hamilton (2017), his filter should be preferred to the HP filter because the latter exhibits a persistence in the cyclical component which is far from from the underlying data generating process.

Considering the two-side HP filter, we calculate g_t^*as:

$$\min_{\{g_t\}_{t=1}^{T}} \left\{ \sum_{t=-1}^{T} (y_t - g_t)^2 + \ddot{\lambda} \times \sum_{t=-1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\} \tag{3.9}$$

By setting $\ddot{\lambda}$, which is the smoothness penalty, we choose the degree in which it is close to the data. Considering quarterly data and t far away from the start or end of the sample (at least 15 years), we can approximate the cyclical component $c_t = y_t - g_t^*$ by the following equation:

$$c_t = \frac{\ddot{\lambda}(1-L)^4}{F(L)} y_{t+2} \tag{3.10}$$

As we can see, this formula generates a stationary series if the fourth differences of our series is stationary. Anyhow, as demonstrated by De Jong and Sakarya (2016), it can be the case the non stationarity may come from the begin or the end of the sample. Moreover, Phillips and Jin (2015) claim that the HP filter may not remove the trend even if the series is $I(1)$. Cogley and Sargent (2005) consider a random walk $y_t = y_{t-1} + \varepsilon_t$, where the first differences are unpredictable and show that equation 3.10 can be written as:

$$c_t = \frac{\ddot{\lambda}(1-L)^3}{F(L)}\varepsilon_{t+2} \tag{3.11}$$

By setting $\ddot{\lambda} = 1600$ (the usual choice for quarterly data), the HP filter leads to a random ε_t and a cycle, which either predict the future as a function of future errors and is predictable as a function of past errors.

Hamilton (2017) highlights that the coefficients of F(L)$^{-1}$ depend on the value of $\ddot{\lambda}$. Consequently, it does not reflect the data generating process, and for this reason there might be persistence of the cycle. In addition, since the filter depends on the future realizations, its ability to predict the future is questionable. He proposes to make a forecast of y_{t+h}, which is made two years in advance and which is based on current and past values. Considering quarterly data, h should be equal to *8* and $p = 4$. The resulting forecast error would be taken as the cycle at time $t + h$ of the probably not stationary series. As a matter of fact, Hamilton (2017) shows that the main reason of most of macroeconomic and financial variables wrong predictions is due to cyclical component. Moreover, as shown by Den Haan (2000), the forecast error should be stationary for many nonstationary processes.

Considering the population linear projection of quarterly data,

$$y_{t+h} = \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 y_{t-3} + v_{t+h} \tag{3.12}$$

Hamilton (2017) shows that if we want to estimate the cycle at time h, and so v_{t+h} , it is not necessary to know the nature of nonstationarity or to have the correct forecasting model. For example, if we have an I(2) series, and considering $p > d$, equation 3.12 (which have $p = 4$) uses two coefficients to get stationary residuals and the other coefficients will be defined by the parameters which characterize the stationary variable v_{t+h}.

Tables

Table 3.1: Sign Restrictions for Identifying the Government Spending Shock

	Shock 1		Shock 2	
Variable	Sign	Periods	Sign	Periods
$\mathrm{EUF}_{\mathrm{it}}$	+	1		
G_{it}	+	1	+	4
$\mathrm{GDP}_{\mathrm{it}}$	+	1	+	4
T_{it}				

Table 3.2: Multipliers Full Sample.[a]

$\mathcal{M}_i$	$x_t = \mathrm{sr}_{t-1}$			
	5^{th}	15^{th}	31.7^{th}	50^{th}
$\mathcal{M}_1$	0.88	0.55	0.63	1.29
$\mathcal{M}_2$	0.68	0.33	0.41	1.10
$\mathcal{M}_3$	0.68	0.34	0.42	1.11
	$x_t = \mathrm{sr}_{t-1}$ and F_t			
	5^{th}	15^{th}	31.7^{th}	50^{th}
$\mathcal{M}_1$	1.41	1.24	1.22	1.39
$\mathcal{M}_2$	1.29	1.11	1.08	1.26
$\mathcal{M}_3$	1.26	1.10	1.09	1.29

[a]Multipliers $\mathcal{M}_i \in \{1, 2, 3\}$ are calculated as outlined in 3.2.4

Table 3.3: Multipliers Peripheral Countries.[a]

$\mathcal{M}_i$	$x_t = \text{sr}_{t-1}$			
	5^{th}	15^{th}	31.7^{th}	50^{th}
$\mathcal{M}_1$	2.05	1.65	1.46	1.46
$\mathcal{M}_2$	2.01	1.59	1.38	1.37
$\mathcal{M}_3$	2.01	1.59	1.39	1.38
	$x_t = \text{sr}_{t-1}$ and F_t			
	5^{th}	15^{th}	31.7^{th}	50^{th}
$\mathcal{M}_1$	0.98	1.14	1.18	1.37
$\mathcal{M}_2$	0.82	1.00	1.05	1.29
$\mathcal{M}_3$	0.83	1.01	1.06	1.31

[a]Peripheral Countries includes Belgium, Ireland, Italy, Portugal and Spain. Multipliers $\mathcal{M}_i \in \{1, 2, 3\}$ are calculated as outlined in 3.2.4.

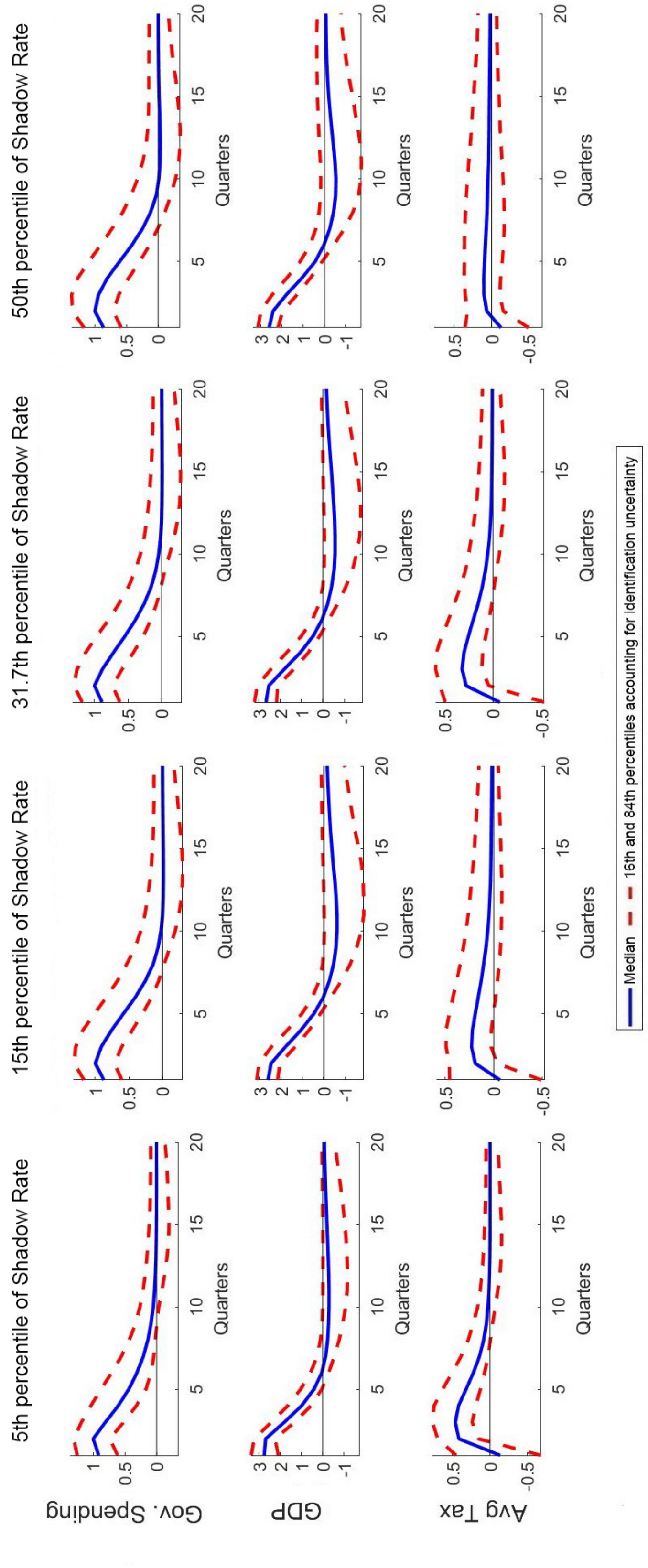
Table 3.4: Multipliers Core Countries.[a]

$\mathcal{M}_i$	$x_t = \text{sr}_{t-1}$			
	5^{th}	15^{th}	31.7^{th}	50^{th}
$\mathcal{M}_1$	3.90	3.22	3.15	4.21
$\mathcal{M}_2$	3.74	3.05	3.01	4.18
$\mathcal{M}_3$	3.74	3.05	3.01	4.18
	$x_t = \text{sr}_{t-1}$ and F_t			
	5^{th}	15^{th}	31.7^{th}	50^{th}
$\mathcal{M}_1$	3.54	3.51	3.26	2.87
$\mathcal{M}_2$	3.39	3.38	3.11	2.69
$\mathcal{M}_3$	3.39	3.39	3.12	2.71

[a]Core Countries includes Austria, Finland, France, Germany and Netherlands. Multipliers $\mathcal{M}_i \in \{1, 2, 3\}$ are calculated as outlined in 3.2.4.

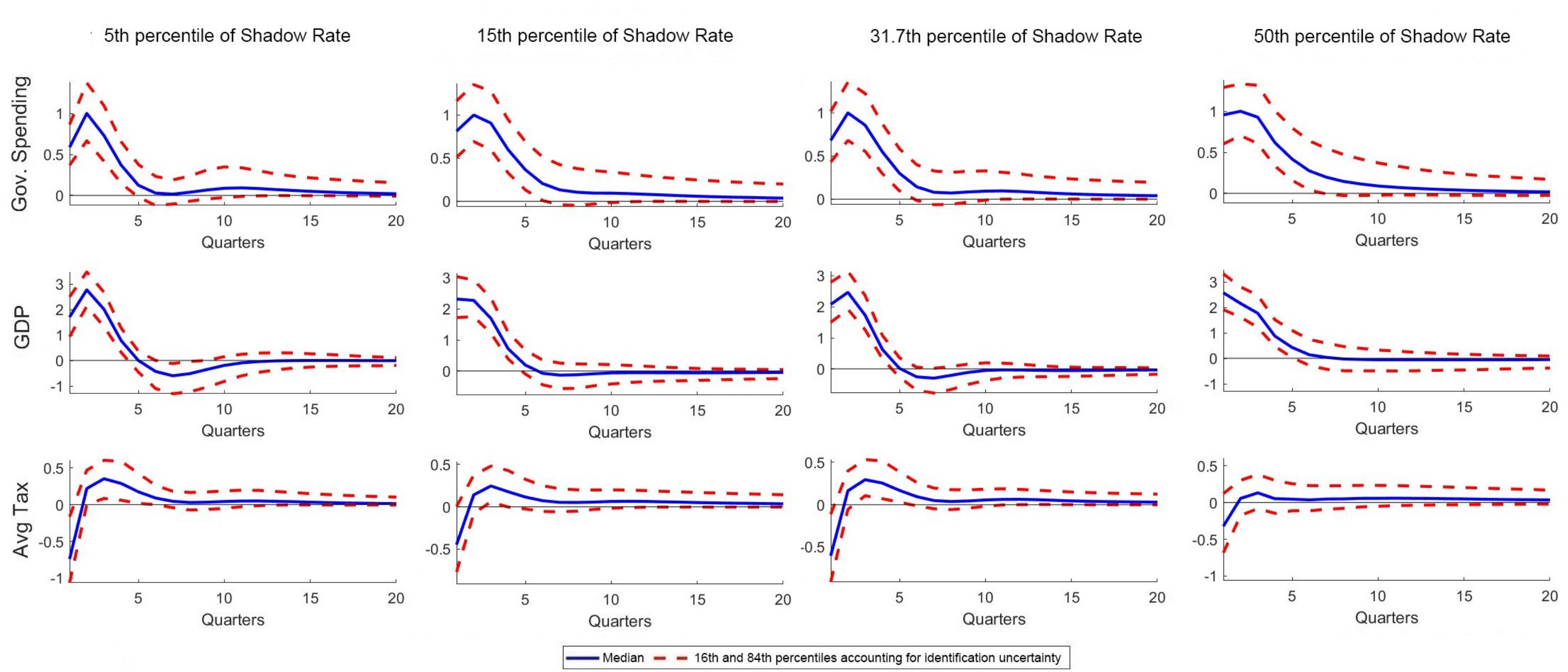
Figures

Figure 3.1: Impulse Response Functions - Full Sample - Baseline Specification



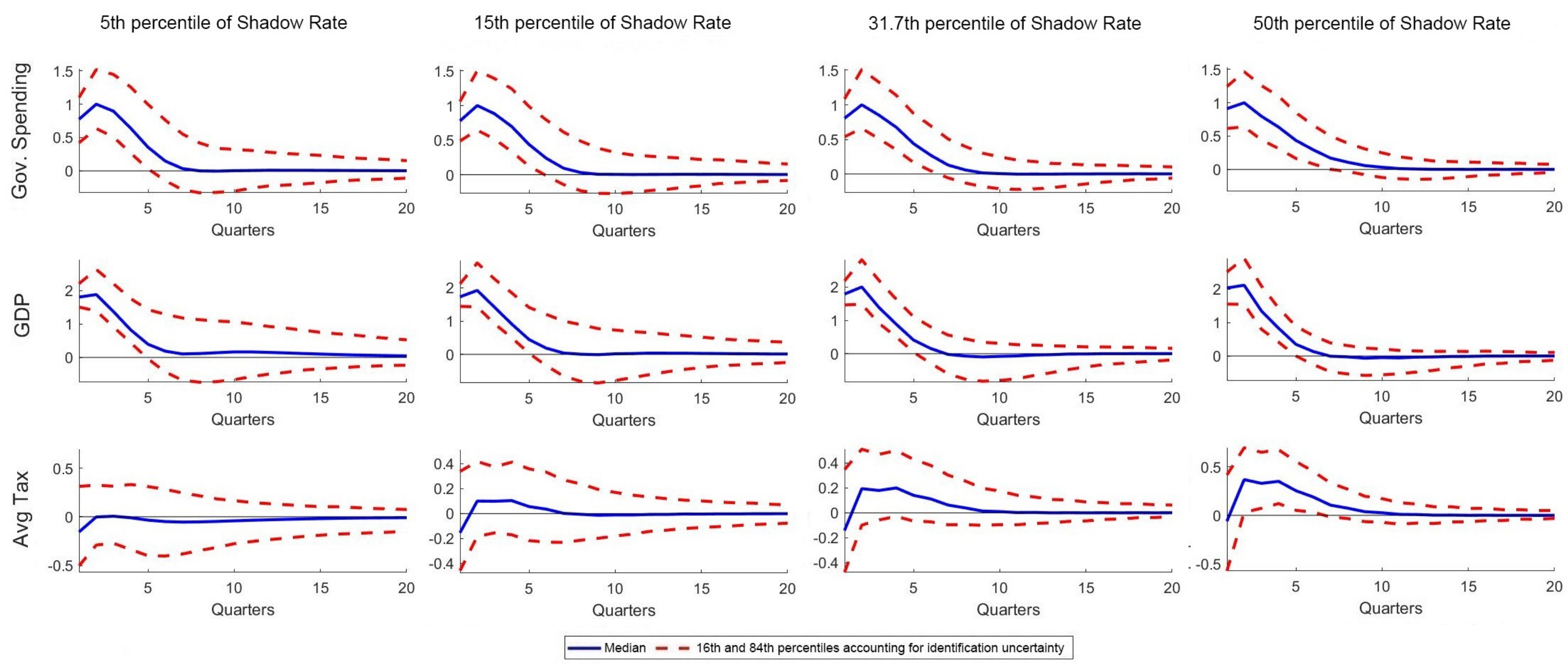
The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.

Figure 3.2: Impulse Response Functions - Full Sample - Specification with F_t



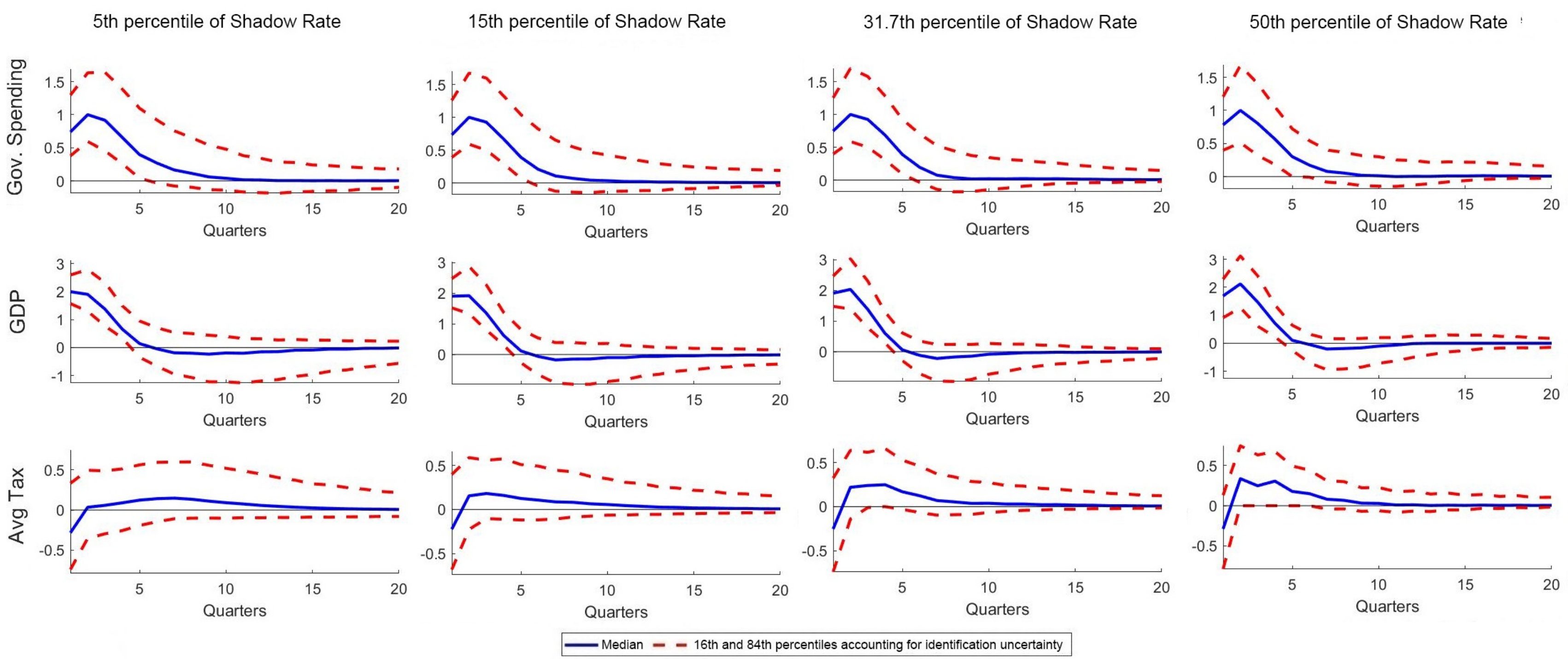
The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.

Figure 3.3: Impulse Response Functions - Peripheral Countries - Baseline Specification



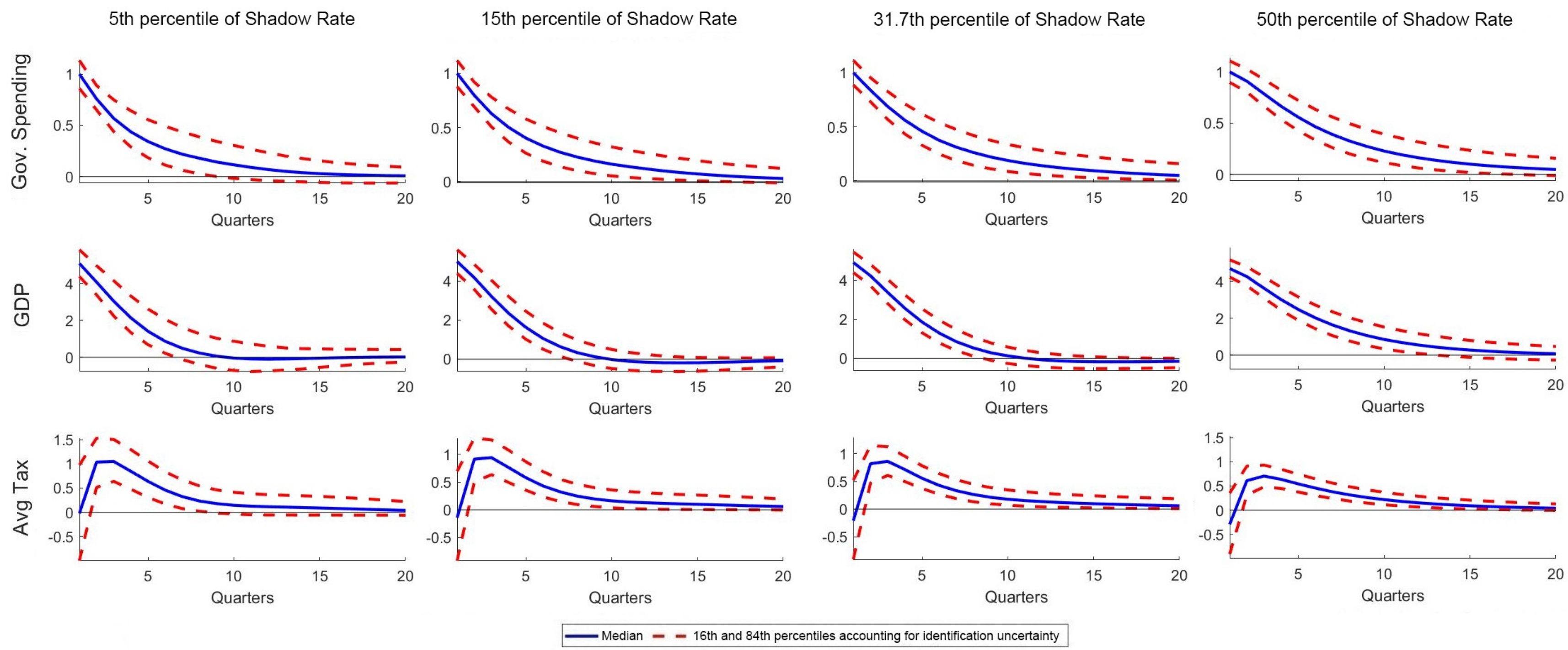
Peripheral Countries includes Belgium, Ireland, Italy, Portugal and Spain. The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.

Figure 3.4: Impulse Response Functions - Peripheral Countries - Specification with F_t



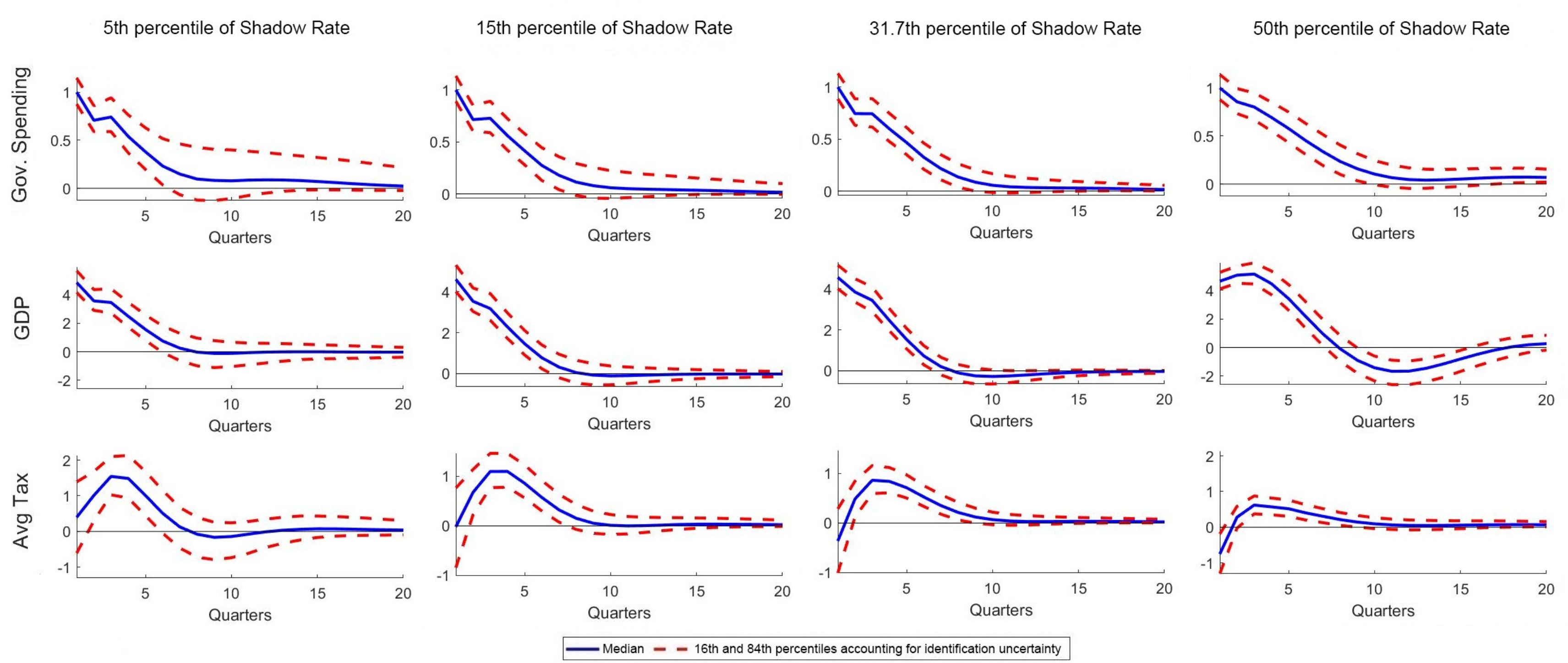
Peripheral Countries includes Belgium, Ireland, Italy, Portugal and Spain. The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.

Figure 3.5: Impulse Response Functions - Core Countries - Baseline Specification



Core Countries includes Austria, Finland, France, Germany and Netherlands. The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.

Figure 3.6: Impulse Response Functions - Core Countries - Specification with F_t



Core Countries includes Austria, Finland, France, Germany and Netherlands. The blue solid lines represent the median of the median distribution of IRFs for each parameter draw, and the red dotted lines report the 16th and 84th of the set of accepted impulse-response functions for all parameter draws.