Abstract

In the last decades, the modeling of crowd motion and pedestrian flow has attracted the attention of applied mathematicians, because of an increasing number of applications, in engineering and social sciences, dealing with this or similar complex systems, for design and optimization purposes.

The crowd has caused many disasters, in the stadiums during some major sporting events as the "Hillsborough disaster" occurred on 15 April 1989 at Hillsborough, a football stadium, in Sheffield, England, resulting in the deaths of 96 people, and 766 being injured that remains the deadliest stadium-related disaster in British history and one of the worst ever international football accidents. Other example is the "Heysel Stadium disaster" occurred on 29 May 1985 when escaping, fans were pressed against a wall in the Heysel Stadium in Brussels, Belgium, as a result of rioting before the start of the 1985 European Cup Final between Liverpool of England and Juventus of Italy. Thirty-nine Juventus fans died and 600 were injured. It is well know the case of the London Millennium Footbridge, that was closed the very day of its opening due to macroscopic lateral oscillations of the structure developing while pedestrians crossed the bridge. This phenomenon renewed the interest toward the investigation of these issues by means of mathematical modeling techniques. Other examples are emergency situations in crowded areas as airports or railway stations. In some cases, as the pedestrian disaster in Jamarat Bridge located in South Arabia, mathematical modeling and numerical simulation have already been successfully employed to study the dynamics of the flow of pilgrims, so as to highlight critical circumstances under which crowd accidents tend to occur and suggest counter-measures to improve the safety of the event.

In the existing literature on mathematical modeling of human crowds we can distinguish two approaches: microscopic and macroscopic models. In model at microscopic scale pedestrians are described individually in their motion by ordinary differential equations and problems are usually set in two-dimensional domains delimiting the walking area under consideration, with the presence of obstacles within the domain and a target. The basic modeling framework relies on classical Newtonian laws of point. The model at the macroscopic scale consists in using partial differential equations, that is in describing the evolution in time and space

of pedestrians supplemented by either suitable closure relations linking the velocity of the latter to their density or analogous balance law for the momentum. Again, typical guidelines in devising this kind of models are the concepts of preferred direction of motion and discomfort at high densities. In the framework of scalar conservation laws, a macroscopic onedimensional model has been proposed by Colombo and Rosini, resorting to some common ideas to vehicular traffic modeling, with the specific aim of describing the transition from normal to panic conditions. Piccoli and Tosin propose to adopt a different macroscopic point of view, based on a measure-theoretical framework which has recently been introduced by Canuto et al. for coordination problems (rendez-vous) of multiagent systems. This approach consists in a discrete-time Eulerian macroscopic representation of the system via a family of measures which, pushed forward by some motion mappings, provide an estimate of the space occupancy by pedestrians at successive time steps. From the modeling point of view, this setting is particularly suitable to treat nonlocal interactions among pedestrians, obstacles, and wall boundary conditions.

A microscopic approach is advantageous when one wants to model differences among the individuals, random disturbances, or small environments. Moreover, it is the only reliable approach when one wants to track exactly the position of a few walkers. On the other hand, it may not be convenient to use a microscopic approach to model pedestrian flow in large environments, due to the high computational effort required. A macroscopic approach may be preferable to address optimization problems and analytical issues, as well as to handle experimental data. Nonetheless, despite the fact that self-organization phenomena are often visible only in large crowds, they are a consequence of strategical behaviors developed by individual pedestrians.

The two scales may reproduce the same features of the group behavior, thus providing a perfect matching between the results of the simulations for the microscopic and the macroscopic model in some test cases. This motivated the multiscale approach proposed by Cristiani, Piccoli and Tosin. Such an approach allows one to keep a macroscopic view without losing the right amount of "granularity," which is crucial for the emergence of some self-organized patterns. Furthermore, the method allows one to introduce in a macroscopic (averaged) context some microscopic effects, such as random disturbances or differences among the individuals, in a fully justifiable manner from both the physical and the mathematical perspective. In the model, microscopic and macroscopic scales coexist and continuously share information on the overall dynamics. More precisely, the microscopic part tracks the trajectories of single pedestrians and the macroscopic part the density of pedestrians using the same evolution equation duly interpreted in the sense of measures. In this respect, the two scales are indivisible.

Starting from model of Cristiani, Piccoli and Tosin we have implemented algorithms to simulate the pedestrians motion toward a target to reach in a bounded area, with one or more obstacles inside. In this work different scenarios have been analyzed in order to find the obstacle configuration which minimizes the pedestrian average exit time. The optimization is achieved using to algorithms. The first one is based on the exhaustive exploration of all positions: the average exit time for all scenarios is computed and then the best one is chosen. The second algorithm is of steepest descent type according to which the obstacle configuration corresponding to the minimum exit time is found using an iterative method. A variant has been introduced to the algorithm so to obtain a more efficient procedure. The latter allows to find better solutions in few steps than other algorithms. Finally we performed other simulations with bounded domains like a classical flat with five rooms and two exits, comparing the results of three different scenario changing the positions of exit doors.