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Essays on Incentive Design for Motivated Agents

Candidato:
Ester Manna

Supervisor:
Ch.mo Prof. Marcello D'Amato

Coordinatore del Dottorato:
Ch.mo Prof. Sergio Destefanis

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Preface

Traditional economic theory assumes that individuals are self-interested. Namely, they are exclusively interested in their own utility disregarding the benefits that their action causes to the others. However, the assumption of selfish individuals proves unable to explain a number of important phenomena and puzzles. Individuals frequently engage in activities that are costly to themselves and mostly benefit others. They vote, volunteer time, help strangers, give to political or charitable organizations, donate blood, and sometimes even risk or sacrifice their life for strangers. A possible explanation for these actions is that the behavior of the individuals can be affected by altruistic, equity and reciprocity considerations (see for example Fehr and Schmidt, 1999). In this sense individuals can be intrinsically motivated: "One is said to be intrinsically motivated to perform an activity when he receives no apparent reward except the activity itself" (Deci, 1971).
In this thesis, I analyze how intrinsic motivation affects individuals' behavior in a number of different situations, such as a public good contribution problem or a school selection of motivated teachers. Furthermore, I am interested in understanding how people respond to different incentives in order to better inform policies.

This dissertation comprises three chapters.

In the first chapter, I elaborate an overview on the impact of intrinsic motivation on the individuals' choice in several economic environments. In the first part of the overview, I consider a public good problem in which individuals can be intrinsically motivated. Each individual can be interested not only in his own utility but also in the utility of the others. Even if the intrinsic motivation has a positive impact on the levels of public good contributed, it seems difficult to achieve the socially optimal level of contribution.
When intrinsic motivations alone are not sufficient to supply the socially optimal level, a principal's intervention is necessary. The intervention usually takes the form of monetary incentives such as payments or regulations. These incentives can crowd out the intrinsic motivations which prompt voluntary actions. If so, a policy may fail to achieve the desired effect.

In the second part of this overview, I analyze the interaction between intrinsic motivation and monetary incentives in order to understand how people respond to incentives. In this sense, this work is also related to the literature on psychological incentives in organizations (Bénabou and Tirole, 2003, 2006; Gneezy and Rustichini, 2000a, 2000b). A key prediction of this literature is that motivation is effective in stimulating effort even in the absence of a monetary compensation.
This review considers implications regarding the design of optimal incentives and public policy, the selection of motivated agents and its interaction with monetary rewards.

In the second chapter, I develop a principal-agent model where the individuals are the (male) agents that are asked to contribute to the public good. The key assumption of this model is that individuals may care not only about their personal interest but also about the well-being of the others. There are only two types of agents: the self-interested agents and the motivated agents. The principal may not have information on the type of agents that contribute to the public good.
With perfect information, the motivated individuals exert higher levels of effort than the unmotivated individuals. The principal offers a transfer that just covers the cost of effort incurred by the agent minus his direct benefit deriving from directly enjoying the public good and the indirect benefit tied to his intrinsic motivation. In this case, both individuals obtain higher benefits than costs by contributing to the public good and they are taxed. In addition, motivated individuals pay higher taxes than the unmotivated individuals. This is because motivated individuals exert a given level of effort even if they pay higher taxes for that.
With incomplete information, the principal gives up information rents in order to induce separation of types. Of course, the magnitude of these rents will be crucially affected by their degree of motivation. Motivated individuals exert higher levels of effort with respect to the first-best. In contrast, unmotivated individuals exert lower levels of effort with respect to the first-best. In addition, both individuals will be taxed but the motivated individuals will be taxed less than the unmotivated individuals. Otherwise, the motivated individual will not provide higher levels of effort.

In the last chapter, I study the “market" for education by developing a model that focuses on the interaction between the public and private educational sectors. The schools offer heterogeneous educational services. Parents and students choose between public and private schools considering the differences in the programmes and the quality of the education provided by the schools. The quality is influenced by the effort exerted by the teachers.
In this model, teachers can take satisfaction from teaching and from developing strategies that are consistent with the best interest of the students. In this sense, teachers can be intrinsically motivated.
The aim of this model is to investigate the impact of intrinsic motivation on the schools'

outcome in terms of quality, price and wage in a mixed duopoly environment (see for an overview De Fraja and Delbono, 1990, and Nett, 1993).

To this end, I develop an oligopolistic model where two schools are positioned at each end of a Hotelling line. The horizontal differentiation reflects the heterogeneity of the programmes offered by the schools. While the private school maximizes its profits, the public school maximizes social welfare. Each school consists of a principal and an agent, both risk neutral. The principal-agent relationship can be interpreted as the relationship between the school-principal who wants to delegate the decision about an outcome in terms of quality to a teacher (the agent).

The two schools offer imperfectly substitutable programmes and they compete against each other on quality and prices. When the degree of substitutability is high, the schools offer similar programmes and there is more competition in the market.

I show that the presence of motivated teachers can benefit or hurt public and private schools depending on the degree of differentiation between the programmes offered by the two schools. When there is high competition in the education "market", both schools prefer to hire motivated teachers in order to attract students. In this case, teachers' motivation plays an important role in the students' choice between schools. In contrast, if the schools offer significantly different programmes, the teachers' intrinsic motivation becomes relatively less important. In that case, both schools obtain higher benefits by hiring self-interested teachers than by hiring motivated teachers. The Nash-equilibrium is the one in which both schools hire selfish teachers.

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Chapter 1

An Overview on Incentive Design for Motivated Agents

I develop an overview on the impact of intrinsic motivation on the individuals' choice in several economic environments. I start by considering a public good problem in which individuals can be intrinsically motivated. Even if the intrinsic motivation has a positive impact on the levels of public good contributed, it seems difficult to achieve the socially optimal level of contribution. When intrinsic motivations alone are not sufficient to supply the socially optimal level, a principal's intervention is necessary. The intervention usually takes the form of monetary incentives. These incentives may conflict with individuals' intrinsic motivation. I analyze the interaction between intrinsic motivation and monetary incentives in order to understand how people respond to incentives. In this sense, this work is related to the literature on psychological incentives in organizations (Bénabou and Tirole, 2003, 2006; Gneezy and Rustichini, 2000a, 2000b). A key prediction of this literature is that motivation is effective in stimulating effort even in the absence of a monetary compensation. This overview is also related to the literature on the self-selection of motivated employees in different sectors (see for instance Besley and Ghatak, 2005, and Prendergast, 2007). This literature suggests that the public sector pays lower incentives to attract motivated employees. This review examines implications regarding the design of optimal incentives and public policy, the selection of motivated agents and its interaction with monetary rewards.

Keywords: Intrinsic Motivation, Monetary and Non-Monetary Incentives, Public Good Problem, Selection of Motivated Employees.

1.1 Introduction

The main objective of this chapter is to develop an overview on the impact of intrinsic motivation on the individuals' choice in several economic environments: "One is said to be intrinsically motivated to perform an activity when he receives no apparent reward except

the activity itself" (Deci, 1971).

The standard models based on the self-interested assumption can sometimes provide misleading predictions. In a public good problem, for example, if individuals only maximize their egoistic payoffs disregarding the well-being of the others, individuals would pursue their own utility through free-riding. Then, there will be under-provision of the public good (as shown in Bergstrom et al., 1986).[1]

Experimental results do not support this prediction (for a review on the experiments' results see Polania-Reyes and Bowles, 2012). In a typical public good experiment, each member is given a sum of money. They may keep this money for themselves or contribute some or all of it to a group fund. Contribution to the fund are doubled, and are then split equally among all members of the group. Total payoff will be maximized if everyone contributes all of their initial sum. However, each individual has an incentive to free-ride (see for example Ledyard, 1995). In contrast, a typical result of the experiments is that many people contribute to a public good supporting activities that are costly to themselves and mostly benefit others.

A possible explanation for these results is that the behavior of individuals can be affected by altruistic, equity, and reciprocity considerations (see for example Fehr and Schmidt, 1999). Economists have developed theoretical models and found empirical evidence showing that individuals can be "intrinsically motivated" (see for example Andreoni, 1990, and Rabin, 1993). Some individuals can be interested not only in their monetary compensation but also in the well-being of the others. Then, they would decide to cooperate, contributing to the public good, even when faced with the opportunity to increase their consumption of the private good.

In the first part of this overview, I consider a public good problem in which individuals can be intrinsically motivated. More specifically, the objective function of each individual depends on his own utility and on the utility of the others. In this model, the consumption outcome critically depends on their degree of intrinsic motivation. In addition, the existence of an efficient outcome requires the highest degree of interest by individuals. Hence, it seems difficult to achieve the socially optimal level of contribution.

When intrinsic motivations alone are not sufficient to supply the socially optimal level, government intervention is necessary. The intervention usually takes the form of monetary incentives such as payments or regulations. These incentives can crowd out the intrinsic motivations which prompt voluntary actions. If so, a policy may fail to achieve the desired effect. It is even possible for a costly policy intervention to lead to a decrease in the overall public good provision, as individuals cease to contribute voluntarily.

In the second part of this overview, I look at the interaction between intrinsic motivation and monetary incentives in order to understand how people respond to incentives to better inform policies. Monetary incentives may come into conflict with intrinsic motivation. This is

[1] In extreme formulations of this model, the Nash equilibrium without intrinsic motivation is simply zero contribution.

because monetary incentives may signal distrust or that achieving a specific goal is difficult as shown in the literature on psychological incentives in organizations (Bénabou and Tirole, 2003, 2006; Gneezy and Rustichini, 2000a, 2000b).
The study of the interaction between monetary incentives and intrinsic motivation also leads to interesting results concerning the selection of employees between public and private sector (see for example Besley and Ghatak, 2005, and Prendergast, 2007). A key prediction of this literature is that intrinsically motivated individuals exert a given level of effort even in the absence of a monetary compensation for that. This leads to self-selection of job applicants with high motivation. Furthermore, this literature suggests that the public sector pays lower incentives than the private sector to attract motivated employees.

The chapter is organized as follows: in section 1.2 I present a public good problem in which individuals can be intrinsically motivated; in section 1.3 I study the intervention of a principal and I discuss some general aspects of how monetary incentives may conflict with intrinsic motivation; section 1.4 is devoted to the study of the effect of monetary incentives on the employees' selection in different sectors; and, finally, concluding remarks are given in section 1.5.

1.2 A Non-Cooperative Public Good Model with Motivated Individuals

I begin by studying a non-cooperative model where each individual chooses his level of consumption independently of the others' decisions.[2] There are two members, A and B consuming goods that are regarded by both as being either private (in q) or public (in Q).[3] They decide over the purchase of a bundle of N private goods at the market price p, and a bundle of K public goods at the prices P_k. The first private good is treated as numeraire and it is assumed that the consumption of the numeraire and all public goods is strictly positive.
I take into account that each individual can differ in his preferences. Then, the preferences of individual A are represented by a utility function:

$$V^A = U^A(q^A, Q) + \beta^A U^B(q^B, Q), \tag{1.1}$$

and the ones of individual B are represented by a utility function:

$$V^B = U^B(q^B, Q) + \beta^B U^A(q^A, Q), \tag{1.2}$$

[2] I consider a non-cooperative model given its theoretical appeal. More specifically, any Nash equilibrium is stable in the sense that no individual can increase his utility by unilaterally changing her/his strategy.

[3] This focus on two individuals is mainly to keep the exposition simple. However, the following analysis can readily be extended to situations with more than two individuals.

Both utility functions are increasing and strictly concave.
β^J measures the degree of interest towards the other individual with $J = A, B$. I assume that it can differ among individuals and $\beta^J \in [0,1]$.
In this construction, $\beta^A = \beta^B = 0$ stands for *egoistic preferences* in which each utility depends on the individuals' own consumption of private goods (q^A and q^B, respectively) and the total amount of public goods (Q), i.e. $V^A = U^A(q^A, Q)$ and $V^B = U^B(q^B, Q)$.
In contrast, when an individual benefits from increasing the utility of the other member, $\beta^J > 0$ stands for *caring preferences*. If $\beta^A = \beta^B = 1$ the two individuals care a lot about each other and their preferences coincide, i.e. $V^A = V^B = U^A(q^A, Q) + U^B(q^B, Q)$.
Each individual J is endowed with an income Y that she/he can independently use for the purchase of public and private goods.
The individuals maximize their objective functions under the following budget constraint:

$$p(q^A + q^B) + PQ \leq Y^J \tag{1.3}$$

and feasibility constraints:

$$Q^A + Q^B = Q; \quad q^A + q^B = q \tag{1.4}$$

Let $\frac{\partial U^J(q^J,Q)}{\partial q_1^J}$ be the marginal utility of the numeraire for individual J, i.e. the first private good, the individual willingness to pay for the public good is equal to $\tau_k^J(q^J, Q) = \frac{\frac{\partial U^J(q^J,Q)}{\partial Q_k}}{\frac{\partial U^J(q^J,Q)}{\partial q_1^J}}$.
Solving both problems simultaneously, I obtain the following result:

Proposition 1.1. *Let (q^A, q^B, Q^A, Q^B) be an equilibrium with degree of caring $\beta^A, \beta^B \in [0,1]$ such that:*

$$\max\{\tau_k^A(q^A, Q) + \beta^B\beta^A\tau_k^B(q^B, Q), \tau_k^B(q^B, Q) + \beta^A\beta^B\tau_k^A(q^A, Q)\} = P_k \tag{1.5}$$

for all public goods k. When $\beta^A\beta^B = 0$, it is a Nash equilibrium of the game with voluntary contributions to public goods without caring. When $\beta^A\beta^B = 1$, it is a Cooperative equilibrium.

Proof. All the mathematical computations are shown in the Appendix A. □

To derive some intuitions out of the above proposition, it is needed to study more in detail the two benchmark cases, i.e. the noncooperative model without caring and the fully cooperative model.
When the individuals are selfish, the objective functions reduce to "egoistic" functions, i.e. $V^A = U^A(q^A, Q)$ and $V^B = U^B(q^B, Q)$. Then the equilibrium reduces to a noncooperative equilibrium without caring:

$$\max\{\tau_k^A(q^A, Q), \tau_k^B(q^B, Q)\} = P_k \tag{1.6}$$

see Cherchye, Demuynck and De Rock (2011).
By contrast, when $\beta^A = \beta^B = 1$, the objective functions V^A and V^B coincide and both individuals optimize the same objective function. By construction, this implies a cooperative equilibrium (i.e. a Pareto optimal allocation):

$$\tau_k^A(q^A, Q) + \tau_k^B(q^B, Q) = P_k \tag{1.7}$$

The Pareto efficient allocation satisfies the Lindahl-Bowen-Samuelson Conditions (see, for example, Samuelson, 1954) in which the sum of the members' marginal willingness to pay is equal to the market prices.

The model captures all possible equilibria between the fully cooperative equilibrium and the non cooperative equilibrium without caring. Indeed, the set of equilibria includes at one extreme the fully cooperative case (the only efficient one) and, at the other extreme, the non-cooperative equilibrium of the "game with voluntary contributions to public goods without caring".
The cooperative equilibrium seems difficult to achieve because "too much care", i.e. $\beta^A = \beta^B = 1$, is needed (Manna, 2012). Then, a cooperative equilibrium is achieved only if each individual puts the same weight on his own utility and on the utility of the other. Looking at a contribution game in general, it seems an exception rather than the rule. If both individuals have low degree of caring, at least one individual increases his utility by deviating from the Pareto optimal allocation.
In conclusion, *the existence of the Pareto efficient equilibrium requires a relatively high degree of caring of the two individuals*. The fully cooperative case, where $\beta^A = \beta^B = 1$, always satisfies this condition, provided that there is full symmetry in the individuals' preferences.

1.3 When intrinsic motivation alone is not enough

In the first part of this overview, I analyzed the individuals' choice in terms of contribution to a public good when individuals can be intrinsically motivated. They made their decision simultaneously and without the intervention of a principal. In some cases, intrinsically motivated individuals are able to supply public goods at efficient levels.[4] However, many public goods are undersupplied.
The failure of voluntary contribution to lead to a Pareto optimal allocation, when people behave non-cooperatively, suggests there may be a role for the government in providing the public good. Governments may provide additional incentives to increase their provision.

[4]For instance, in Australia people voluntary supply sufficient blood donations despite the absence of any formal incentive (see Reeson, 2008).

An incentive is anything (monetary and not monetary) that motivates a person to undertake a particular action or choose an alternative instead of another. People can be motivated to contribute by formal incentives. Payments may be used to subsidize contributions, or regulations to mandate a minimum contribution. Such incentives are extrinsic, that is they are dependent on external rewards or sanctions, as opposed to intrinsic incentives which are inherent to an individual (Deci, 1971).

If each individual's preferences are publicly known, the principal can easily implement the efficient level of public-good provision in so improving the outcome under voluntary provision. If preferences are private information, the principal has to elicit the information needed to approach the optimum. Because this information has to come from the individuals who hold it, the question is how to give these individuals incentives to properly reveal this information.

In this part of the overview, I discuss some general aspects of how extrinsic incentives may come into conflict with intrinsic motivation. The reason why there might be an interaction between extrinsic incentives and intrinsic motivation is that these elements operate on different psychological mechanisms. The psychological sources of intrinsic motivation are non-selfish motives, like altruism and inequity aversion. By contrast, monetary incentives give to the agent a selfish motive to act.[5] Moreover, explicit incentive contracts may signal distrust and introduce an element of control that might undermine effort exerted by motivated individuals (Falk and Kosfeld, 2006). Monetary incentives from principals may change how tasks are perceived by agents. If incentives are not large enough, this change in perception can lead to undesired effects on behavior. In other cases, incentives might have the desired effects in the short term, but they still weaken intrinsic motivations.

The negative effects of monetary incentives are due to the fact that people act not only to acquire economic goods and services but also to constitute themselves as dignified, autonomous and moral individuals. By contrast, implicit incentives may support the actions of the individuals because the strategic incentives do not contradict individuals' intrinsic motivation (Gächter, Kessler and Königstein, 2011[6]).

In the following subsections, I will analyze more in detail the literature on psychology incentives, the impact of intrinsic motivation for performing a particular task and the interaction between monetary incentives and intrinsic motivation.

[5] In the public good experiment designed by Falkinger et al. (2000), subjects at first experience a monetary incentive system which proves very effective in increasing contributions to the public good. Later they play the same game without incentives and they contribute the 26% less than subjects who have not been exposed to the incentives.

[6] In this paper, they also show that there is no reason why the voluntary cooperation of non-selfish people should be adversely affected by implicit incentives and that they reinforce reciprocal motivations.

1.3.1 The relation between extrinsic and intrinsic incentives

Economic theory emphasizes the importance of incentives. Providing rewards contingent upon effort or positive performance directly encourages positive effort. While there is substantial evidence consistent with this point of view, there is also an argument that explicit rewards may have the adverse consequence of crowding out the agent's intrinsic motivation for performing the task.[7] Monetary incentives have two kinds of effects: the standard direct price effect, which makes the incentivized behavior more attractive, and an indirect psychological effect. In some cases, the psychological effect works in an opposite direction with respect to the price effect and can crowd out the incentivized behavior. Several papers in recent years have shown that such crowding-out effects can be handled with fairly standard economic modeling of principal-agent problems that use intrinsic motivation assumptions. Bénabou and Tirole (2003) assume that the principal has more information than the agent. Any incentive contract offered by the principal has the potential to convey this information to the agent. For example, if the principal has superior information about the difficulty of the project, then a contract that promises a high-reward contingent on a success might convey the message to the agent that the job is difficult to perform. In their basic model, increasing compensation increases the probability that an agent will supply effort, but also signals to the agent that the job is distasteful or that effort is unlikely to lead to a success. They show that rewards may be only weak reinforcers in the short term and that, as stressed by psychologists, they may have hidden costs, in that they become negative reinforcers once they are withdrawn. The idea is that by offering low-powered incentives, the principal signals that she trusts the agent. Conversely, rewards (extrinsic motivation) have a limited impact on current performance, and reduce the agent's motivation to undertake similar tasks in the future. Then, they use the same logic to show that empowering the agent is likely to increase his intrinsic motivation. Similarly, help offered by others may be detrimental to one's self-esteem and create a dependence. The "crowding out" case requires the agent be less knowledgeable in some dimension than the principal. Furthermore, sorting condition must hold, in that the principal has to be more inclined to offer a reward when the agent has limited ability or the task is unattractive. Otherwise, there will be "crowding in". Thus, when concerned about a potential negative impact of rewards, one should first check whether the reward provider has private information about the task or the agent's talent. One should then, as the agent does, think through the provider's ulterior motivation and how her payoff from giving a contingent reward is affected by her knowledge.

In Bénabou and Tirole (2006), individuals have a utility function with three main components: they value extrinsic rewards, enjoy doing an activity, and care about their image vis-à-vis themselves or others. The image component depends on the value they or someone else attributes to their intrinsic and extrinsic motivation as a function of their effort level and

[7] See Deci and Ryan (1985) for a survey.

incentives. This image motivation depends on how much individuals care for their reputation and may be affected by how public such an image is. Individual preferences for the enjoyment of tasks and for the image component of their utility may differ between people and are assumed to be private information. In this model, extrinsic incentives reduce other motives for undertaking the task if a higher personal benefit associated with a certain level of pro-social behavior affects the reputational value attributed to a person's intrinsic and extrinsic motivation. That is, decreasing the signal about a person's pro-social preferences and increasing the signal about a person's greediness may result in lower image motivation. In such cases, offering higher material rewards may backfire if the effect on image motivation is stronger than the standard price effect. This effect may depend on the extent to which these signals are public.
The models, explained in these two articles, *illustrate how the principal's incentive gives a bad information to the agents and can affect agents' decisions about effort.* In a private-good context without image concerns and in which the principal is better informed than the agent, the principal chooses a reward level based on several factors, including how the principal views the difficulty or attractiveness of the task to be performed and how the principal views the intrinsic motivation or ability of the agent. For example, offering incentives for improved academic performance in schools may signal that achieving a specific goal is difficult, that the task is not attractive, or that the agent is not well-suited for it (and thus needs the additional incentive of a reward). Alternatively, offering incentives could signal that the principal does not trust the agent's intrinsic motivation. This signal will be "bad news" for the agent and can lower the intrinsic motivation of the agent to undertake the task. Considering the image concerns, extrinsic incentives may have a negative impact on the performance of the individuals. This is because the monetary incentives reduce the moral approbation behind the action and replace it with a simple monetary compensation.

Gneezy and Rustichini (2000a and 2000b) present evidence for explicit incentives having counterintuitive influences in laboratory and natural experiments. Gneezy and Rustichini (2000a) show that the imposition of an explicit penalty for failing to pick up a child on time at a day-care center leads to a decrease in the number of people who pick up their child on time. They suggest that a mechanism similar to the one described by Benabou and Tirole is at work. The incentive scheme conveys information to the agent, which leads to a counterintuitive response. The authors argue that the fine eliminated the moral disapprobation associated with arriving late and replaced it with a simple monetary cost which some parents decided was worth incurring. Their results show that the effect of price changes can be quite different than in economic theory when behavior has moral components which wages and prices alter. In the day-care setting of Gneezy and Rustichini (2000b), they show that the level of performance in a task is not monotonic in monetary rewards. Imposing a modest fine for late pick ups could lower the agent's subjective probability that an even more severe

penalty would be imposed.
The psychology literature suggests that crowding out does not depend on the ability of incentive schemes to convey information about the task, but instead argues that monetary incentives change preferences in systematic ways. These reasons illustrate possible behavioral effects of incentives and create implications for the design of incentives.

1.3.2 When do monetary incentives crowd out or crowd in intrinsic motivation? (Bowles and Hwang, 2008)

Of course the crowding-out evidence discussed above does not imply that using incentives to obtain behavioral changes will always be counterproductive. Experimental evidence indicates that conventional economic incentives and intrinsic motivation may be either complements or substitutes.
Bowles and Hwang (2008) analyze theoretically when monetary incentives crowd in or crowd out intrinsic motivation. Furthermore, they investigate if a social planner, aware of the crowding out problem, would make more or less use of the monetary incentives relative to a naive planner who assumes that economic and intrinsic motives are separable.
To achieve this objective, they develop a model in which individuals have a set of ethical values that may motivate the individuals' behavior and let these values be influenced (positively or negatively) by the use of explicit incentives.
They consider a community of identical individuals indexed by $i = 1,...,n$ who may contribute to a public project by taking an action, $a^i \in [0,1]$, at a cost $g(a^i)$ which is non-negative, increasing and convex in its argument. The output of the project depends on individual contribution, $\psi(a^1, a^2, ..., a^n)$, and explicit incentives take the form of a subsidy s that is positive or equal to 0 and proportional to the amount contributed.[8]
The individual i's utility is:

$$u^i = \psi(a^1, a^2, ..., a^n) + sa^i - g(a^i) + v(a^i, s) \tag{1.8}$$

where $v(a^i, s)$ represents the ethical values and has the following explicit form: $v = a^i(\underline{v} + \lambda s)$ and the marginal effect of i's contributing on i's values is $v_{a^i} = \underline{v} + \lambda s$. The classical separability assumption maintains that the level of monetary incentives does not influence the marginal utility of contributing: that is $\lambda = 0$. Varying a^i to maximize u^i for given values of s and the others' contributions, the individual's best response a^i is given by:

$$g'(a^i) = \psi_{a^i} + s + \underline{v} + \lambda s \tag{1.9}$$

[8]They do not consider the case of taxes (i.e. $s < 0$) because motivational crowding is not symmetric: in experiments, both bonuses and fines crowd out social preferences (though typically in different degree) so one cannot reverse the crowding effect by adopting taxes rather than subsidies.

where the left hand side represents the marginal cost of contributing and right hand side terms represent the marginal benefits arising from the project and from subsidies, and the marginal benefits associated with the individual's contribution, respectively.
Thanks to the implicit function theorem, the effect of the subsidy on the individual's contribution (given the contributions of the others) is then

$$\frac{\delta a^i}{\delta s} = \frac{1+\lambda}{g'' - \psi_{a^i a^i}} \tag{1.10}$$

where the denominator is positive by the second order condition of the individual's optimization problem (in the case of a convex benefit function for the public project, requiring that the marginal costs of contributing be rising faster than the marginal private material benefits). It is possible to see that there is either crowding in ($\lambda > 0$) or crowding out ($\lambda < 0$). Under crowding in, ethical values and incentives are complements, as increased use of the incentive enhances the marginal effect of contributing on one's values and the effect of the subsidy increases the individuals' action. Crowding out makes incentives and ethical values substitutes, reducing the effect of incentives on the individuals' behavior. If $\lambda < -1$ there is "strong" crowding out and the incentives reduce contributions. Moreover, a positive response by subjects to explicit incentives does not indicate that crowding out is absent but only that $\lambda > -1$.
If crowding out is so strong that the incentive has an effect opposite to its intent, the social planner would reduce the monetary incentives. By contrast, where the effectiveness of incentives is blunted but not reversed, the implications for the optimal use of incentives are far from obvious. The reduced effectiveness of the incentive associated with crowding out would entail a larger incentive for a planner designing a subsidy to ensure compliance with a quantitative target. As a result, crowding out makes the incentive less effective, so that to attain the target, more incentive is needed.
In conclusion, while explicit incentives do a tolerably good job in many situations, in others performance would be improved if mechanism design took account of the effects of incentives on preferences.
A potential extension to this article could be to study the case in which the population is made up of both self-interested and motivated individuals, as it is suggested by experimental evidence. In this case some mechanisms provide incentives that induce even the motivated individuals to act as if they were selfish and self-interested individuals to act as if they were motivated.

1.3.3 Some Economic Applications

In the following subsections, I discuss the impact of monetary incentives on the individuals' choice in different circumstances. I analyze a theoretical model of household recycling, in

which the public policy has an impact on the individuals' perception of the morally ideal action. Then, I examine the effect of monetary incentives in education. In this case, monetary incentives can be successful if the incentives are well-specified and well-target.

Image Motivation in Household Recycling (A theoretic Model of Brekke, Kverrndokk and Nyborg, 2003)

Economically household recycling activities contribute to the production of public goods such as improved environmental quality. Recent decades have seen the introduction of sanctions and payments in an attempt to maintain the supply of environmental goods and services. However, the possibility of crowding out indicates that the policy can be less effective than expected, or even trigger a reduction in supply.[9] This is because individuals are willing to supply many environmental goods and services even in absence of monetary incentives. Such voluntary contributions are due to intrinsically motivated individuals.
Brekke, Kverndokk and Nyborg (2003) assume that consumers want to think of themselves as socially responsible, which is in turn determined by the distance between one's actual behavior and a morally ideal action.
In mathematical terms, the self-image of the individuals is given by the following loss function:

$$S_i^d = -K(a_i - a_i^*)^2 \tag{1.11}$$

where S_i^d is the self-image as a responsible individual, $K > 0$, a_i is the actual behavior, and a_i^* is the individual's perception of the morally ideal behavior. With the chosen functional form, the individual's self-image attains a global maximum equal to zero at $a_i = a_i^*$. This means that the function is concave in a_i: it is increasing in a_i for $a_i < a_i^*$, and falling in a_i for $a_i > a_i^*$. Further, self-image is decreasing in a_i^* for $a_i < a_i^*$, and increasing in a_i^* for $a_i > a_i^*$.
From i's perspective, the morally ideal contribution is the action which would hypothetically maximize social welfare if everybody else had acted as she herself did. Then, a_i^* can be interpreted as the individual's perception of her responsibility as a responsible citizen which may be determined by moral or other considerations. If she contributes less than this level, she does not fulfill her duty, and thus experiences a utility loss in terms of a reduced self-image.
Their model has similarity to the "warm glow" model of Andreoni (1990), because the cynical view on moral behavior as simply serving the private interest of having good conscience. Andreoni (1990) introduces a generalization of the altruistic model. In his article, he elaborates the idea that giving produces a pleasurable feeling, the "warm glow", which is formulated as a preference for giving per sé, distinct from the benefit enjoyed by the recipient. The idea is

[9] This can be particularly likely if payments are relatively small as is the case for many environmental incentives.

that one's own contribution to a public good produces a private good, warm glow, as a by-product of contributing to the public good. Andreoni refers to an individual with preferences for such a warm glow as an "impure altruist". Furthermore, he assumes that the individuals are not indifferent between gifts made by themselves and gifts made by the other individuals or the government. While the warm glow (or impure altruist) individual contributes for the pure pleasure of the giving, the duty-oriented person contributes to keep her self-image as a responsible citizen. Moreover, the impure altruist derives a strictly positive (non-pecuniary) benefit from making a contribution, the duty-oriented person suffers a utility loss if he contributes less than he believes he ought to. Consequently, if his perceived duty increases, thus making the goal harder to reach, his utility will be reduced, but he may still increase his contributions (see Bruvoll and Nyborg, 2004). Both models of Andreoni (1990) and Brekke et al. (2003) reach the same conclusion regarding the under-provision of public good, even if the reason for under-provision is different.

Brekke et al. (2003) demonstrate with their model that monetary incentives may decrease morally motivated contributions. Public policy affects behavior not only through its effect on relative prices and budget, but also through the policy's effects on individuals' perception of the morally ideal action.[10]

To test the empirical relevance of the theoretical model, they use Statistic Norway's Omnibus survey for November-December 1999 concerning with recycling of household waste. The survey data on recycling and voluntary community work are consistent with the idea that public policy has an impact on individuals' perception of the morally ideal action. If duty-based motives are important for voluntary contributions, any policy designed to increase contributions should take into account that although people may contribute out of a feeling of responsibility, they may also try to avoid situations in which this feeling arises. Then, the duty-oriented people have been placed in a situation where they feel to have a responsibility for others and they act accordingly. However, when my responsibility is being exploited by another person, my duty towards that disappears. A proper understanding of the causes of social interaction effects in voluntary contributions to public goods is potentially important for policy analysis.

[10] In a given context, the individual may be unsure of what is and what is not his responsibility; and if so, he may look at the others to infer what is demanded of a "proper" citizen. That is, informational social influence can come into play, causing social interaction in individual contributions (Nyborg et al., 2005). Nyborg et al. (2005) assume that when the individual is unsure of what his responsibility is, he (at least partially) learns this from the observation of others' behavior.

Can Monetary Incentives Enhance Educational Outcomes?

It may seem that designing incentive mechanisms to improve education should be relatively unambiguous. Students may invest too little effort in their own education because they overly discount the future, have time-inconsistent preferences, or underestimate the return on education. Monetary incentives can then provide immediate returns that provide an extra motivation to study. Similarly, incentives can give parents and teachers additional reasons to put more effort into educating children or simply making sure the kids go to school (Glewwe, Ilias, and Kremer, 2010).

However, monetary incentives may crowd out other underlying reasons for educational decisions. Kohn (1999) refers to incentives provided in education as “bribes". He shows that while manipulating people with incentives seems to work in the short run,[11] it is a strategy that ultimately fails and even does lasting harm. Kohn demonstrates that people actually do inferior work when they are enticed with money, grades, or other incentives. “The more we use artificial inducements to motivate people, the more they lose interest in what we’re bribing them to do. Parents and teachers who care about helping students to learn, meanwhile, should be doing everything possible to help them forget that grades exist".[12]

Experimental evidence provides important insights about when such incentives are more likely to work. In particular, the empirical evidence shows that incentives work well in increasing attendance and enrollment and they have mixed results on effort and achievements.

Regarding the incentives in attendance and enrollment, the program Progresa in Mexico is an example (Behrman, Sengupta, and Todd, 2005; Schultz, 2004). This program covered the high opportunity costs of sending kids to school for the poor families. The program paid on average 55$ per month (over one-fifth of the average family income) for families whose children attended school. The evaluation of the first years of the program show earlier ages of school entry, less grade repetition, better grade progression, lower drop-out rates, and higher school reentry among drop-outs. Particularly noteworthy are the reduction of drop-out rates during the transition from primary to secondary school, and that grade progression occurred even from younger children who do not receive educational incentives through the program. This last finding suggests a forward-looking behavior on the part of the parents.

Therefore, *the evaluation of programs using incentives to reward enrollment and school attendance is positive.* This is due to specific characteristic of the programs. More specifically, the programs offer incentives for concrete tasks. Students either attend school and receive the reward, or not. These programs do not involve a complicated objective that students may

[11] *Promising gifts to children for good behavior can never produce anything more than temporary obedience* (Kohn, 1999).

[12] Many educators believe paying students is morally wrong. They argue that one of the goals of schools is to increase the importance of intrinsic motivation.

not know how to achieve, and neither there are difficulties in measuring and rewarding the achievement of the objective. Furthermore, incentives are offered to families and not to the children specifically, and thus the incentives do not directly affect the motivation of those being educated. In this way, the possibility that children may substitute their desire to learn for their desire to receive the reward is of less concern.
Compared with the evidence on attendance and enrollment, the evidence on incentives offered for academic performance is more mixed and depends on the characteristics of the task being rewarded. Bettinger (2010) shows that to give direct incentives for higher grades in primary schools in Coshocton, Ohio, increased math scores but not those of other subjects, such as reading or social science. One possible interpretation of these results, compatible with research in psychology, suggests external incentives may be more effective in concrete subjects, such as primary school math, than in more conceptual topics, such as reading and social sciences (Rouse, 1998). This can also mean that incentives seem to work well for some students but not for others.
In conclusion, monetary incentives seem to have moderate success when the incentives are well-specified and well-targeted (“read these books" rather than “read books"), and a principal (the government, for example) has to be very careful when designing the incentives to prevent adverse changes in children’s behavior.

1.4 Incentives in the Selection of Motivated Employees

Another related topic is the self-selection of motivated employees in different sectors. It is often argued that preferences and work motivations of employees differ depending by the nature of the jobs. Many jobs involve helping people in need or contributing to society at large, delivering these jobs attractive to people who have a strong willingness to serve others or the public interest (see Buurman, Dur and Van den Bossche, 2009).
There is a large literature that suggests that employees want more than just monetary compensation for their work and they differ in their intrinsic motivation (see for example Francois, 2004, Besley and Ghatak, 2005, and Prendergast, 2007). Most articles in this literature are principally focused on working relationship between the public and private sectors and on the matching of employees with different motivation in different sectors.
Besley and Ghatak (2005) emphasize the importance of the precise definition of the target of a public sector institution in increasing organizational efficiency. They find that it is possible to achieve a perfect matching between workers and firms in which motivated employees work in the non-profit sectors making their preferred projects. In this case, motivated individuals improve their levels of effort, even if they receive a low compensation for that. They conclude that monetary incentives are likely to be lower-powered in the non-profit than in

the for-profit firms.

The use of high powered incentives in the public or non-profit sector has long been questionable. For example, Holmström and Milgrom (1991) argue that that if an employee performs several tasks and it is not easy measure how well these tasks are performed, then introducing monetary incentives is not always efficient as such incentives are often associated with less effort and less attention being placed on tasks of high social value. They emphasize the distinction between output and the agent's measured performance. In general, it is shown that if the principal wishes the agent to allocate effort towards a task that is not easily measurable then incentives on the measurable tasks must be weakened. When the agent has an outside task option, optimal incentives are also weaker the more that effort is substitutable between outside and inside tasks. Holmström and Milgrom further describe how the subset of tasks performed within an individual job and the method of pay for that job, are jointly determined. It is predicted that subsets of tasks will be grouped around the costs of measuring and rewarding performance. Some workers will perform a set of easy to measure tasks and will be paid based on measured performance. Others will perform a set of difficult to measure tasks and will receive a fixed wage. A good example is teaching. Teachers have the choice of whether to invest in effort that will improve the test scores of students or effort that gives students other skills such as inquisitiveness and curiosity whose performance is difficult to measure. The authors argue that if teachers are given incentive pay based on exam performance then they will exert less of the second type of effort. Thus, the overall outcome of incentive pay may be an undesirable one and it could simply be a first best option to offer teachers a flat salary. The clear implication of this article is that a performance related pay scheme should be used less in jobs that involve many, complex or difficult-to-measure tasks.[13] However, they do not consider the case in which individuals can be intrinsically motivated. Intrinsically motivated teachers could have less incentive to shirk either type of effort.

In a related paper by Francois (2004), he analyzes selection into production of a public good. It is assumed that a single firm has a large pool of potential applicants to choose from. The workers differ in their valuation of the public good, and their motivation is purely outcome related. The main purpose of the paper is to compare performance-based pay with a flat-rate salary. If payment is made conditional on delivery, only individuals with motivation

[13] Dixit (1997) extends Holmström and Milgrom's (1991) bilateral multi-tasking model to the case where several principals contract with a single agent over a range of tasks. The agent's decision rule is to choose the optimal effort level to exert on each task according to his private costs and benefits given the full set of contracts. When the principals collude to provide a single contract to the agent, the same result is found as in the Holmström-Milgrom model. When the principals cannot collude however, each principal sets a reward schedule that is privately optimal given each of the other principal's reward schedules and the agent's optimal decision rule. But the lack of collusion leads to a negative externality weakening incentives relative to the collusive case. The implication of this work is that providing incentives for a government agent to act in alignment with the social optimum is more costly when his reward depends on satisfying several non-collusive principals.

above a certain cutoff value will apply for the position, and these individuals will also deliver. When a salary is used, the situation is more complex. A contract offered will typically attract two types of applicants: shirkers, who have a very low valuation of the public good, and those who have a particularly high valuation of the good. One interesting point is that it is not straightforward to derive the comparative statics for how these two cutoff points change when the salary is increased: it depends on the distribution of worker types. Due to this ambiguity, it is not possible to determine whether performance-related pay or salaries are preferred without putting further restrictions on the distribution of types. However, total output will not be very sensitive to salaries: an increase in wages causes an influx of committed workers, but also more shirkers find it worthwhile to work for the higher salary. For this reason, wages will be lower in the public sector.
In Prendergast (2007), many individuals are motivated to exert effort because they care about their jobs, rather than because there are monetary consequences to their actions. He assumes that workers differ in altruism for clients and shows that government prefers to attract different workers types for different agencies. Moreover, he shows that, when agents' types are not observable, agencies are likely to attract both the most preferred and the least preferred workers.

A key prediction of this literature is that we should observe relatively low pay and weak monetary incentives in jobs with high degree of individuals' intrinsic motivation, as these lead to self-selection of job applicants with high motivation. Furthermore, this literature suggests that the public sector pays lower monetary incentives than the private sector to attract motivated employees.

1.4.1 Career Concerns or Implicit Incentives

The theory of the incentives generated by career concerns, or implicit incentives, has been relatively overlooked in relation to the large literature on explicit incentives generated, within firms, by alternative methods of pay. The central idea behind implicit incentives is that the worker exerts effort in order to influence the market's beliefs about his talent. Thus even when an employee is paid a fixed wage, he may be motivated by the effect his effort has on future wages.
Holmström (1982) presents a model in which wages depend on expected productivity which is a function of observed performance in previous periods. This creates an "implicit contract" linking contemporaneous performance to future wages. There are two parties in the model: the market and the worker. Both are assumed to know the distribution of talent across workers, the distribution of noise in the production process and the worker's optimal decision rule. When talent is fixed, the firm sequentially infers a more precise belief about

the worker's talent with each observation on output. The more precise is the firms belief, the less incentive there is for the worker to exert effort. As the number of observations becomes large, optimal effort tends towards zero. When there is no uncertainty, there is no effect of effort on beliefs and hence no effort will be exerted. This extreme result depends on the initial assumption that talent is fixed. Then, Holmström assumes that talent evolves over time due to effects such as learning by doing; this introduces extra uncertainty about talent at every observation on output. In this case, the precision of the market's belief about an agent's talent tends to a constant state where the learning through observations on output exactly offsets the talent shocks. This is associated with a positive optimal level of effort for the worker.
Two key results are generated from this model. First, career concerns are more effective if the evolution of talent is more stochastic or if the observations on output are more accurate. The former captures the intuition that when there is more uncertainty about how potentially talented he is, an employee will work harder since his effort has a stronger impact on future wages by influencing this belief. The more accurate are the observations on output the larger is the return on effort in a single period since more will be inferred by the market as a result of each observation. The second key result is that, assuming that the precision of the markets belief on talent converges upwards to its stationary state, effort levels converge monotonically downwards to their stationary state. That is, young workers will work harder than older workers since they are more able to establish a reputation through effort when the market's information is more diffuse as it is thought to be for younger workers. This result also holds intuitive appeal.

The nature of incentives generated through career concerns may be particularly important in the public sector. Dewatripont, Jewitt and Tirole (1999) extend Holmström's model (1982) by including multiple tasks and analyze the incentives of government agency officials. Of particular motivation to their analysis are a set of observations by Wilson (1989). Wilson stresses three key differences between government agencies and private firms. The first is the preponderance of career concerns over financial incentives facing government officials. Second, the objectives of the government agency are often unclear and, third, they often operate with limited autonomy in relation to private firms. Wilson also notes that successful government agencies have a clear focus on a specific mission. A formal analysis using the multitask career concerns model generates a set of results consistent with these observations. It is derived that expanding the set of tasks pursued by the worker typically reduces total effort. This is because the link from performance to the market's inference about talent becomes weaker with more tasks. Uncertainty over the nature of tasks pursued by the worker or the effort allocation between them is also shown to reduce total effort.
These two results taken together support the observation by Wilson that successful agencies pursue a narrow and clear mission. They also lend support to the idea that hiring profes-

sionals who focus on a specific task can be relied upon for greater effort than a generalist bureaucrat and should receive more autonomy. One of the key insights from this literature is that there is scope for improving performance in the public sector simply through organizational design. Improving the clarity of missions and allocating a minimal number of tasks to public sector officials may substantially improve incentives via individuals' concern for their careers and clarity of goals.

1.5 Conclusions

In this overview, I have studied the impact of intrinsic motivation on the individuals' behavior. In particular, I have considered the case in which individuals contribute to a public good. In this case, models that consider individuals' intrinsic motivation provide better predictions than standard models based on the self-interest assumption, as many experiments show. However, even if the individuals' intrinsic motivation has a positive impact on the levels of contribution, the Pareto efficient allocation is not always achieved. Then, the intervention of a principal is needed. The principal can offer different incentives to different types of agents in order to elicit higher levels of effort. I have studied the impact of monetary incentives on the individual choice when the individuals have different degrees of intrinsic motivation. Furthermore, I have investigated how monetary incentives can reduce the individuals' performance under some circumstances. In education, for example, monetary incentives seem to have moderate success when the incentives are well-specified and well-targeted ("read these books" rather than "read books") but they may crowd out other underlying reasons for educational decisions. Moreover, monetary incentives can signal distrust or difficulty of achieving a specific goal.

Another related topic is the self-selection of motivated employees between public and private sector. Intrinsically motivated individuals exert a given level of effort even in the absence of a monetary compensation for that and this leads to self-selection of job applicants with high motivation. This literature also suggests that the public sector offers lower incentives than the private sector to attract motivated employees.

1.6 Appendix A

The marginal rate of substitution τ^J represents the individual willingness to pay for the public good. Let $\frac{\partial U^J(q^J,Q)}{\partial q_1^J}$ be the marginal utility of the numeraire for member J, i.e. the first private good, the member J's marginal willingness to pay for an additional unit of the public good k at the allocation $\{q^J, Q\}$ is as follows:

$$\tau_k^J(q^J, Q) \equiv \frac{\frac{\partial U^J}{\partial Q_k}}{\frac{\partial U^J}{\partial q_1^J}}\Big|_{(q^J,Q)} \tag{1.12}$$

Lemma 1.1. *An equilibrium is characterized by the following system of inequalities for $J = A, B$.*

$$\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial q_1^A} \leq p\lambda^A \tag{1.13}$$

$$\frac{\partial V^B}{\partial U^A}\frac{\partial U^A}{\partial q_1^A}\beta^B \leq p\lambda^B \tag{1.14}$$

$$\frac{\partial V^A}{\partial U^B}\frac{\partial U^B}{\partial q_1^B}\beta^A \leq p\lambda^A \tag{1.15}$$

$$\frac{\partial V^B}{\partial U^B}\frac{\partial U^B}{\partial q_1^B} \leq p\lambda^B \tag{1.16}$$

$$\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial Q_k} + \frac{\partial V^A}{\partial U^B}\frac{\partial U^B}{\partial Q_k}\beta^A \leq \lambda^A P_k \tag{1.17}$$

$$\frac{\partial V^B}{\partial U^A}\frac{\partial U^A}{\partial Q_k}\beta^B + \frac{\partial V^B}{\partial U^B}\frac{\partial U^B}{\partial Q_k} \leq \lambda^B P_k \tag{1.18}$$

where λ^A and λ^B are the Langrange multipliers of the respective budget constraints. An inequality becomes an equality for any private or public good which is positively consumed. Furthermore, the price of the numeraire is equal to 1*, i.e. $p = 1$.*

Proof. Follow directly by doing the first order conditions of the optimization problem of individual A and of individual B with respect to the numeraire and the public goods. □

1.6.1 Proof of Proposition 1.1.

Proof. In both cases, each individual maximizes a strictly concave utility function under linear constraints.
If the equations (1.13) and (1.15) hold with equality:

$$\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial q_1^A} = \frac{\partial V^A}{\partial U^B}\frac{\partial U^B}{\partial q_1^B}\beta^A = \lambda^A$$

Then, it is possible to rewrite the equation (1.17) in the following way:

$$\frac{\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial Q_k}}{\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial q_1^A}} + \frac{\frac{\partial V^A}{\partial U^B}\frac{\partial U^B}{\partial Q_k}\beta^A}{\frac{\partial V^A}{\partial U^B}\frac{\partial U^B}{\partial q_1^B}\beta^A} \leq \frac{\lambda^A}{\lambda^A}P_k \Rightarrow$$

$$(\tau_k^A(q^A, Q) + \tau_k^B(q^B, Q)) \leq P_k$$

Furthermore, we have that:

$$\frac{\partial V^B}{\partial U^A}\frac{\partial U^A}{\partial Q_k}\beta^B + \frac{\partial V^B}{\partial U^B}\frac{\partial U^B}{\partial Q_k} \leq \lambda^B(\tau_k^A(q^A, Q) + \tau_k^B(q^B, Q)) \leq \lambda^B P_k$$

As one of the two conditions (1.17) and (1.18) must hold with equality, we have that

$$\tau_k^A(q^A, Q) + \tau_k^B(q^B, Q) = P_k \tag{1.19}$$

as k was arbitrary, this holds for every public good.
When (1.14) and (1.16) hold with equality, I obtain analogous results to the previous case.
When (1.13) and (1.16) hold with equality:

$$\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial q_1^A} = \lambda^A$$

$$\frac{\partial V^B}{\partial U^B}\frac{\partial U^B}{\partial q_1^B} = \lambda^B$$

Then, we can rewrite the condition (1.17) as:

$$\lambda^A\frac{\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial Q_k}}{\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial q_1^A}} + \beta^A\lambda^B\frac{\frac{\partial V^A}{\partial U^B}\frac{\partial U^B}{\partial Q_k}}{\frac{\partial V^B}{\partial U^B}\frac{\partial U^B}{\partial q_1^B}} \leq \lambda^A P_k \Rightarrow$$

$$\lambda^A\tau_k^A(q^A, Q^A) + \beta^A\lambda^B\frac{\frac{\partial V^A}{\partial U^B}}{\frac{\partial V^B}{\partial U^B}}\tau_k^B(q^B, Q^B) \leq \lambda^A P_k \tag{1.20}$$

and the condition (1.18) as:

$$\beta^B\lambda^A\frac{\frac{\partial V^B}{\partial U^A}\frac{\partial U^A}{\partial Q_k}}{\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial q_1^A}} + \lambda^B\frac{\frac{\partial V^B}{\partial U^B}\frac{\partial U^B}{\partial Q_k}}{\frac{\partial V^B}{\partial U^B}\frac{\partial U^B}{\partial q_1^B}} \leq \lambda^B P_k \Rightarrow$$

$$\beta^B\lambda^A\frac{\frac{\partial V^B}{\partial U^A}}{\frac{\partial V^A}{\partial U^A}}\tau_k^A(q^A, Q^A) + \lambda^B\tau_k^B(q^B, Q^B) \leq \lambda^B P_k \tag{1.21}$$

I define $\beta^A = \frac{\lambda^A}{\lambda^B}\frac{\frac{\partial V^B}{\partial U^A}}{\frac{\partial V^A}{\partial U^A}}$ and $\beta^B = \frac{\lambda^B}{\lambda^A}\frac{\frac{\partial V^A}{\partial U^B}}{\frac{\partial V^B}{\partial U^B}}$. Substituting λ^A and λ^B, I obtain the following:

$$\beta^A = \frac{\frac{\partial V^B}{\partial U^A}}{\frac{\partial V^B}{\partial U^B}}\frac{\frac{\partial U^A}{\partial q_1^A}}{\frac{\partial U^B}{\partial q_1^B}}$$

and

$$\beta^B = \frac{\frac{\partial V^A}{\partial U^B}\frac{\partial U^B}{\partial q_1^B}}{\frac{\partial V^A}{\partial U^A}\frac{\partial U^A}{\partial q_1^A}}$$

Then, β^A represents the ratio of member B's marginal valuation for a unit increase of the numeraire quantity for member A (which enters the object function V^B through U^A) relative to his marginal valuation for the same increase of the numeraire quantity for his own (which enters V^B through U^B). Likewise, β^B represents the ratio of A's marginal valuation for a unit increase of the numeraire quantity for B relative to her marginal valuation for the same quantity increase for her own. These parameters capture the degree of caring at the equilibrium allocation.

Substituting the values of these parameters in the equations (1.20) and (1.21), I obtain the following:

$$\tau_k^A(q^A, Q^A) + \beta^A\beta^B\tau_k^B(q^B, Q^B) \leq P_k \tag{1.22}$$

and

$$\beta^A\beta^B\tau_k^A(q^A, Q^A) + \tau_k^B(q^B, Q^B) \leq P_k \tag{1.23}$$

Then, one of these two conditions must hold with equality and I obtain the result in proposition 1.

$$\max\{\tau_k^A(q^A, Q) + \beta^B\beta^A\tau_k^B(q^B, Q), \tau_k^B(q^B, Q) + \beta^A\beta^B\tau_k^A(q^A, Q)\} = P_k \tag{1.24}$$

□

I have shown that taking the first order condition for a household equilibrium with degree of caring $\beta \in [0, 1]$, it coincides with the Lindahl-Bowen-Samuelson Condition when $\beta^A = \beta^B = 1$ for any k and with a Nash Equilibrium of the game with voluntary contributions to public goods when $\beta^A = \beta^B = 0$ for any k.

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Chapter 2

Incentive Design for Motivated Agents in a Public Good Problem

I develop a multi-agent model where the individuals are the agents who are asked to contribute to the public good. The individuals may be interested not only in their own utility but also in the well-being of the others. Namely, individuals may or may not be intrinsically motivated. Whether they are motivated or not is their private information. I investigate the possibility of implementing a mechanism compatible with individual incentives that simultaneously results in efficient decisions, the voluntary participation of the individuals, and the feasibility of the budget. Under asymmetric information on the individuals' intrinsic motivation, the principal has to offer different transfers to different agents in order to attain separation of types. The transfers are made as a function of their degree of motivation. Both individuals will be taxed but motivated individuals pay lower taxes than unmotivated individuals. This is because the principal pays an information rent to motivated individuals in order to elicit higher levels of effort. Monetary incentives are necessary to compensate motivated individuals in order to increase the levels of contribution.

Keywords: Intrinsic Motivation, Caring Preferences, Public Good Problem, Incomplete Information.

2.1 Introduction

Intrinsic motivation alone may be not sufficient to supply the socially optimal level of contribution in a public good problem. Then, the intervention of a principal is needed. The intervention usually takes the form of monetary incentives such as payments or regulations. Payments may be used to subsidize contributions, whereas regulations to mandate a minimum contribution.

If each individual's preferences are publicly known, the principal can easily implement the efficient level of public-good provision in so improving the outcome under voluntary provision. If preferences are private information, the principal has to elicit the information needed to approach the optimum. Because this information has to come from the individuals who hold it, the question is how to give these individuals incentives to properly reveal this information.

I consider a principal-agent model in which individuals contribute to the public good by exerting costly effort. The principal-agent relationship can be interpreted as the relationship between the government that delegates the decision of contribution to a public good to the individuals (the agents). The principal can motivate individuals through a tax-subsidy system.

As said, in this model individuals provide a non-monetary contribution. In general, provision of public goods may be accomplished either monetarily or by exerting effort. For example, when the walls of a community center need to be painted, the citizenry either decides to work together to paint the walls themselves or collects money to hire someone to do the job. There are many other examples of non-monetary contributions. For instance, consider the household recycling activities in which the individuals contribute by exerting effort in order to improve environmental quality.

The key assumption of my model is that individuals may have caring preferences.[1] More specifically, they may care not only about their personal interest but also about the well-being of the others.[2] There are only two types of (male) agents: self-interested agents and motivated agents. In contrast, individuals have the same valuation of the public good.

The principal may not have information on the type of agent that contributes to the public good. I investigate the possibility of finding a mechanism which fulfills some desirable requirements. Specifically, the implementation of the efficient decisions compatible with the incentives and the voluntary participation of the individuals that satisfy the feasibility of the budget (taxes plus subsidies less or equal than zero).[3]

With perfect information, the principal does not have to offer an incentive to the individuals because it has all the necessary information to implement the efficient levels of contribution. More specifically, the principal offers a transfer which just covers the cost of effort incurred by the agent minus his direct benefit deriving from directly enjoying the public good and the indirect benefit tied to his intrinsic motivation. In this case, both individuals will be taxed. This is because both individuals obtain higher marginal benefits than marginal costs.

[1] These preferences are able to explain altruism, empathy and responsibility towards the others.

[2] Most economic models have shown that individuals can be affected by fairness, equity and altruistic considerations (see for example Fehr and Schmidt, 1999). For instance, lots of people invest time and money in improving the environment and supporting activities that are costly to themselves and mostly benefit others.

[3] This chapter is related to the works by Arrow (1979) and d'Aspremont and Gèrard-Varet (1979). The mechanism proposed by Arrow (1979) and d'Aspremont and Gèrard-Varet (1979) satisfies the budget balancing property but need not be individually rational.

In addition, motivated individuals pay higher taxes than selfish individuals. The economic intuition behind this result is the following: motivated individuals provide a given level of effort for a higher taxation. The principal extracts all the surplus of the public good through taxation.

With asymmetric information, a motivated agent benefits from pretending to be a low type. The principal gives up information rents in order to induce separation of types. The transfers are made as a function of their degree of motivation. In this case, the taxes paid by motivated individuals are lower than in the first-best. Monetary incentives are necessary to compensate the motivated individuals in order to elicit higher levels of effort and to increase the levels of contribution. In contrast, unmotivated individuals pay the same amount of taxes under first-best and second-best. Furthermore, motivated individuals pay lower taxes than unmotivated individuals. This result stands in contrast to the previous literature in which motivation is effective in stimulating work effort even in the absence of monetary incentives (see for example Gneezy and Rustichini, 2000a,b and Benabou and Tirole, 2003, 2006).[4]

I show that under asymmetric information with respect to the individuals' intrinsic motivation the total amount of taxes paid by the individuals is reduced. This is because the principal pays a rent to motivated individuals in order to increase the levels of contribution. This result is obtained at the expenses of an increase in the decision inefficiency: the total level of contribution is lower than the first-best total level of contribution. Motivated individuals contribute more than in the first-best, while the unmotivated individuals contribute less.

This article is related to the literature on contribution games for Peer-to-Peer Networks (see for example Ballester, Calvó-Armengol and Zenou, 2006, and Corbo, Calvó-Armengol and Parkes, 2006). This literature investigates how individuals provide public goods when there are externalities from other individuals' play. I share with this literature the complementarities among the levels of effort exerted by the individuals. However, they do not consider differences in the individuals' preferences.

The plan of the chapter is as follows: in section 2.2 I present the general framework of the model; section 2.3 is devoted to the analysis of the results under asymmetric information; and concluding remarks are offered in section 2.4.

[4]This literature shows that monetary incentives can influence negatively the individuals' behavior in terms of their levels of contribution. The reason is that monetary incentives give the agent a selfish motive to operate. Explicit incentives from principals may change how tasks are perceived by agents (Gneezy and Rustichini, 2000a) and they may also reduce the value of generous or civic minded acts as a signal of one's moral character (Benabou and Tirole, 2003, 2006).

2.2 The model

2.2.1 The Set-Up of the Model

I develop a principal-agent model between a principal and two agents, A and B. The principal-agent relationship can be interpreted as the relationship between the government who delegates the decision of contribution to a public good to the individuals (the agents).
The principal's utility function is given by:

$$\Omega = -(t_A + t_B), \tag{2.1}$$

where t_A and t_B are the transfers given respectively to agent A and to agent B. These transfers can be positive or negative depending on the individuals' levels of contribution. The transfers are feasible if $t_A + t_B \leq 0$.[5]
In order to provide the public good, the agents have to exert effort and incur in a quadratic cost $\frac{1}{2}g_J^2$ with $J = A, B$. The individuals care not only about their own contribution but also about the contribution provided by the others. This is because the levels of contribution of each individual can affect the marginal benefit of others' individual contributions. The measure of this benefit depends on the parameter θ_J that represents their valuation of the public good and it is strictly positive.
In addition, the agents' utility might positively depend on the benefits that the other individual obtains. The measure of this utility depends on the parameter α that represents the intrinsic motivation of the agent. There are only two types of agents: the self-interested agents with $\alpha = 0$ and the motivated agents with $\alpha > 0$. The participation of the individuals is voluntary.
The utility of the individual A is given by the following:

$$V_A = t_A + \theta_A f(g_A, g_B) - \frac{1}{2}g_A^2 + \alpha_A \left[\theta_B f(g_A, g_B) - \frac{1}{2}g_B^2\right] \tag{2.2}$$

and the utility of the individual B is represented by:

$$V_B = t_B + \theta_B f(g_A, g_B) - \frac{1}{2}g_B^2 + \alpha_B \left[\theta_A f(g_A, g_B) - \frac{1}{2}g_A^2\right] \tag{2.3}$$

where $f(g_A, g_B)$ is an increasing, concave and twice continuously differentiable production function with $f(0,0) = 0$. To obtain an explicit solution, the function will take the following form: $f(g_A, g_B) = \sqrt{g_A g_B}$. This function considers the complementarity among the levels of effort exerted by the individuals (as shown in Ballester, Calvó-Armengol and Zenou, 2006, and Corbo, Calvó-Armengol and Parkes, 2006).
The utility of each agent consists of a part depending on his own "egoistic" utility and a part

[5] In this case, the principal obtains a benefit by the provision of the public good.

depending on the utility of the other individual. In this construction, $\alpha_A = \alpha_B = 0$ stands for *egoistic preferences* in which each agent cares only about his own benefit from the contribution to the public good. In contrast, $\alpha > 0$ stands for *caring preferences* in which each agent cares also about the benefit obtained by the other agent. If $\alpha_A = \alpha_B = 1$ the two individuals care a lot about each other and their preferences coincide, i.e. $V_A = V_B$.
In the following sections, I analyze the case in which the principal has perfect information on the agents' intrinsic motivation and the case in which it has not (α_J with $J = A, B$ can be not observable and not contractible by the principal).

The timing of the game is as follows. *At the initial stage* 0, agents are informed about their type; *at stage* 1, the principal offers a menu of contracts consisting of the individuals' levels of contribution and the individuals' transfers; *at stage* 2, the agents decide whether to participate or not. If they decide to participate, they choose their contributions simultaneously, which are then observed by the principal; finally, *at stage* 3, transfers are collected and the public good is provided.

2.2.2 The Benchmark Case

I study the optimal mechanism when the government intervenes. It adopts incentives in presence of a public goods problem, in which the level of contribution of each individual can affect the marginal benefit of other individuals' contributions.
With perfect information, the principal need not offer an incentive to the individuals because it has all the necessary information to implement the efficient levels of contribution. The transfer will just cover the cost of effort incurred by the agent minus his direct benefit deriving from directly enjoying the public good and the indirect benefit tied to his intrinsic motivation:

$$t_A = \frac{1}{2}g_A^2 - \theta_A\sqrt{g_A g_B} - \alpha_A\left[\theta_B\sqrt{g_A g_B} - \frac{1}{2}g_B^2\right] \tag{2.4}$$

and

$$t_B = \frac{1}{2}g_B^2 - \theta_B\sqrt{g_A g_B} - \alpha_B\left[\theta_A\sqrt{g_A g_B} - \frac{1}{2}g_A^2\right] \tag{2.5}$$

The transfer makes each agent indifferent between accepting and rejecting the contract, given the required effort to contribute.[6] Regarding the transfers, it is possible to note that a high θ_J with $J = A, B$ reduces them. This means that agents with a high valuation of the public good receive lower incentive pay at the optimum. Take, for instance, $\theta_B > \theta_A$. In this case, the principal will offer a bigger transfer to individual A who is less interested to the public good than individual B. This is because the more involved individual contributes to

[6]The principal offers a transfer to each agent which makes them at least as well off as their outside option. The outside option of the agent in the absence of contribution is normalized to zero.

the provision even if he receive a low compensation. The economic intuition behind this result is the following: *if agents care more about the public good, there will be less need to use transfers and to distort the outcome away from efficiency.* In other words, the principal will offer a higher transfer to the agent with a low valuation and a lower transfer to the agent with a high valuation. Analogously, if the egoistic utilities are positive, the principal will offer a bigger transfer to the less motivated individual.

Under complete information the individuals' efficient levels of contribution are obtained by equating the government's marginal value to the sum of individuals' marginal cost. This result will coincide with the one obtained when the government is a "benevolent principal" who maximizes the social welfare. So the government maximizes the following:

$$\Omega = [(1+\alpha_A)\theta_B + (1+\alpha_B)\theta_A]\sqrt{g_A g_B} - \frac{1}{2}\left[(1+\alpha_B)g_A^2 + (1+\alpha_A)g_B^2\right] \tag{2.6}$$

Henceforth, I focus on the case in which the individuals are homogeneous in their valuation of the public good, i.e. $\theta_A = \theta_B = \theta$. Applying the first order condition to equation (2.6) with respect to g_A and g_B, the first-best levels of contribution are represented by the following two equations:

$$g_A^* = \frac{\theta[(1+\alpha_B)^{9/4}(1+\alpha_A)^{3/4}](2+\alpha_A+\alpha_B)}{2(1+\alpha_B)^3(1+\alpha_A)} \tag{2.7}$$

and

$$g_B^* = \frac{\theta[(1+\alpha_A)^{9/4}(1+\alpha_B)^{3/4}](2+\alpha_A+\alpha_B)}{2(1+\alpha_A)^3(1+\alpha_B)} \tag{2.8}$$

The left-hand side is the private marginal cost of contributing and the right-hand side is the marginal benefit of contributing to the public good. This marginal benefit consists of the marginal benefit due to the individuals' valuation of public good and to their intrinsic motivation. As usual, the first-best induces the agents to exert the level of contribution that maximizes the the social welfare. Hence, the resulting contribution levels are socially optimal.[7]

Proposition 2.1. *If $\alpha_A = \alpha_B = \alpha$ $g_A^* = g_B^* = \theta$. In contrast, if $\alpha_A \neq \alpha_B$ and $\alpha_A > \alpha_B$ $g_A^* > g_B^*$.*

Proof. By substituting in equations (2.7) and (2.8) α_A and α_B with α, I obtain the following:

$$g^* = \frac{\theta[(1+\alpha)^{\frac{9}{4}}(1+\alpha)^{\frac{3}{4}}][(2+2\alpha)]}{2(1+\alpha)^3(1+\alpha)} = \frac{\theta[(1+\alpha)^3][2(1+\alpha)]}{2(1+\alpha)^3(1+\alpha)} = \theta$$

$$\text{While } g_A^* > g_B^* \text{ if } \frac{\theta[(1+\alpha_B)^3(1+\alpha_A)]^{\frac{3}{4}}}{2(1+\alpha_B)^3(1+\alpha_A)} > \frac{\theta[(1+\alpha_A)^3(1+\alpha_B)]^{\frac{3}{4}}}{2(1+\alpha_A)^3(1+\alpha_B)}$$

That is,

$$[(1+\alpha_B)^{9/4}(1+\alpha_A)^{3/4}](1+\alpha_A)^3(1+\alpha_B) > [(1+\alpha_A)^{9/4}(1+\alpha_B)^{3/4}](1+\alpha_B)^3(1+\alpha_A)$$

After some computations, it is easy to see that the inequality holds if $\alpha_A > \alpha_B$. □

[7] Since the participation constraints bind regardless of the agents' type, the principal extracts all the surplus above the agents' reservation utility, and therefore it has the incentive to maximize it.

If both individuals are also homogeneous in their degree of motivation, i.e. $\alpha_A = \alpha_B = \alpha$, the optimal levels of contributions correspond to their valuation of the public good, i.e. $g_A^* = g_B^* = \theta$.
In contrast, when the degree of intrinsic motivation is different among individuals, i.e. $\alpha_A \neq \alpha_B$, the individual with the highest α increases his contribution. In contrast, the individual with lowest α reduces his contribution.[8] This is because an increase of the individual degree of motivation has a positive impact on his own level of contribution.[9] While it has a negative effect on the level of contribution of the other individual.[10] Obviously, the effect of their valuation of the public good on the levels of contribution is positive for both individuals, i.e. $\frac{\partial g_A}{\partial \theta} > 0$ and $\frac{\partial g_B}{\partial \theta} > 0$. Individuals that value more the public good are willing to exert higher levels of effort.

Substituting the levels of contribution provided into equations (2.4) and (2.5), I obtain the optimal levels of transfers:

$$t_A^* = (2+\alpha_A+\alpha_B)\theta^2\left[\frac{(2+\alpha_A+\alpha_B)}{8\sqrt{1+\alpha_A}(1+\alpha_B)^{3/2}} + \frac{\alpha_A(2+\alpha_A+\alpha_B)}{8\sqrt{1+\alpha_B}(1+\alpha_A)^{3/2}} - \frac{1}{2}\frac{(1+\alpha_A)}{\sqrt{(1+\alpha_A)(1+\alpha_B)}}\right] \tag{2.9}$$

$$t_B^* = (2+\alpha_A+\alpha_B)\theta^2\left[\frac{(2+\alpha_A+\alpha_B)}{8\sqrt{1+\alpha_B}(1+\alpha_A)^{3/2}} + \frac{\alpha_B(2+\alpha_A+\alpha_B)}{8\sqrt{1+\alpha_A}(1+\alpha_B)^{3/2}} - \frac{1}{2}\frac{(1+\alpha_B)}{\sqrt{(1+\alpha_A)(1+\alpha_B)}}\right] \tag{2.10}$$

Proposition 2.2. *$t_A^*, t_B^* < 0$ for any values of $\theta > 0$ and $\alpha \in [0,1]$. If $\alpha_A > \alpha_B$ $t_A^* < t_B^* < 0$. In words, individual A will be taxed more.*

Proof. I want to show that both individuals are always taxed, i.e. $t_A^*, t_B^* < 0$ for any values of $\theta > 0$ and $\alpha \in [0,1]$. The proof unfolds in two steps. First, I study the marginal effect of the degree of motivation α on the optimal levels of transfers. Secondly, I show that the above inequality holds also in a limit case and this completes the proof.
Deriving equations (2.9) and (2.10) with respect to α_A, α_B, I obtain that the effect of these

[8] If I consider the extreme case in which the individual A is highly motivated, i.e. $\alpha_A = 1$, and the individual B is selfish, i.e. $\alpha_B = 0$, the optimal levels of contribution are the following: $g_A^* = 1.26\theta$ and $g_B^* = 0.89\theta$.

[9] The impact of the individual A's intrinsic motivation on his level of contribution is equal to:

$$\frac{\partial g_A}{\partial \alpha_A} = \frac{\theta[(1+\alpha_A)(1+\alpha_B)^3]^{\frac{3}{4}}}{2(1+\alpha_A)(1+\alpha_B)^3} + \frac{3\theta(2+\alpha_A+\alpha_B)}{8(1+\alpha_A)[(1+\alpha_A)(1+\alpha_B)^3]^{\frac{1}{4}}} - \frac{\theta[(1+\alpha_A)(1+\alpha_B)^3]^{\frac{3}{4}}(2+\alpha_A+\alpha_B)}{2(1+\alpha_A)^2(1+\alpha_B)^3} > 0$$

if $\alpha_A > \frac{1}{3}(\alpha_B - 2)$. This inequality always holds true. The objective functions of the individuals are symmetric. Then, I also have that $\frac{\partial g_B}{\partial \alpha_B} > 0$.

[10] The impact of the individual B's intrinsic motivation on the level of contribution of the other is:

$$\frac{\partial g_A}{\partial \alpha_B} = \frac{\theta[(1+\alpha_A)(1+\alpha_B)^3]^{\frac{3}{4}}}{2(1+\alpha_A)(1+\alpha_B)^3} + \frac{9\theta(2+\alpha_A+\alpha_B)}{8(1+\alpha_B)[(1+\alpha_A)(1+\alpha_B)^3]^{\frac{1}{4}}} - \frac{3\theta[(1+\alpha_A)(1+\alpha_B)^3]^{\frac{3}{4}}(2+\alpha_A+\alpha_B)}{2(1+\alpha_A)(1+\alpha_B)^4} < 0$$

if $\alpha_B < 2 + 3\alpha_A$. This inequality always holds true. Then, I also have $\frac{\partial g_B}{\partial \alpha_A} < 0$.

variables on the transfers is negative, i.e. $\frac{\partial t_J}{\partial \alpha_A} < 0$ and $\frac{\partial t_J}{\partial \alpha_B} < 0$ with $J = A, B$. Moreover, I obtain the same result regarding the impact of the valuation θ, i.e. $\frac{\partial t_J}{\partial \theta} < 0$. If agents are motivated and/or have a high valuation of the public good, there will be less need to use transfers and to distort the outcome away from efficiency.
Then, I consider a limit case in which both individuals are selfish, i.e. $\alpha_A = \alpha_B = 0$. Since the impact of motivation on the transfers is negative, if the individuals will be taxed in this limit case, they will be always taxed for higher values of these parameters. If both individuals are selfish, the transfers are equal to $t_A = t_B = -\frac{1}{2}\theta^2$.[11] □

This result is due to the fact that the benefit of contributing to the public good are higher than the cost of contributing. Then, individuals are always taxed and more motivated individuals pay higher taxes. There is *no budget-balance*: $t_A + t_B \neq 0$. However, the transfers are feasible.

2.3 Implementation of the second best

In the second best, the individuals' caring parameters are their private information. For simplicity, I assume that α_J with $J = A, B$ can take only two values: $\underline{\alpha}_J = 0$ and $\overline{\alpha}_J > 0$. In contrast, the valuation of the public good is common knowledge with $\theta_A = \theta_B = \theta > 0$.
The principal will offer different contracts designed for different types. The offered transfer will depend on the contribution of the agent himself, i.e. $t_J(g_J)$. Under asymmetric information, the motivated agent has an advantage to pretend to be low type. To induce separation of types, the principal will pay an information rent to the motivated agent. The magnitude of these rents will be crucially affected by the parameter of asymmetric information α.
I will compare the levels of contribution of the agents and the rents offered to them with the benchmark case in order to see how the asymmetric information on the caring parameters and the heterogeneity of α among individuals have an impact on the individuals' choice in terms of contribution. In addition, I will investigate when the principal would make more or less use of monetary incentives under asymmetric information on the degree of caring.
The asymmetric information with respect to the individuals' intrinsic motivation reduces the total amount of taxes paid by the individuals with respect to the first-best. This result is obtained at the expenses of an increase in the decision inefficiency: the total level of contribution is lower than the first-best total level of contribution.

[11] If both individuals are highly motivated, i.e. $\alpha_A = \alpha_B = 1$, the transfers are equal to $t_A = t_B = -2\theta^2$. This is also true when individuals are heterogenous in their motivation. Moreover, more motivated individuals pay higher taxes. If only individual A is highly intrinsically motivated, while the individual B is self-interested, the optimal transfers will be equal to: $t_A = -1.72\theta^2$ and $t_B = -0.659\theta^2$.

2.3.1 Asymmetric Information on the Degree of Caring

In this subsection, I present the case in which the individuals' caring parameter is their private information. There are only two types of individuals: self-interested agents $\underline{\alpha}_J = 0$ and motivated agents $\overline{\alpha}_J > 0$.[12] In contrast, the valuation of the public good is common knowledge and strictly positive for both individuals, i.e. $\theta_A = \theta_B > 0$.
The agent A does not know if the agent B is low-type or high-type. The conditional probability distribution for the individual A is as follows:

$$\mu = Pr(\alpha_B = \overline{\alpha}_B | \alpha_A = \overline{\alpha}_A) = Pr(\alpha_B = \underline{\alpha}_B | \alpha_A = \underline{\alpha}_A);$$

$$(1-\mu) = Pr(\alpha_B = \overline{\alpha}_B | \alpha_A = \underline{\alpha}_A) = Pr(\alpha_B = \underline{\alpha}_B | \alpha_A = \overline{\alpha}_A).$$

Then, the conditional probability of having individuals of the same type is equal to μ, while that of having individuals of different type is $1-\mu$.[13] I assume $\mu \geq \frac{1}{2}$. This means that the probability of having individuals of the same type is higher or equal than the probability of having individuals of different type.
The principal offers different contract schemes to the agents depending on their levels of contribution and maximizes the following:

$$\Omega = \frac{\mu}{2}\left[-\underline{t}_A(\underline{g}_A) - \underline{t}_B(\underline{g}_B) - \overline{t}_A(\overline{g}_A) - \overline{t}_B(\overline{g}_B)\right] + \frac{(1-\mu)}{2}\left[-\overline{t}_A(\overline{g}_A) - \underline{t}_B(\underline{g}_B) - \underline{t}_A(\underline{g}_A) - \overline{t}_B(\overline{g}_B)\right] \tag{2.11}$$

subject to incentives and non negative participation constraints. The incentive constraints require that is a Bayesian equilibrium for each agent to report her type truthfully. While participation constraints require that each type of each agent have to be at least as well off by participating as they would be by not participating in the contribution of the public good. Given that the mechanism is voluntary, the individual choice must not only induce truthful revelation of information but must also satisfy the participation constraints. I assume that the outside options are exogenously given and normalized to zero.
I can simplify the number of relevant constraints. As usual, I obtain that the incentive constraint for the $\overline{\alpha}$-agent is binding (because the difficulty comes from a $\overline{\alpha}$-agent willing to claim that he is inefficient rather than the reverse). Under complete information the motivated individual pays higher taxes. Under asymmetric information, this individual can pretend to be low type. The principal has to offer him an incentive to reveal his type. In contrast, the participation constraint for the $\underline{\alpha}$-agent is the binding one (because if a menu

[12] I made this assumption only to simplify the computation but my results do not change qualitatively if both parameters are between 0 and 1.

[13] In probability theory, the conditional probability of $\overline{\alpha}_A$ given $\overline{\alpha}_B$ is the probability of $\overline{\alpha}_A$ if $\overline{\alpha}_B$ is known to occur (or have occurred). Mathematically, it is defined for $Pr(\overline{\alpha}_B) \neq 0$ as $Pr(\overline{\alpha}_A | \overline{\alpha}_B) = Pr(\overline{\alpha}_A \cap \overline{\alpha}_B)/Pr(\overline{\alpha}_B)$. In my case, $\mu = Pr(\overline{\alpha}_A \cap \overline{\alpha}_B)/Pr(\overline{\alpha}_B)$ where the $Pr(\overline{\alpha}_B) = \frac{1}{2}$ (the probability to extract between the low and the high type). Then, $Pr(\overline{\alpha}_A \cap \overline{\alpha}_B) = \frac{\mu}{2}$. Given that it is simple to compute the other probabilities.

of contracts enables a unmotivated agent to reach his status quo utility level, it will be also the case for a motivated one). All the mathematical computations and the proof of the main results are showed in the Appendix A.
Using the participation constraints and the incentive constraints, the optimal transfers will have to satisfy the following equations if the individuals are low-type:

$$\begin{aligned} \underline{t}_A &= \frac{1}{2}\underline{g}_A^2 - \theta[\mu\sqrt{\underline{g}_A\underline{g}_B} + (1-\mu)\sqrt{\underline{g}_A\overline{g}_B}] \\ \underline{t}_B &= \frac{1}{2}\underline{g}_B^2 - \theta[\mu\sqrt{\underline{g}_A\underline{g}_B} + (1-\mu)\sqrt{\underline{g}_B\overline{g}_A}] \end{aligned} \tag{2.12}$$

and the following equations if the individuals are high-type:

$$\begin{aligned} \overline{t}_A = \frac{1}{2}\overline{g}_A^2 - (1+\overline{\alpha}_A)\theta[\mu\sqrt{\overline{g}_A\overline{g}_B} + (1-\mu)\sqrt{\overline{g}_A\underline{g}_B}] + \\ \underbrace{+(1+\overline{\alpha}_A)\theta[\mu\sqrt{\underline{g}_A\overline{g}_B} + (1-\mu)\sqrt{\underline{g}_A\underline{g}_B}] - \theta[\mu\sqrt{\underline{g}_A\underline{g}_B} + (1-\mu)\sqrt{\underline{g}_A\overline{g}_B}]}_{=R\text{: Information Rent}} \end{aligned} \tag{2.13}$$

$$\begin{aligned} \overline{t}_B = \frac{1}{2}\overline{g}_B^2 - (1+\overline{\alpha}_B)\theta[\mu\sqrt{\overline{g}_A\overline{g}_B} + (1-\mu)\sqrt{\underline{g}_A\overline{g}_B}] + \\ \underbrace{+(1+\overline{\alpha}_B)\theta[\mu\sqrt{\overline{g}_A\underline{g}_B} + (1-\mu)\sqrt{\underline{g}_A\underline{g}_B}] - \theta[\mu\sqrt{\underline{g}_A\underline{g}_B} + (1-\mu)\sqrt{\overline{g}_A\underline{g}_B}]}_{=R\text{: Information Rent}} \end{aligned}$$

The transfer offered to the $\underline{\alpha}$-agent has to cover his cost of exerting effort and he does not earn any information rent. While the $\overline{\alpha}$-agent obtain an information rent R. This rent is obtained through the amount contributed by the individuals.
Substituting the transfers in the principal's utility and doing the first order condition, I obtain the following system of equations:

$$\begin{aligned} \overline{g}_J &= (1+\overline{\alpha})\theta\mu + \theta\frac{\underline{g}_J}{\sqrt{\underline{g}_J\overline{g}_J}}\left[(1-\mu) + \frac{(1+\overline{\alpha})(1-2\mu)}{2}\right] \\ \underline{g}_J &= 2\theta\mu - (1+\overline{\alpha})\theta(1-\mu) + \theta\frac{\overline{g}_J}{\sqrt{\underline{g}_J\overline{g}_J}}\left[(1-\mu) + \frac{(1+\overline{\alpha})(1-2\mu)}{2}\right] \end{aligned} \tag{2.14}$$

with $J = A, B$.

Solving the system of equations simultaneously and setting $\mu = \frac{1}{2}$[14], I obtain the following result:

Proposition 2.3. *Let $(\overline{g}_J^{*SB}, \underline{g}_J^{*SB})$ with $J = A, B$ be an equilibrium where individuals value positively the public good, i.e. $\theta > 0$, and there is asymmetric information on the degree of caring:*

- $\overline{g}_J^{*SB} > \overline{g}_J^{*}$ *and* $\underline{g}_J^{*SB} < \underline{g}_J^{*}$ *for any values of* $\overline{\alpha} > 0$.

Proof. It is shown in the Appendix B. □

The motivated individual contributes more than under complete information, while the unmotivated individual contributes less. This result is shown in figure 2.1. In the first graph, there is the comparison between the levels of contribution of the motivated individual under complete and incomplete information. The blue line represents his levels of contribution in the first-best, while the purple line the levels of contribution in the second-best. The blue line is always below the purple line. In the second graph, I compare the levels of contribution of the unmotivated individual under complete and incomplete information. The blue line represents his levels of contribution under first-best, while the purple his levels of contribution under second-best. The blue line is always above the purple line. Furthermore, if the

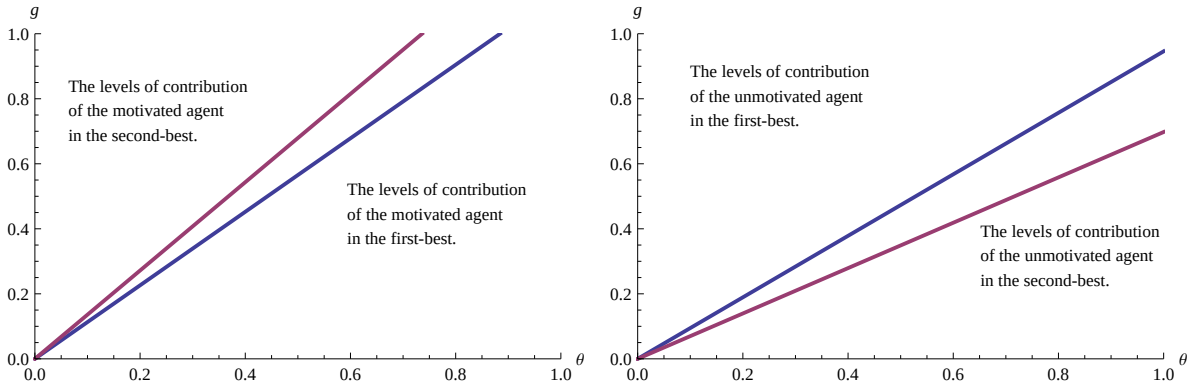

Figure 2.1: Comparison on the levels of contribution when $\overline{\alpha} = 1$ and $\mu = \frac{1}{2}$.

individuals are heterogeneous (an individual is motivated, while the other is selfish) the total level of contribution obtained under asymmetric information is lower than the total level of

[14] I consider $\mu = \frac{1}{2}$ to simplify the computations. However, the result in the proposition 2.3 still holds for higher values of μ.

contribution obtained under complete information. There is in other words *inefficiency on the decision choice.*[15]

Substituting the levels of contribution into equations (2.12) and (2.13), I obtain the levels of transfer in the second -best and I obtain the following result:

Proposition 2.4. $\underline{t}_J^{*SB} < \overline{t}_J^{*SB} < 0$. *Furthermore,* $\underline{t}_J^{*} \cong \underline{t}_J^{*SB} < 0$ *and* $\overline{t}_J^{*} < \overline{t}_J^{*SB} < 0$ *for any values of* θ *and* $\overline{\alpha} > 0$.

Proof. It is shown in the Appendix B. □

Under asymmetric information, the motivated agent pays lower taxes than the unmotivated agent. This depends on the fact that a higher degree of caring has two opposite effects on the transfer given to the motivated agent. The first effect is positive and it represents the effect of the individual degree of caring on the marginal cost of exerting effort. A high α has a positive impact on the information rent given to the motivated agent. The principal has to offer a positive transfer in order to cover the cost of effort and to motivate the involved individual by contributing efficient quantities. The second effect is negative and it represents the effect of the caring parameter on the monetary incentive. Individuals with a high degree of caring will be willing to exert a higher level of effort even if they receive a lower compensation or have to pay higher taxes for that. This is because when the individual has a high caring parameter, he could provide spontaneously high levels of effort and the principal does not need to give them a high rent. However, the first effect dominates the second one and the motivated individual pays lower taxes in the second-best than in the first-best.[16] In contrast, the unmotivated individual pays the same amount of taxes under first and second-best.
Hence, *the total amount of taxes paid by the individuals is lower than in the benchmark case.*

[15] If there is equal probability that the individuals are of the same type, i.e. $\mu = \frac{1}{2}$, I obtain the following levels of contribution: $\underline{g}_A^{*SB} = \underline{g}_B^{*SB} = 0.7\theta$ and $\overline{g}_A^{*SB} = \overline{g}_B^{*SB} = 1.36\theta$. It is possible to note that the highly motivated individual contributes more than under complete information, while the unmotivated individual contributes less. If the individual A is motivated and the individual B is selfish, the total level of contribution will be equal to $\overline{g}_A^{*SB} + \underline{g}_B^{*SB} = 1.36\theta + 0.7\theta = 2.06\theta$. Under complete information, the total level of contribution was equal to $g_A^* + g_B^* = 1.26\theta + 0.89\theta = 2.15\theta$.

[16] This result stands in contrast to the previous literature in which motivation is effecting in stimulating work effort even in absence of monetary incentives.

2.4 Conclusions

I have developed a principal-agent model in which individuals contribute to the public good by exerting costly effort. In addition, the individuals are interested not only in their monetary compensation but also in the impact of their action on the well-being of the others. There are only two types of agent: self-interested agents and motivated agents. The degree of individuals' motivation could be their private information.

The main objective of this chapter was to investigate the possibility of finding a mechanism which fulfills some desirable requirements. Specifically, the implementation of the efficient decisions compatible with the incentives and the voluntary participation of the individuals that satisfy the feasibility of the budget.

With complete information, the principal offers a transfer which just covers the cost of effort incurred by the (male) agent minus his direct benefit deriving from directly enjoying the public good and the indirect benefit tied to his intrinsic motivation. In this case, both individuals are taxed because they obtain higher benefits than costs by contributing to the public good. Furthermore, motivated individuals pay higher taxes than the unmotivated individuals. This is because motivated individuals exert a given level of effort even if they pay higher taxes for that.

With incomplete information, the principal pays a rent to motivated individuals in order to attain separation of types. In this case, I obtained the opposite result than in the first-best. Motivated individuals pay lower taxes than selfish individuals. This is because the principal has to reduce the taxes paid by motivated individuals in order to compensate them to elicit higher levels of effort. Moreover, motivated individuals pay lower taxes than in the first-best, while selfish individuals pay the same amount. Then, the total amount of taxes paid by the individuals was reduced. This result was obtained at the expenses of an increase in the decision inefficiency.

For future research, it might be also interesting to analyze the case in which the individuals are heterogeneous with respect to their valuation of the public good. Moreover, in my model I have only two types of individuals but it is interesting to consider a continuum of individuals.

This simple mechanism leads to a new insight concerning the individuals' contributions to public goods, the interaction between monetary incentives and intrinsic motivation, but it could be extended also in private organizations. In organizations, motivated agents would improve the performance of the firm and then they would obtain a higher payment than the unmotivated agents.

2.5 Appendix A

2.5.1 Which constraints are binding?

I show that, as usual, the incentive constraint for the $\overline{\alpha}$-agent is binding. While the participation constraint for the $\underline{\alpha}$-agent is the binding one.

Proof. I have the following incentive and participation constraints for the individual A:

$$\begin{aligned}(IC_H): \overline{t}_A - \frac{1}{2}\overline{g}_A^2 + (1+\overline{\alpha}_A)\theta\left[\mu(\overline{g}_A\overline{g}_B)^{\frac{1}{2}} + (1-\mu)(\overline{g}_A\underline{g}_B)^{\frac{1}{2}}\right] \geq \\ \underline{t}_A - \frac{1}{2}\underline{g}_A^2 + (1+\overline{\alpha}_A)\theta\left[\mu(\underline{g}_A\overline{g}_B)^{\frac{1}{2}} + (1-\mu)(\underline{g}_A\underline{g}_B)^{\frac{1}{2}}\right]\end{aligned} \tag{2.15}$$

$$\begin{aligned}(IC_L): \underline{t}_A - \frac{1}{2}\underline{g}_A^2 + \theta\left[\mu(\underline{g}_A\underline{g}_B)^{\frac{1}{2}} + (1-\mu)(\underline{g}_A\overline{g}_B)^{\frac{1}{2}}\right] \geq \\ \overline{t}_A - \frac{1}{2}\overline{g}_A^2 + \theta\left[\mu(\overline{g}_A\underline{g}_B)^{\frac{1}{2}} + (1-\mu)(\overline{g}_A\overline{g}_B)^{\frac{1}{2}}\right]\end{aligned} \tag{2.16}$$

$$(PC_H): \overline{t}_A - \frac{1}{2}\overline{g}_A^2 + (1+\overline{\alpha}_A)\theta\left[\mu(\overline{g}_A\overline{g}_B)^{\frac{1}{2}} + (1-\mu)(\overline{g}_A\underline{g}_B)^{\frac{1}{2}}\right] \geq 0 \tag{2.17}$$

$$(PC_L): \underline{t}_A - \frac{1}{2}\underline{g}_A^2 + \theta\left[\mu(\underline{g}_A\underline{g}_B)^{\frac{1}{2}} + (1-\mu)(\underline{g}_A\overline{g}_B)^{\frac{1}{2}}\right] \geq 0 \tag{2.18}$$

The incentive and participation constraints for the individual B are symmetric.
Let's determine which constraints bind.
First, note that if the equations (2.15) and (2.18) are satisfied, then

$$\begin{aligned}\overline{t}_A - \frac{1}{2}\overline{g}_A^2 + (1+\overline{\alpha}_A)\theta\left[\mu(\overline{g}_A\overline{g}_B)^{\frac{1}{2}} + (1-\mu)(\overline{g}_A\underline{g}_B)^{\frac{1}{2}}\right] \geq \\ (1+\overline{\alpha}_A)\theta\left[\mu(\underline{g}_A\overline{g}_B)^{\frac{1}{2}} + (1-\mu)(\underline{g}_A\underline{g}_B)^{\frac{1}{2}}\right] - \theta\left[\mu(\underline{g}_A\underline{g}_B)^{\frac{1}{2}} + (1-\mu)(\underline{g}_A\overline{g}_B)^{\frac{1}{2}}\right] \geq 0\end{aligned} \tag{2.19}$$

The equation (2.19) reflects the fact that the type $\overline{\alpha}$ receives more surplus from the contribution than type $\underline{\alpha}$. Then, the participation constraint for the individual with high-valuation:

$$\overline{t}_A - \frac{1}{2}\overline{g}_A^2 + (1+\overline{\alpha}_A)\theta\left[\mu(\overline{g}_A\overline{g}_B)^{\frac{1}{2}} + (1-\mu)(\overline{g}_A\underline{g}_B)^{\frac{1}{2}}\right] \geq 0$$

is satisfied as well. Furthermore, it will not be binding unless $\underline{g} = 0$. In contrast, the participation constraint for the low-type must be binding.
Next, the incentive compatibility constraint for the high-type (equation 2.15) must be binding, that is:

$$\begin{aligned}\overline{t}_A = \frac{1}{2}\overline{g}_A^2 - (1+\overline{\alpha}_A)\theta\left[\mu(\overline{g}_A\overline{g}_B)^{\frac{1}{2}} + (1-\mu)(\overline{g}_A\underline{g}_B)^{\frac{1}{2}}\right] + \\ +(1+\overline{\alpha}_A)\theta\left[\mu(\underline{g}_A\overline{g}_B)^{\frac{1}{2}} + (1-\mu)(\underline{g}_A\underline{g}_B)^{\frac{1}{2}}\right] - \theta\left[\mu(\underline{g}_A\underline{g}_B)^{\frac{1}{2}} + (1-\mu)(\underline{g}_A\overline{g}_B)^{\frac{1}{2}}\right]\end{aligned}$$

If this incentive were not binding, the principal could increase $\overline{t}_A$ slightly and keep all constraints satisfied. And the incentive constraint for the low-type, that is

$$\underline{t}_A \geq \frac{1}{2}\underline{g}_A^2 - \theta\left[\mu(\underline{g}_A\underline{g}_B)^{\frac{1}{2}} + (1-\mu)(\underline{g}_A\overline{g}_B)^{\frac{1}{2}}\right]$$

cannot be binding given that $\theta\left[\mu(\underline{g}_A\underline{g}_B)^{\frac{1}{2}} + (1-\mu)(\underline{g}_A\overline{g}_B)^{\frac{1}{2}}\right] \geq 0$ has to be satisfied. □

2.6 Appendix B

2.6.1 Proof of Proposition 2.3. and 2.4.

I want to show that the levels of contribution of the motivated individuals under the second-best are higher than the levels of contribution under the first-best, i.e. $\overline{g}_J^{*SB} - \overline{g}_J^* > 0$. While I have the opposite result regarding the unmotivated individuals, i.e. $\underline{g}_J^{*SB} - \underline{g}_J^* < 0$.
Given the explicit forms of the levels of the contributions under asymmetric information are very large, it is not immediate to see that the inequalities hold.[17] For this reason, I report this result graphically for different values of $\overline{\alpha}$.
In the figure 2.2, I obtain the levels of the contribution for the selfish agent when the probability to have individuals of the same type is equal to $\frac{1}{2}$. The first graph illustrates the levels of the contribution when $\overline{\alpha} = \frac{1}{2}$, while the second graph illustrates the same when $\overline{\alpha} = 1$. The blue line is the levels of the contribution in the first-best, while the second line the levels of the contribution in the second-best. It is possible to note that the blue line is always above the purple line. Furthermore, higher is the degree of motivation of the other individual, lower will be the contribution of the selfish individual.
In the figure 2.3, there are the levels of the contribution for the motivated agent. Again, the first graph illustrates the levels of the contribution when $\overline{\alpha} = \frac{1}{2}$, while the second graph illustrates that when $\overline{\alpha} = 1$. The blue line is the levels of the contribution in the first-best, while the purple line the levels of the contribution in the second-best. It is possible to note that the blue line is always below the purple line. Furthermore, higher is the degree of motivation, higher will be the contribution of the motivated individual. These results still hold for

[17] If, for example, I consider the extreme case in which $\mu = \frac{1}{2}$ and $\overline{\alpha} = 1$, the levels of contribution are given by:

$$\overline{g}^{*SB} = \frac{9\theta}{16} + \sqrt{\frac{9\theta^2}{64} + \frac{(-72+6\sqrt{471})^{1/3}\theta^2}{6(3^{2/3})} - \frac{\theta^2}{(-72+5\sqrt{471})^{1/3}} + \frac{4\theta^3}{256\sqrt{\frac{9\theta^2}{64} + \frac{(-72+5\sqrt{471})^{1/3}\theta^2}{8(3^{2/3})} - \frac{13\theta^2}{8(3(-72+5\sqrt{471}))^{1/3}}}}}$$

and

$$\underline{g}^{*SB} = \frac{(1276-147\sqrt{2})\theta^2 + (96+32\sqrt{2})\theta^4}{348\theta(3+\sqrt{2})}$$

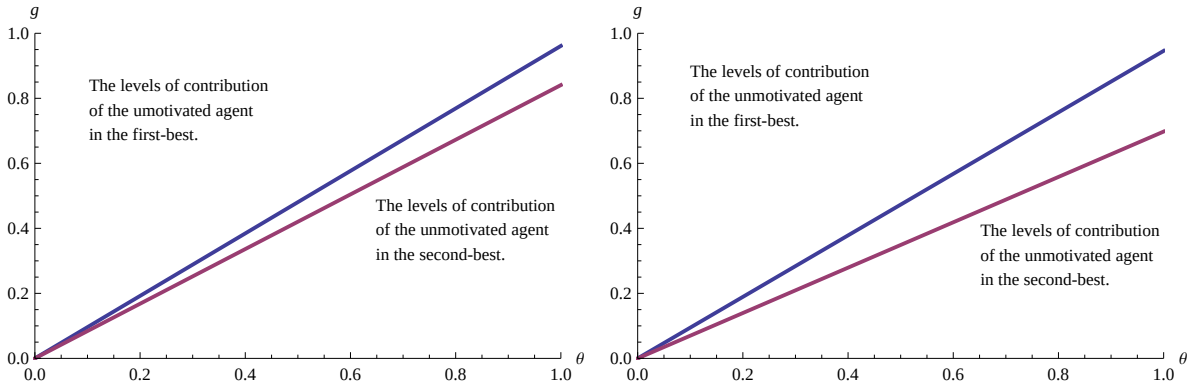

Figure 2.2: Comparison on the levels of contribution when the agent is selfish.

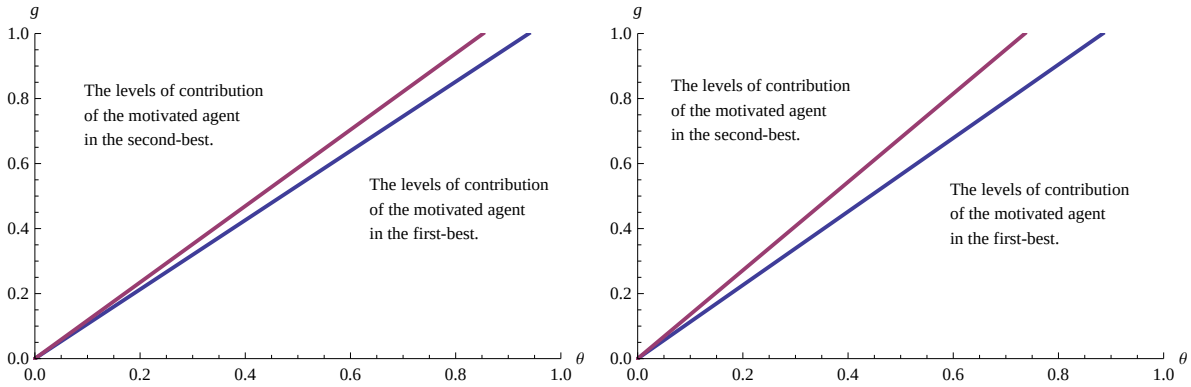

Figure 2.3: Comparison on the levels of contribution when the agent is motivated.

different values of $\overline{\alpha}$.

Substituting the values of the contribution when, for example, $\overline{\alpha} = 1$ into equations (2.12) and (2.13), I obtain the transfers in the second-best. Comparing these transfers with the ones obtained in the first-best, I obtain the following graphical representations for the selfish and the motivated individual, respectively:

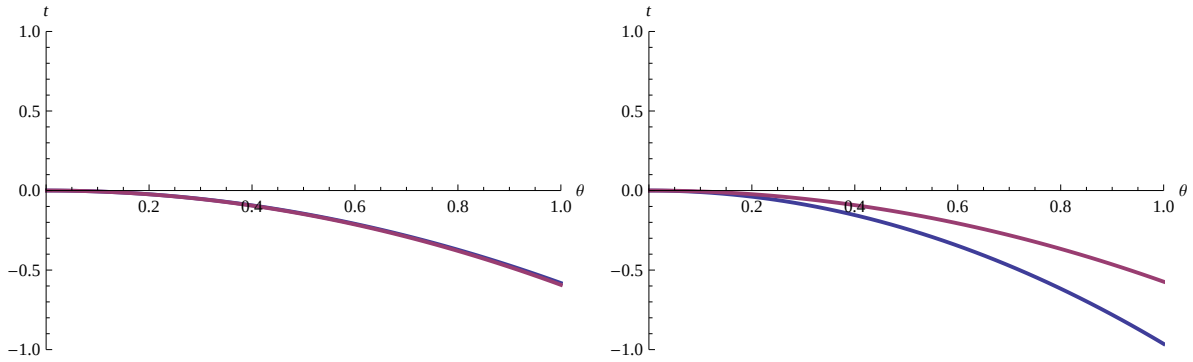

Figure 2.4: Comparison on the levels of taxes.

The blue line represents the taxes paid in the first-best, while the purple line the taxes paid in the second-best.
It is possible to note that the amount of taxes paid by selfish individuals coincide in the first and in the second-best (first graph in figure 2.4). In contrast, motivated individuals pay lower taxes in the second-best than in the first-best (second graph in figure 2.4).
In addition, motivated individuals pay lower taxes than the selfish individual in the second-best.
The results still hold with different values of $\overline{\alpha}$.

All the simulations and comparison are checked using Mathematica and they are available under request.

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Chapter 3

Mixed Duopoly with Motivated Teachers

I study whether the presence of motivated teachers benefits public and private schools in a competitive environment. The quality is influenced by the effort exerted by the teachers. Teachers' motivation may have a positive impact on the levels of effort and, then, on the quality. The effect of motivation strictly depends on the degree of differentiation of the programmes offered by the two schools. When both schools offer similar programmes, the Nash equilibrium is the one in which both schools hire motivated teachers. This is because teachers' motivation plays an important role in the students' choice between the two school. In contrast, when the two schools offer significantly different programmes, the Nash equilibrium is the one in which both schools hire selfish teachers. The increase in the students' utility due to the higher quality provided by the motivated teachers is more than offset by the reduction in the profits earned by the two schools.

Keywords: Intrinsic Motivation, Mixed Duopoly, School Choice, Hotelling Model.

3.1 Introduction

It is often argued that preferences and work motivation of employees differ depending on the nature of the jobs. Many jobs involve helping people in need or contributing to society at large, making these jobs attractive to people who have a strong willingness to serve the others or the public interest (see Buurman, Dur and Van den Bossche, 2009). An appropriate example is teaching. Teachers can take satisfaction from teaching and from developing strategies that are consistent with the best interest of the students because they believe in the virtue of the education for the society. In this sense, *teachers can be intrinsically motivated*.[1]

[1]Another suitable example could be the health-care system. Doctors may be interested not only in their monetary compensation but also in the impact of their work on the well-being of their patients (see for example Ma, 2004).

In many countries the educational services are provided by both public and private institutions. This article analyzes the “market" for education by developing a model that focuses on the interaction between the public and private educational sectors.[2] The educational services offered by schools are *heterogeneous*. In Italy, for example, most private schools have a strong religious connotation[3] while public schools are typically secular. Furthermore, there is a number of foreign and international schools that are private, including American, French and British schools. Many international schools offer bi-lingual programmes, as well as English as a Foreign Language (EFL) exams if the students' first language is not English. Then, private schools can be appealing to parents who want to give to their children an education closer to the international standards. On top of that, most private schools can offer a more caring and protective atmosphere, as well as the opportunity of additional or intensive lessons, which some parents believe to be more conducive to learning. Parents in some of the wealthier areas of the major cities may also send their children to a private school simply for its exclusivity. Thus, there exists some degree of horizontal differentiation between schools.

Parents and students consider the differences in the educational services offered when they choose between public and private schools. Another feature which is taken into account in making the school choice is the *quality* of the education provided by the schools. A high level of quality can play a crucial role in the students' decision between different schools. The quality is influenced by the effort exerted by the teachers. Teachers' motivation may have a positive impact on the levels of effort and, then, on the quality. Therefore, there exists also some degree of vertical differentiation between schools which is endogenous and depends on the incentives provided to the teachers.

The aim of this model is to investigate the impact of intrinsic motivation on the schools' outcome in terms of quality, price and wage in a mixed duopoly environment (see De Fraja and Delbono, 1990, and Nett, 1993, for general reviews of the mixed oligopoly markets).

To this end, I develop an oligopolistic model where two schools are positioned at each end of a Hotelling line. The horizontal differentiation reflects the heterogeneity of the programmes offered by the schools. While the private school maximizes its profits, the public school maximizes social welfare. The social welfare is given by the sum of the students' utility, the teachers' utility and the profits obtained by both schools. Each school consists of a principal and an agent, both risk neutral. The principal-agent relationship can be interpreted as the relationship between the school-principal who wants to delegate the decision about an outcome in terms of quality to a teacher (the agent).

The two schools offer imperfectly substitutable programmes and they compete against each other on quality and prices. When the degree of substitutability is high, the schools offer

[2] Epple and Romano (1998) also study a model of competition between public and private schools. However, their model stresses competition for students among public and private schools.

[3] Most private schools are run by religious organisations (the majority by the Jesuits).

similar programmes and there is more competition in the market.
The key assumption of the model is that teachers can be intrinsically motivated. There are only two types of agents: the selfish teacher and the motivated teacher.
I show that the presence of motivated teachers can benefit or hurt public and private schools depending on the degree of differentiation between the programmes offered by the two schools.
If the programmes offered by the schools are only slightly different and both principals hire self-interested teachers, a principal finds it profitable to deviate by hiring a motivated teacher. By doing so, the public principal increases the social welfare and the private principal obtains a comparative advantage in terms of demand and price. Then, if a principal hires a motivated teacher the best response of the other principal is to hire a motivated agent as well. *The Nash equilibrium is the one in which both schools hire motivated teachers.* Teachers' motivation plays an important role in the students' choice between the two schools. Individuals' intrinsic motivation increases the quality of the educational services offered by both schools. To elicit higher levels of effort to increase the quality, both principals have to offer higher incentives to their agents. Then, motivation has a positive impact on the wages offered to the teachers. This result does not support the previous literature in which motivation is effective in stimulating work effort even in absence of monetary rewards (see for example Gneezy and Rustichini, 2000a,b and Bénabou and Tirole, 2003, 2006).[4]
If the schools offer very different programmes, there is less scope for competition in the market. The private school obtains higher benefits by hiring self-interested teachers than by hiring motivated teachers. This is because teachers' motivation is relatively less important. Considering the public school, the higher the degree of differentiation of the programmes offered by the schools, the lower the utility of the students. This effect has a negative impact on the social welfare. When the schools offer different educational programmes, there is less competition in the market and this has a negative impact on the students' utility reducing social welfare. In addition, the higher the degree of differentiation of the programmes, the higher the profits obtained by the schools. This effect has a positive impact on the social welfare. In this case, the increase in the students' utility due to the higher quality provided by the motivated teachers is more than offset by the reduction in the profits earned by the two schools. In other words, if the programmes offered by the schools are very different, the role of agents' intrinsic motivation is less important. Hence, the public school also hires the selfish teacher and the students choose the school with a programme closer to their necessity. *The Nash equilibrium is the one in which both schools hire selfish teachers.*

[4]This literature shows that monetary incentives can influence negatively the individuals' behavior in terms of their levels of contribution. The reason is that monetary incentives give the agent a selfish motive to operate. Explicit incentives from principals may change how tasks are perceived by agents (Gneezy and Rustichini, 2000a) and they may also reduce the value of generous or civic minded acts as a signal of one's moral character (Bénabou and Tirole, 2003, 2006). If extrinsic incentives are not large enough, this change in perception can even lead to undesired effects on behavior (Gneezy and Rustichini, 2000b).

This chapter of the thesis is related to two strands of the literature: the literature on the effects of competition on managerial incentives and the literature on psychological incentives in organizations.

Regarding the literature on the effects of competition on managerial incentives, while some articles argue that competition reduces managerial incentives (see, for example, Hart, 1983[5]), others show that the relationship is ambiguous (see, for example, Scharfstein, 1988, and Hermalin, 1992[6]). Schmidt (1997) explains that greater competition may lead to stronger incentives for agents because greater effort is required to avert the threat of bankruptcy. Raith (2003) examines how the degree of competition among firms in an industry with free entry and exit affects the incentives for their managers. Then, the effect of competition on incentives and effort takes place through a change in the equilibrium number of firms in the industry. His results suggest an unambiguous positive relationship between competition and incentives. Baggs and De Bettignies (2007) also conclude that competition increases managerial incentives. These articles do not consider potential differences in the agents' preferences or in the principals' objectives.

Regarding the literature on psychological incentives in organizations (see for instance Bénabou and Tirole, 2003, 2006; Gneezy and Rustichini, 2000a, 2000b), most articles are principally focused on the impact of monetary incentives on the individuals' effort when they are intrinsically motivated. In this literature, some articles also study the matching of motivated employees between public and private sector (see for example Besley and Ghatak, 2005, and Prendergast, 2007).

I try to link together these two strands of the literature. I consider differences in the agents' preferences and I study the impact of competition and intrinsic motivation on the schools' outcome in presence of a mixed duopoly environment.

The plan of the paper is as follows: in section 3.2 I present the set-up of the model; section 3.3 is devoted to characterize the equilibrium of the model; in section 3.4 I study the solution of the game; and concluding remarks are given in section 3.5.

[5]Hart (1983) is the first to formalize this idea by modeling the effect of competition on the agency problems between a firm's owner and a manager.

[6]Scharfstein (1988) reconsiders Hart's model while relaxing the assumption of infinitely risk-adverse managers. Hermalin (1992) considers additional effects of competition on the agency problem, all of which are of potentially ambiguous sign.

3.2 The Set-Up of the Model

I build a mixed duopoly model where two schools are positioned at each end of a Hotelling line, with locations $x_i = 0$ and $x_j = 1$, respectively. The school i is public and the school j is private. The public school maximizes the social welfare. In contrast, the private school maximizes his profits. Each school is constituted of a principal and an agent, both of whom are risk neutral. The principal-agent relationship can be interpreted as the relationship between the school-principal that delegates the decision about an outcome in terms of quality q to an agent (the teacher). A teacher working in the public sector cannot participate in the private sector too, and viceversa. There are a continuum of students of mass 1 distributed uniformly along the line.

The agents are wealth constrained with zero initial wealth and have a reservation wage of zero. The agents have quadratic effort costs, which are observable to the principal. The exerted effort ϵ determines the quality of the educational services offered by the two schools. For expositional convenience, I assume that quality q depends linearly on the teachers' effort: $q = \epsilon$ in both schools. There is no asymmetric information between the principal and the agent. Since quality is verifiable, the principals do not need to offer an incentive to the agents because they have all the necessary information to implement the efficient levels of quality.

In addition, the teachers' utilities might positively depend on the benefits of the students. The teachers can be also interested in the impact of their work on the students' utility. The measure of this utility depends on the parameter θ that represents the intrinsic motivation of the agent. It influences the optimal levels of quality and price. There are only two types of teachers: the self-interested teachers with $\underline{\theta} = 0$ and the motivated teachers with $\overline{\theta} > 0$. There is an infinite number of teachers of both types. The principals offer a wage that covers the cost of effort paid by the teacher minus his intrinsic motivation.

After the employment decision, the two schools offer imperfectly substitutable services, competing against each other on quality q and prices p.

The timing of the model is as follows. *At the initial stage 0*, each principal decides whether to hire a motivated teacher or a non motivated teacher. *At stage 1*, each principal makes an offer (ω, q) to their agent. The teachers accept any contract with an expected utility of at least their reservation utility, which I normalize to 0. If the agents accept the contract, they exert the required levels of effort; *At stage 2*, after agents have exerted effort determined by the contract, principals simultaneously choose prices; *At stage 3*, the students choose between the two school.

3.2.1 The Objective Functions

A student enjoys a utility

$$\begin{aligned} U_i &= q_i - p_i - tx \ \text{ from the service offered by the school } i \text{ and} \\ U_j &= q_j - p_j - t(1-x) \ \text{ from the service offered by the school } j. \end{aligned} \tag{3.1}$$

At location x, a student i incurs a transport cost tx for traveling to school i and a cost $t(1-x)$ to school j. For given p_i, p_j, q_i, q_j there is a cutoff $\overline{x}$, such that all students with $x < \overline{x}$ choose the service of school i, and with $x > \overline{x}$ choose school j. The parameter t represents the degree of differentiation of the educational services offered by the two schools. When t is low the schools offer similar programmes, implying fierce competition.
The key assumption of this model is that teachers can be intrinsically motivated. The teachers' utility function consists of their own "egoistic" payoff, given by the difference between wage and effort costs, and of their intrinsic motivation. There are two types of teachers: self-interested teachers with $\underline{\theta} = 0$ and motivated teachers with $\overline{\theta} > 0$.
The agents' utility function from working in the public and private schools, respectively, can be written as:

$$\begin{aligned} V_i &= \omega_i - \frac{1}{2} q_i^2 + \theta_i \overline{U}_i \\ V_j &= \omega_j - \frac{1}{2} q_j^2 + \theta_j \overline{U}_j \end{aligned} \tag{3.2}$$

where $\overline{U}_i$ and $\overline{U}_j$ are the utilities of the average student deciding for school i and school j, respectively, and are equal to: $\overline{U}_i = q_i - p_i - t\,\frac{\overline{x}}{2}$ and $\overline{U}_j = q_j - p_j - t\,\frac{(1-\overline{x})}{2}$.
The public school i maximizes the social welfare while the private school j maximizes profits. Moreover, both schools obtain a benefit given by the product of price and demand minus the wage given to the teacher.
More specifically, the public school i maximizes the following:

$$\pi_i = \overline{U}_i + \overline{U}_j + V_i + V_j + p_i\, d_i - \omega_i + p_j\, d_j - \omega_j + \overline{v} - \overline{v} \tag{3.3}$$

While the private school j maximizes the following profit function:

$$\pi_j = p_j\, d_j - \omega_j + \overline{v} \tag{3.4}$$

$\overline{v}$ is strictly positive and represents the funding offered to the private school.[7]
The principals maximize their objective functions under the following participation constraints:

$$\begin{aligned} \omega_i - \frac{1}{2} q_i^2 - \theta_i\, \overline{U}_i \geq 0 \\ \omega_j - \frac{1}{2} q_j^2 - \theta_j\, \overline{U}_j \geq 0 \end{aligned} \tag{3.5}$$

[7] Most private schools in Italy are either authorised or given legal recognition by the state and many receive state funding.

The participation constraints guarantee that both teachers do not choose their outside option.

Assumption 3.1. *I make the following assumption to guarantee an interior solution.*

- $t \in \left(0, \frac{1}{3}\right)$;
- *and* $\overline{\theta} \in \left(0, \frac{1-3t}{2(1+t)}\right]$.

The role of this assumption will become clear in the next sections.

3.3 The Characterization of the Equilibrium

The equilibrium is determined by backward induction.
At stage 3, the students choose the school. A student located at $\overline{x}$ is indifferent between the public school i and the private school j if and only if $U_i = U_j$, or equivalently $q_i - p_i - t\overline{x} = q_j - p_j - t(1-\overline{x})$. $\overline{x}$ represents the demand for the public school i and $(1-\overline{x})$ the demand for the private school j:

$$\begin{aligned} \overline{x} = d_i(q_i, q_j, p_i, p_j, t) &= \frac{1}{2} + \frac{(q_i - q_j) + (p_j - p_i)}{2t} \\ (1-\overline{x}) = d_j(q_i, q_j, p_i, p_j, t) &= \frac{1}{2} + \frac{(q_j - q_i) + (p_i - p_j)}{2t} \end{aligned} \tag{3.6}$$

At stage 2, the principals choose their prices to maximize their objective functions, taking qualities and wages as given.
The public school maximizes the social welfare:

$$\max_{p_i} \pi_i = (1+\theta_i)\,\overline{U}_i + (1+\theta_j)\,\overline{U}_j + d_i\,p_i - \frac{1}{2}q_i^2 + d_j\,p_j - \frac{1}{2}q_j^2 \tag{3.7}$$

while the private school maximizes its profits:

$$\max_{p_j} \pi_j = d_j\,p_j - \omega_j + \overline{v} \tag{3.8}$$

Taking the first order conditions of equations (3.7) and (3.8) with respect to p_i and p_j, respectively, I obtain the following equilibrium prices:

$$\begin{aligned} p_i &= \frac{(q_j - q_i)}{2} + \frac{1}{4}\,t(2 - 3\theta_i - 7\theta_j) \\ p_j &= (q_j - q_i) + t\left(1 - \frac{3}{2}\theta_j\right) \end{aligned} \tag{3.9}$$

Substituting equilibrium prices into the equations (3.7) and (3.8), I obtain an expression for social welfare and private profits as a function of the levels of quality and wages offered by the two schools:

$$\begin{aligned}\pi_i = (1+\theta_i)\,\overline{U}_i + \left[\frac{2(q_i-q_j)+t(6+3\theta_i+\theta_j)}{8t}\right]\left[\frac{(q_j-q_i)}{2}+\frac{1}{4}t(2-3\theta_i-7\theta_j)\right]-\frac{1}{2}\,q_i^2+\\ +(1+\theta_j)\,\overline{U}_j+\left[\frac{2(q_j-q_i)+t(2-3\theta_i-\theta_j)}{8t}\right]\left[(q_j-q_i)+t\left(1-\frac{3}{2}\theta_j\right)\right]-\frac{1}{2}\,q_j^2\end{aligned} \tag{3.10}$$

$$\pi_j = \left[(q_j-q_i)+t\left(1-\frac{3}{2}\theta_j\right)\right]\left[\frac{2(q_j-q_i)+t(2-3\theta_i-\theta_j)}{8t}\right]-\omega_j+\overline{v} \tag{3.11}$$

At stage 1, these functions are maximized with respect to ω_i, q_i and ω_j, q_j, respectively.
I obtain the optimal levels of quality:

$$\begin{aligned}q_i^* &= \frac{32-56t+19\theta_i-44t\theta_i+13\theta_j-36t\theta_j}{8(3-4t)}\\ q_j^* &= \frac{32-16t+19\theta_i+12t\theta_i+13\theta_j+20t\theta_j}{8(3-4t)}\end{aligned} \tag{3.12}$$

with wages:

$$\begin{aligned}\omega_i^* = \frac{1}{2}\left(\frac{32-56t+\theta_i(19-44t)+\theta_j(13-36t)}{8(3-4t)}\right)^2+\\ -\theta_i\left(\frac{64-184t+56t^2+\theta_i(38-103t-36t^2)+\theta_j(26-33t-108t^2)}{16(3-4t)}\right)\end{aligned} \tag{3.13}$$

$$\begin{aligned}\omega_j^* = \frac{1}{2}\left(\frac{32-16t+\theta_i(19+12t)+\theta_j(13+20t)}{8(3-4t)}\right)^2+\\ -\theta_j\left(\frac{64-176t+72t^2+\theta_i(38-93t-12t^2)+\theta_j(26-11t-100t^2)}{16(3-4t)}\right)\end{aligned}$$

The chosen prices at stage 2 are equal to:

$$\begin{aligned}p_i^* &= \frac{t(16-8t+5\theta_i+12t\theta_i-7\theta_j+28t\theta_j)}{4(3-4t)}\\ p_j^* &= \frac{t(16-8t+14\theta_i+5\theta_j+12t\theta_j)}{2(3-4t)}\end{aligned} \tag{3.14}$$

At stage 3 the demands are equal to

$$\begin{aligned}d_i^* &= \frac{8-24t-5\theta_i-12t\theta_i-11\theta_j-4t\theta_j}{8(3-4t)}\\ d_j^* &= \frac{16-8t+5\theta_i+12t\theta_1+11\theta_j+4t\theta_j}{8(3-4t)}\end{aligned} \tag{3.15}$$

and the outcomes obtained by the schools are realized:

$$\pi_i = (1+\theta_i)\,\overline{U}_i^* + (1+\theta_j)\,\overline{U}_j^* + d_i^*\,p_i^* - \frac{1}{2}\,q_i^{*2} + d_j^*\,p_j^* - \frac{1}{2}\,q_j^{*2} \tag{3.16}$$

$$\pi_j = d_j^* \; p_j^* - \omega_j^* + \overline{v} \tag{3.17}$$

There are only two types of agents: the self-interested teachers with $\underline{\theta} = 0$ and the motivated teachers with $\overline{\theta} > 0$. At stage 1, I characterize the equilibrium for different degrees of intrinsic motivation and I obtain different payoffs:

- when both teachers are self-interested, i.e. $\theta_i = \theta_j = \underline{\theta} = 0$ (the equilibrium outcomes will be denoted by π_i^*, π_j^*, respectively);
- when teachers are homogeneous and motivated, i.e. $\theta_i = \theta_j = \overline{\theta} > 0$ (the equilibrium outcomes will be denoted by $\overline{\pi}_i^*, \overline{\pi}_j^*$, respectively);
- when only the teacher i is motivated, i.e. $\theta_i = \overline{\theta} > 0$ and $\theta_j = \underline{\theta} = 0$ (the equilibrium outcomes will be denoted by $\widehat{\pi}_i^*, \underline{\pi}_j^*$, respectively);
- and, finally, when only the agent j is motivated, i.e. $\theta_j = \overline{\theta} > 0$ and $\theta_i = \underline{\theta} = 0$ (the equilibrium outcomes will be denoted by $\underline{\pi}_i^*, \widehat{\pi}_j^*$, respectively).

The characterization of the equilibrium for different degrees of intrinsic motivation are in the Appendix A.

3.4 The Type Choice Game

In stage zero, both firms choose simultaneously which type of agent to hire. Given prices, quantities and wages, the type choice reduces to the following game:

	$\underline{\theta}_j$	$\overline{\theta}_j$
$\underline{\theta}_i$	(π_i^*, π_j^*)	$(\underline{\pi}_i^*, \widehat{\pi}_j^*)$
$\overline{\theta}_i$	$(\widehat{\pi}_i^*, \underline{\pi}_j^*)$	$(\overline{\pi}_i^*, \overline{\pi}_j^*)$

Figure 3.1: The Type-Choice Game

I start comparing the benefits obtained by hiring self-interested teachers with those obtained by hiring only a motivated teacher when there is high competition in the market. Furthermore, I also compare the benefits obtained by hiring motivated teachers with those obtained by hiring a non-motivated agent when the rival school hires a motivated agent.

Lemma 3.1. *If* $t < 0.165$*, then*

- $\pi_i^* < \widehat{\pi}_i^*$ *and* $\pi_j^* < \widehat{\pi}_j^*$*;*
- $\overline{\pi}_i^* > \underline{\pi}_i^*$ *and* $\overline{\pi}_j^* > \underline{\pi}_j^*$*.*

Proof. See Appendix B. □

If there is high competition between schools (t is small) and both principals hire self-interested teachers, a principal finds it profitable to deviate by hiring a motivated teacher. By doing so, the public principal increases the social welfare and the private principal obtains a comparative advantage in terms of demand and price. In addition, if a principal hires a motivated teacher the best response of the other principal is to hire a motivated agent as well.

This leads to the following proposition.

Proposition 3.1. *When* $t < 0.165$*, the unique Nash equilibrium is the one in which both schools hire motivated agents.*

Proof. See lemma 3.1. □

When the public and the private schools offer similar programmes, there is high competition in the scholastic market. In this case, the public school hires a motivated teacher because the agent's intrinsic motivation has a positive impact on the social welfare. Then, the best response of the private school is to follow suit and to hire a motivated teacher as well. However, I find that the profits obtained by the private school when both teachers are selfish would always be higher than the profits obtained when both teachers are motivated, i.e. $\pi_j^* > \overline{\pi}_j^*$. This result is illustrated in the first graph of figure 3.2. The blue line represents the profits obtained by the private school when both teachers are self-interested. While the purple line represents the profits obtained by the private school when both teachers are motivated. Then, the presence of the public school, and its choice to hire a motivated teacher, "pushes" the private principal to hire a motivated agent too. This result is illustrated in the second graph of figure 3.2. Now, the blue line represents the profits of the private school by hiring a selfish agent when the public school hires a motivated agent. While the purple line represents always the profits of the private school when both teachers are motivated. The purple line $\overline{\pi}_j^*$ is above the blue line $\underline{\pi}_j^*$ until t is low enough. Hence, the private school finds it profitable to hire a motivated agent as well.

Lemma 3.2. *If* $0,165 < t < 0.2$*, then*

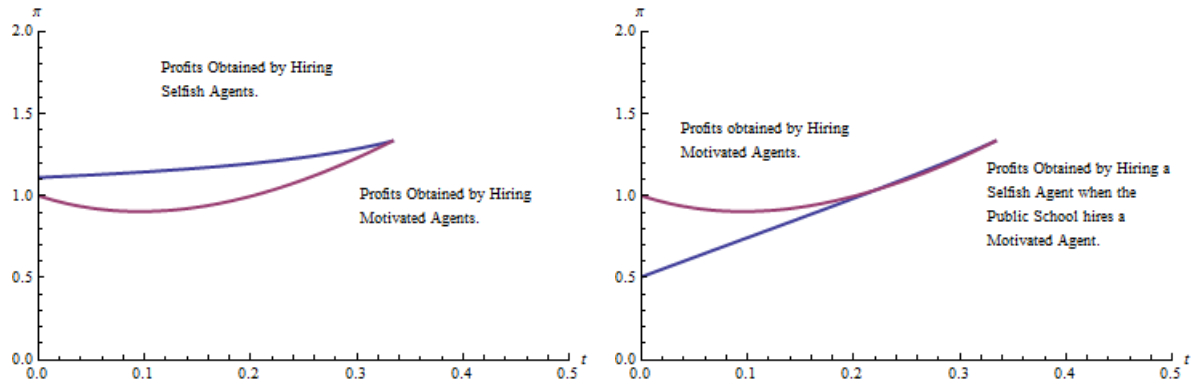

Figure 3.2: Comparison Profits of the Private School.

- $\pi_i^* > \widehat{\pi}_i^*$ *and* $\pi_j^* < \widehat{\pi}_j^*$.
- $\overline{\pi}_i^* < \underline{\pi}_i^*$ *and* $\overline{\pi}_j^* > \underline{\pi}_j^*$.

Proof. See Appendix B. □

This leads to the following proposition.

Proposition 3.2. *When* $0,165 < t < 0.2$*, the unique Nash equilibrium is the one in which the public school hires a selfish teacher, while the private school hires a motivated teacher.*

Proof. See lemma 3.2. □

As the schools tend to have differentiated educational programmes, the students' utility is reduced leading to a reduction of the social welfare. In contrast, the higher the degree of differentiation of the programmes offered by the schools, the higher the profits obtained by the schools. This effect has a positive impact on the social welfare. There is a level of t for which the increase in the students' utility due to the higher quality provided by the motivated teachers is more than offset by the reduction in the profits earned by the two schools. In this case, the public principal hires the selfish teacher. In contrast, the private principal obtains a comparative advantage by hiring a motivated agent when the public principal hires a selfish one.

Lemma 3.3. *If* $t > 0.2$*, then*

- $\pi_i^* > \widehat{\pi}_i^*$ *and* $\pi_j^* > \widehat{\pi}_j^*$.
- $\overline{\pi}_i^* < \underline{\pi}_i^*$ *and* $\overline{\pi}_j^* < \underline{\pi}_j^*$.

Proof. See Appendix B. □

This leads to the following proposition.

Proposition 3.3. *When $t > 0.2$, the unique Nash equilibrium is the one in which both schools hire selfish teachers.*

Proof. See lemma 3.3. □

If the schools offer very different educational services, there is less scope for competition in the market and the teachers' intrinsic motivation becomes relatively less important. In that case, both schools obtain higher benefits by hiring self-interested teachers than by hiring motivated teachers. Then, the students will choose the school with a programme closer to their necessity without considering the teachers' intrinsic motivation.

3.5 Conclusions

In this article, I have shown that the effect of teachers' intrinsic motivation strictly depends on the the degree of differentiation of the programmes offered by the two schools. When there is high competition in the education "market", both schools hire motivated teachers in order to attract students. In this case, teachers' motivation plays an important role in the students' choice between schools. In contrast, if the schools offer different programmes, the teachers' intrinsic motivation becomes relatively less important. In that case, both schools obtain higher benefits by hiring self-interested teachers than by hiring motivated teachers. The Nash-equilibrium is the one in which both schools hire selfish teachers.
For future research, it might be interesting to analyze the optimal position of the schools on the Hotelling line and to study different models of competition. Moreover, a possible extension of the model is the one in which the quality is unobservable.

3.6 Appendix A

3.6.1 Characterization of the Equilibrium when Both Teachers are Self-Interested

I begin by characterizing the equilibrium when teachers are self-interested, i.e. $\theta_i = \theta_j = 0$.
At stage 1, the optimal levels of quality and wages are determined.
The quality's levels are equal to:

$$\begin{aligned} q_i^* &= \frac{(4-7t)}{(3-4t)} \\ q_j^* &= \frac{2(2-t)}{(3-4t)} \end{aligned} \tag{3.18}$$

with wages

$$\begin{aligned} \omega_i^* &= \frac{1}{2}\left(\frac{(4-7t)}{(3-4t)}\right)^2 \\ \omega_j^* &= \frac{1}{2}\left(\frac{2(2-t)}{(3-4t)}\right)^2 \end{aligned} \tag{3.19}$$

By hiring selfish individuals, the quality of the educational services is lower in the public school than in the private school. For this reason, the wage paid to the public teacher has to be lower than the wage paid to the teacher working in the private school.
The chosen prices at stage 2 are:

$$\begin{aligned} p_i^* &= \frac{2t(2-t)}{(3-4t)} \\ p_j^* &= \frac{4t(2-t)}{(3-4t)} \end{aligned} \tag{3.20}$$

When the schools offer similar programmes (low t) there is more competition in the market that leads to a low price for both schools. Moreover, with a higher quality offered by the private school, the private principal sets higher price than the public school, i.e. $p_j^* > p_i^*$.
And at stage 3, the demands are realized with

$$\begin{aligned} d_i^* &= \frac{(1-3t)}{(3-4t)} \\ d_j^* &= \frac{(2-t)}{(3-4t)} \end{aligned} \tag{3.21}$$

The improvement of the quality permits to the private principal to "steal" the market to the public school, i.e. $d_j^* > d_i^* > 0$.
And the public and private principals obtain, respectively, the following:

$$\begin{aligned} \pi_i^* = &\left[\frac{(1-3t)}{(3-4t)}\right]\left[\frac{2t(2-t)}{(3-4t)}\right] - \frac{1}{2}\left(\frac{(4-7t)}{(3-4t)}\right)^2 + \left[\frac{8-23t+7t^2}{2(3-4t)}\right] + \\ &+ \left[\frac{(2-t)}{(3-4t)}\right]\left[\frac{4t(2-t)}{(3-4t)}\right] - \frac{1}{2}\left(\frac{2(2-t)}{(3-4t)}\right)^2 + \left[\frac{8-22t+9t^2}{2(3-4t)}\right] \end{aligned} \tag{3.22}$$

$$\pi_j^* = \left[\frac{(2-t)}{(3-4t)}\right]\left[\frac{4t(2-t)}{(3-4t)}\right] - \frac{1}{2}\left(\frac{2(2-t)}{(3-4t)}\right)^2 + \overline{v} \tag{3.23}$$

3.6.2 Characterization of the Equilibrium when Teachers are Homogeneous and Motivated

In this subsection, I determine the equilibrium when both teachers are homogeneous and intrinsically motivated, i.e. $\theta_i = \theta_j = \overline{\theta} > 0$.
The optimal levels of quality are the following:

$$\begin{aligned} \overline{q}_i^* &= \frac{(4-7t+4\overline{\theta}-10t\overline{\theta})}{(3-4t)} \\ \overline{q}_j^* &= \frac{2(2-t+2\overline{\theta}+2t\overline{\theta})}{(3-4t)} \end{aligned} \tag{3.24}$$

The quality of the services offered by the public school is always lower than the quality offered by the private school, i.e. $\overline{q}_i^* < \overline{q}_j^*$. The teachers' intrinsic motivation has a positive impact on the levels of quality offered by the two schools, i.e. $\frac{\partial \overline{q}_i^*}{\partial \overline{\theta}} > 0$ and $\frac{\partial \overline{q}_j^*}{\partial \overline{\theta}} > 0$. The impact of teachers' intrinsic motivation on the levels of quality is higher in the public school than the private school if the programmes offered by the schools are similar, i.e. $\frac{\partial \overline{q}_i^*}{\partial \overline{\theta}} > \frac{\partial \overline{q}_j^*}{\partial \overline{\theta}}$ if $t < \frac{1}{6} = 0.167$.
The wages are given by:

$$\begin{aligned} \overline{\omega}_i^* &= \frac{1}{2}\left(\frac{(4-7t+4\overline{\theta}-10t\overline{\theta})}{(3-4t)}\right)^2 - \overline{\theta}\left(\frac{8-23t+7t^2+8\overline{\theta}-17t\overline{\theta}-18t^2\overline{\theta}}{2(3-4t)}\right) \\ \overline{\omega}_j^* &= \frac{1}{2}\left(\frac{2(2-t+2\overline{\theta}+2t\overline{\theta})}{(3-4t)}\right)^2 - \overline{\theta}\left(\frac{8-22t+9t^2+8\overline{\theta}-13t\overline{\theta}-14t^2\overline{\theta}}{2(3-4t)}\right) \end{aligned} \tag{3.25}$$

The teachers' intrinsic motivation has a countervailing effect on the wages. On the one hand, the teachers' intrinsic motivation has a positive impact on the wages. This is because a high $\overline{\theta}$ leads to high levels of quality of the educational services. Then, both principals pay high wages in the way to cover the cost of effort and to improve quality performance. On the other hand, teachers' intrinsic motivation has a negative impact on the wages. This is because motivated teachers provide a given level of quality even if they receive a low compensation for that. *The overall effect is positive* in both schools. This result does not support the previous literature in which motivation is effective in stimulating work effort even in absence of monetary rewards (see for example Gneezy and Rustichini, 2000a,b and Benabou and Tirole, 2003, 2006). Furthermore, the effect of the agents' intrinsic motivation on wages is higher in the private school than in the public school, i.e. $\frac{\partial \overline{\omega}_j^*}{\partial \overline{\theta}} > \frac{\partial \overline{\omega}_i^*}{\partial \overline{\theta}} > 0$. This is because given that the quality offered by the private school is higher, the teacher j has to receive a higher compensation for his work than the teacher i, i.e. $\omega_i^* < \omega_j^*$.

Prices are given by:

$$\begin{aligned} \overline{p}_i^* &= \frac{t(8-4t-\overline{\theta}+20t\overline{\theta})}{2(3-4t)} \\ \overline{p}_j^* &= \frac{t(16-8t+19\overline{\theta}+12t\overline{\theta})}{2(3-4t)} \end{aligned} \tag{3.26}$$

The price of the public school is lower than the price of the private school, i.e. $\overline{p}_j^* > \overline{p}_i^* > 0$. Moreover, the impact of the teachers' intrinsic motivation is different in the two school. More specifically, the impact of $\overline{\theta}$ on the price of the public school is negative, unless the programmes offered by the schools are very similar, i.e. $t < \frac{1}{20}$. In contrast, the impact of the teachers' intrinsic motivation on the price of the private school is positive. A higher value of $\overline{\theta}$ increases the gap between the levels of quality offered by the schools increasing the price of the private school and reducing the price of the public one.[8]

The demand in the private and public school is respectively equal to:

$$\begin{aligned} \overline{d}_i^* &= \frac{(1-3t-\overline{\theta}(1+t))}{(3-4t)} \\ \overline{d}_j^* &= \frac{(2-t+2\overline{\theta}(1+t))}{(3-4t)} \end{aligned} \tag{3.27}$$

A higher quality offered by the private school leads to an increase of its demand.

And the profits are realized:

$$\begin{aligned} \overline{\pi}_i^* = &\left[\frac{(1-3t-\overline{\theta}(1+t))}{(3-4t)}\right]\left[\frac{t(8-4t-\overline{\theta}+20t\overline{\theta})}{2(3-4t)}\right] - \frac{1}{2}\left(\frac{(4-7t+4\overline{\theta}-10t\overline{\theta})}{(3-4t)}\right)^2 + \\ &+\left[\frac{(2-t+2\overline{\theta}(1+t))}{(3-4t)}\right]\left[\frac{t(16-8t+19\overline{\theta}+12t\overline{\theta})}{2(3-4t)}\right] - \frac{1}{2}\left(\frac{2(2-t+2\overline{\theta}+2t\overline{\theta})}{(3-4t)}\right)^2 + \\ &+(1+\overline{\theta})\left[\frac{8-23t+7t^2+8\overline{\theta}-17t\overline{\theta}-18t^2\overline{\theta}}{2(3-4t)}\right] + (1+\overline{\theta})\left[\frac{8-22t+9t^2+8\overline{\theta}-13t\overline{\theta}-14t^2\overline{\theta}}{2(3-4t)}\right] \end{aligned} \tag{3.28}$$

$$\begin{aligned} \overline{\pi}_j^* = &\left[\frac{(2-t+2\overline{\theta}(1+t))}{(3-4t)}\right]\left[\frac{t(16-8t+19\overline{\theta}+12t\overline{\theta})}{2(3-4t)}\right] + \overline{v} + \\ &-\left[\frac{1}{2}\left(\frac{2(2-t+2\overline{\theta}+2t\overline{\theta})}{(3-4t)}\right)^2 - \overline{\theta}\left(\frac{8-22t+9t^2+8\overline{\theta}-13t\overline{\theta}-14t^2\overline{\theta}}{2(3-4t)}\right)\right] \end{aligned} \tag{3.29}$$

[8] This result is due to the fact that the schools maximize different objective functions. In my previous article (Manna, 2013), I show that if both principals maximize their profits, the effect of $\overline{\theta}$ on the prices is positive. Motivation has a positive impact on the quality offered by the firms. It implicitly reduces the product differentiation between firms stiffening competition and reducing prices. With higher qualities, the degree of differentiation of the product becomes relatively less important, leading to fiercer competition.

3.6.3 Characterization of the Equilibrium when Only the Public Teacher i is Motivated

Now, suppose that only the agent i is intrinsically motivated, i.e. $\theta_i = \overline{\theta}$ and $\theta_j = 0$.
At stage 1, agents exert effort and the optimal levels of quality are determined:

$$\begin{aligned} \widehat{q}_i^* &= \frac{(32-56t+19\overline{\theta}-44t\overline{\theta})}{8(3-4t)} \\ \underline{q}_j^* &= \frac{(32-16t+19\overline{\theta}+12t\overline{\theta})}{8(3-4t)} \end{aligned} \tag{3.30}$$

Again, the quality of the services offered by the public school is lower than the quality offered by the private school, i.e. $\widehat{q}_i^* < \underline{q}_j^*$. In addition, the intrinsic motivation of the teacher hiring by the public school has a positive impact on the levels of quality offered by the two schools, i.e. $\frac{\partial \widehat{q}_i^*}{\partial \overline{\theta}} > 0$ and $\frac{\partial \underline{q}_j^*}{\partial \overline{\theta}} > 0$. This is due to the fact that an increase of the level of quality offered by the school i "pushes" the principal j to elicit higher agent effort in order to improve quality. [9]

The principals pay the following wages to their agents:

$$\begin{aligned} \widehat{\omega}_i^* = \frac{1}{2}\left(\frac{(32-56t+19\overline{\theta}-44t\overline{\theta})}{8(3-4t)}\right)^2 - \overline{\theta}\left(\frac{64-184t+56t^2+38\overline{\theta}-103t\overline{\theta}-36t^2\overline{\theta}}{16(3-4t)}\right) \\ \underline{\omega}_j^* = \frac{1}{2}\left(\frac{(32-16t+19\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right)^2 \end{aligned} \tag{3.31}$$

To maintain the comparative advantage in terms of quality, the principal j provides more incentives. The private school produces higher levels of quality and provides stronger incentives, i.e. $\widehat{\omega}_i^* < \underline{\omega}_j^*$.
The prices are given by:

$$\begin{aligned} \widehat{p}_i^* &= \frac{t(16-8t+5\overline{\theta}+12t\overline{\theta})}{4(3-4t)} \\ \underline{p}_j^* &= \frac{t(8-4t+7t\overline{\theta})}{(3-4t)} \end{aligned} \tag{3.32}$$

A high agent i's degree of motivation has a positive effect on the price of both schools. A high $\overline{\theta}$ produces an improvement of the quality offered by both schools, which increases its price. This effect has a positive impact on the marginal profits obtained by the private school and a negative impact on the students' utility.
The demands will be equal to:

$$\begin{aligned} \widehat{d}_i^* &= \frac{(8-24t-5\overline{\theta}-12t\overline{\theta})}{8(3-4t)} \\ \underline{d}_j^* &= \frac{(16-8t+5\overline{\theta}+12t\overline{\theta})}{8(3-4t)} \end{aligned} \tag{3.33}$$

[9] If the two schools maximized the same objective functions, I would have obtained that the school i has a comparative advantage with respect to the private school.

The private school j gains a quality comparative advantage over the public school, and obtains the "business stealing effect": an increase of quality permits to "steal" the market share to the rival firm. This effect has a positive impact on the marginal profits obtained by the private school j.
At stage 3, the benefits obtained by the schools are realized:

$$\begin{aligned}\widehat{\pi}_i^* = &\left[\frac{(8-24t-5\overline{\theta}-12t\overline{\theta})}{8(3-4t)}\right]\left[\frac{t(16-8t+5\overline{\theta}+12t\overline{\theta})}{4(3-4t)}\right]-\frac{1}{2}\left(\frac{(32-56t+19\overline{\theta}-44t\overline{\theta})}{8(3-4t)}\right)^2+\\ &+\left[\frac{(16-8t+5\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right]\left[\frac{t(8-4t+7t\overline{\theta})}{(3-4t)}\right]-\frac{1}{2}\left(\frac{(32-16t+19\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right)^2+\\ &+(1+\overline{\theta})\left(\frac{64-184t+56t^2+38\overline{\theta}-103t\overline{\theta}-36t^2\overline{\theta}}{16(3-4t)}\right)+\left(\frac{64-176t+72t^2+38\overline{\theta}-93t\overline{\theta}-12t^2\overline{\theta}}{16(3-4t)}\right)\end{aligned} \tag{3.34}$$

$$\underline{\pi}_j^* = \left[\frac{(16-8t+5\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right]\left[\frac{t(8-4t+7t\overline{\theta})}{(3-4t)}\right]-\frac{1}{2}\left(\frac{(32-16t+19\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right)^2+\overline{v} \tag{3.35}$$

Even if only the agent i is motivated, the private school maintain his comparative advantage in terms of quality.

3.6.4 Characterization of the Equilibrium when Only the Private Teacher j is Motivated

Finally, when only the agent j is intrinsically motivated, i.e. $\theta_i = \underline{\theta} = 0$ and $\theta_j = \overline{\theta}_j > 0$, agents exert effort and the following levels of quality are determined:

$$\begin{aligned}\underline{q}_i^* &= \frac{(32-56t+13\overline{\theta}-36t\overline{\theta})}{8(3-4t)}\\ \widehat{q}_j^* &= \frac{(32-16t+13\overline{\theta}+20t\overline{\theta})}{8(3-4t)}\end{aligned} \tag{3.36}$$

The quality of the services offered by the public school is lower than the quality offered by the private school, i.e. $\underline{q}_i^* < \widehat{q}_j^*$. In addition, the intrinsic motivation of the teacher hiring by the public school has a positive impact on the levels of quality offered by the two schools, i.e. $\frac{\partial \underline{q}_i^*}{\partial \overline{\theta}} > 0$ and $\frac{\partial \widehat{q}_j^*}{\partial \overline{\theta}} > 0$. In addition, this impact of $\overline{\theta}$ on the levels of quality is higher in the private school than in the public one.

The principals pay the following wages to their agents:

$$\begin{aligned} \underline{\omega}_i^* &= \frac{1}{2}\left(\frac{(32-56t+13\overline{\theta}-36t\overline{\theta})}{8(3-4t)}\right)^2 \\ \widehat{\omega}_j^* &= \frac{1}{2}\left(\frac{(32-16t+13\overline{\theta}+20t\overline{\theta})}{8(3-4t)}\right)^2 - \overline{\theta}\left(\frac{64-176t+72t^2+26\overline{\theta}-11t\overline{\theta}-100t^2\overline{\theta}}{16(3-4t)}\right) \end{aligned} \tag{3.37}$$

In this case, the agents' intrinsic motivation has a positive impact on the wage given to the unmotivated teacher i. In contrast, the impact of $\overline{\theta}$ on the wage of the motivated teacher j depends on the value of t and $\overline{\theta}$. Furthermore, the wage of the public teacher can be higher than the wage of the private teacher if $t < 0.25$. If there is competition in the scholastic market, the public principal has to pay a high wage to the unmotivated teacher i to produce a larger output. The motivated teacher j provides a given level of effort even if he receives a low compensation for that.

The prices are given by:

$$\begin{aligned} \underline{p}_i^* &= \frac{t(16-8t+7\overline{\theta}-28t\overline{\theta})}{4(3-4t)} \\ \widehat{p}_j^* &= \frac{t(16-8t+5\overline{\theta}-12t\overline{\theta})}{2(3-4t)} \end{aligned} \tag{3.38}$$

A high agent i's degree of motivation has a positive effect on the price of the private school. A high $\overline{\theta}$ produces an improvement of the quality offered by the private school, which increases its price. This effect has a positive impact on the marginal profits obtained by the private school and a negative impact on the students' utility. Regarding the public school, the impact of $\overline{\theta}$ on its price is positive if $t < 0.25$. In this case, the public teacher receives a higher wage than the private teacher. Then, the public school has to increase the price to repay his agent. The demands will be equal to:

$$\begin{aligned} \underline{d}_i^* &= \frac{(8-24t-11\overline{\theta}-4t\overline{\theta})}{8(3-4t)} \\ \widehat{d}_j^* &= \frac{(16-8t+11\overline{\theta}+4t\overline{\theta})}{8(3-4t)} \end{aligned} \tag{3.39}$$

The private school j gains a quality comparative advantage over the public school, and obtains the "business stealing effect": an increase of quality permits to "steal" the market share to the rival firm. This effect has a positive impact on the marginal profits obtained by the private school j.

At stage 3, the profits are realized:

$$\underline{\pi}_i^* = \left[\frac{(8-24t-11\overline{\theta}-4t\overline{\theta})}{8(3-4t)}\right]\left[\frac{t(16-8t+7\overline{\theta}-28t\overline{\theta})}{4(3-4t)}\right]-\frac{1}{2}\left(\frac{(32-56t+13\overline{\theta}-36t\overline{\theta})}{8(3-4t)}\right)^2+$$
$$+\left[\frac{(16-8t+11\overline{\theta}+4t\overline{\theta})}{8(3-4t)}\right]\left[\frac{t(16-8t+5\overline{\theta}-12t\overline{\theta})}{2(3-4t)}\right]-\frac{1}{2}\left(\frac{(32-16t+13\overline{\theta}+20t\overline{\theta})}{8(3-4t)}\right)^2+$$
$$+\left(\frac{64-184t+56t^2+26\overline{\theta}-33t\overline{\theta}-108t^2\overline{\theta}}{16(3-4t)}\right)+(1+\overline{\theta})\left(\frac{64-176t+72t^2+26\overline{\theta}-11t\overline{\theta}-100t^2\overline{\theta}}{16(3-4t)}\right) \tag{3.40}$$

$$\widehat{\pi}_j^* = \left[\frac{(16-8t+5\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right]\left[\frac{t(8-4t+7t\overline{\theta})}{(3-4t)}\right]-\frac{1}{2}\left(\frac{(32-16t+19\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right)^2+$$
$$-\overline{\theta}\left(\frac{64-176t+72t^2+26\overline{\theta}-11t\overline{\theta}-100t^2\overline{\theta}}{16(3-4t)}\right)+\overline{v} \tag{3.41}$$

3.7 Appendix B

3.7.1 Proof lemma 3.1., 3.2. and 3.3.

In the first part of these lemmas, I compare the profits when both principals hire self-interested agents with the one in which only a principal hires a motivated agent. In the lemma 1, I show that $\pi_i^* < \widehat{\pi}_i^*$ and $\pi_j^* < \widehat{\pi}_j^*$ for any value of $\overline{\theta}$ and for a value of t small enough.
I start by considering the explicit expression for the public school:

$$\widehat{\pi}_i^* - \pi_i^* > 0 \text{ if}$$
$$\left[\frac{(8-24t-5\overline{\theta}-12t\overline{\theta})}{8(3-4t)}\right]\left[\frac{t(16-8t+5\overline{\theta}+12t\overline{\theta})}{4(3-4t)}\right]-\frac{1}{2}\left(\frac{(32-56t+19\overline{\theta}-44t\overline{\theta})}{8(3-4t)}\right)^2+$$
$$+\left[\frac{(16-8t+5\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right]\left[\frac{t(8-4t+7t\overline{\theta})}{(3-4t)}\right]-\frac{1}{2}\left(\frac{(32-16t+19\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right)^2+$$
$$+(1+\overline{\theta})\left(\frac{64-184t+56t^2+38\overline{\theta}-103t\overline{\theta}-36t^2\overline{\theta}}{16(3-4t)}\right)+\left(\frac{64-176t+72t^2+38\overline{\theta}-93t\overline{\theta}-12t^2\overline{\theta}}{16(3-4t)}\right)+$$
$$-\left[\left[\frac{(1-3t)}{(3-4t)}\right]\left[\frac{2t(2-t)}{(3-4t)}\right]-\frac{1}{2}\left(\frac{(4-7t)}{(3-4t)}\right)^2+\left[\frac{8-23t+7t^2}{2(3-4t)}\right]\right]+$$
$$-\left[\left[\frac{(2-t)}{(3-4t)}\right]\left[\frac{4t(2-t)}{(3-4t)}\right]-\frac{1}{2}\left(\frac{2(2-t)}{(3-4t)}\right)^2+\left[\frac{8-22t+9t^2}{2(3-4t)}\right]\right]>0 \tag{3.42}$$

It is not immediate to see that the inequality holds. The proof unfolds in two steps. First, I study the marginal effect of the degree of differentiation of the educational services t and of the agent's degree of motivation $\overline{\theta}$ on the difference between benefits. Secondly, I show that the above inequality holds also in a limit case and this completes the proof.
I begin by studying the effect of t on the difference in the benefits obtained by the public school. Differentiating equation (3.42) with respect to t, I obtain the following:

$$\frac{\partial(\widehat{\pi}_i^* - \pi_i^*)}{\partial t} < 0 \tag{3.43}$$

When t is high, the market is less competitive. This reduces the students' utility and, then, has a negative impact on the social welfare. In addition, the overall effect of t on the differential benefits is negative. This is because when the public teacher i is motivated, the degree of differentiation is relatively less important. In other words, this reduction of the social welfare is lower than in the case in which both teachers are selfish.
I also analyze the marginal effect of the agent's intrinsic motivation $\overline{\theta}$ on the benefits. Deriving equation (3.42) with respect to $\overline{\theta}$, I obtain the following:

$$\frac{\partial(\widehat{\pi}_i^* - \pi_i^*)}{\partial \overline{\theta}} > 0 \text{ if } t < 0.165 \tag{3.44}$$

A higher motivation influences positively the social welfare when the public school is the only one to hire the motivated agent but only if t is small enough. In contrast, it has no effect on the profits of the firm when both principals hire self-interested agents. Then, the sign of this derivative is positive for $t < 0.165$ and negative in other case.
To consider a limit case, I take the maximum value for t in the interval in which it affects negatively the differential benefits and the minimum value for $\overline{\theta}$, since their impact on the differential benefits is positive. If inequality (3.42) holds in this limit case, it will be always satisfied for other values of these parameters in the interval $t \in (0; 0.165)$. I set $t = 0.164$ and $\overline{\theta} = 0.005$ and I obtain that $\pi_i^* = 0.4214 < 0.4218 = \widehat{\pi}_i^*$.

Now, I consider the explicit expression for the private school:

$$\widehat{\pi}_j^* - \pi_j^* > 0 \text{ if}$$

$$\left[\frac{(16-8t+5\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right]\left[\frac{t(8-4t+7t\overline{\theta})}{(3-4t)}\right] - \frac{1}{2}\left(\frac{(32-16t+19\overline{\theta}+12t\overline{\theta})}{8(3-4t)}\right)^2 +$$

$$-\overline{\theta}\left(\frac{64-176t+72t^2+26\overline{\theta}-11t\overline{\theta}-100t^2\overline{\theta}}{16(3-4t)}\right) - \left[\left[\frac{(2-t)}{(3-4t)}\right]\left[\frac{4t(2-t)}{(3-4t)}\right] - \frac{1}{2}\left(\frac{2(2-t)}{(3-4t)}\right)^2\right] > 0 \tag{3.45}$$

I study the effect of t on the difference in the profits obtained by the private school. Differentiating equation (3.45) with respect to t, I obtain the following:

$$\frac{\partial(\widehat{\pi}_j^* - \pi_j^*)}{\partial t} < 0 \tag{3.46}$$

The overall effect of the degree of differentiation of the product on the differential profits is negative. This is because t has a positive impact on the profits of the firm when both principals hire self-interested teachers but a negative impact when only a principal hires a motivated teacher. When t increases the comparative advantage by hiring a motivated agent is reduced.

I also analyze the marginal effect of the agent's intrinsic motivation $\overline{\theta}$ on the private school's profits. Deriving equation (3.45) with respect to $\overline{\theta}$, I obtain the following:

$$\frac{\partial(\widehat{\pi}_j^* - \pi_j^*)}{\partial\overline{\theta}} > 0 \text{ if } t < 0.2 \tag{3.47}$$

A higher motivation influences positively the profits when the private school is the only one to hire the motivated agent but only if t is small enough. This is because the private school obtains a comparative advantage with respect to the rival school in terms of demand and price. In contrast, it has no effect on the profits of the firm when both principals hire self-interested agents. Then, the sign of this derivative is positive for $t < 0.2$ and negative in other case.

To consider a limit case, I take the maximum value for t in the interval in which it affects negatively the differential benefits and the minimum value for $\overline{\theta}$, since their impact on the differential benefits is positive. If inequality (3.45) holds in this limit case, it will be always satisfied for other values of these parameters in the interval $t \in (0;0.2)$. I set $t = 0.19$ and $\overline{\theta} = 0.005$ and I obtain that $\pi_j^* = -0.81 + \overline{v} < 0.809 + \overline{v} = \widehat{\pi}_j^*$.

When $t < 0.165$, both schools obtain higher benefits by hiring a motivated agent when the other school hires a selfish agent. When $0.165 < t < 0.2$, only the private school obtains higher profits by hiring a motivated teacher when the public school hires a selfish one. When $t > 0.2$, both schools obtain higher profits by hiring selfish individuals.

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