PhD thesis:"**Old and New Problem in Continuum Structural Materials**"
by
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SUMMARY

The PhD thesis in Mathematics titled "Old and New Problems in continuum structural masterials " is divided into two parts. In Part I (Chapter I and II relative to the " Old Problems") is studied in particular the classic behavior of materials from the mechanical point of view. In Chapter I were determined tensors of stress and strain in a solid having the shape of a hollow cylindrical , that is not simply connected , when it does act on it a displacement field able to induce all six elementary distortions of Volterra in the case where the material constituting the solid is homogeneous , linearly elastic and transversely isotropic . Unlike Volterra considers an isotropic body, characterized by two elastic constants : (E and G) , presents in the constitutive relations of the material become five (A, C, F, L and N). The result obtained is that the functions of displacement [u_1 (x,y,z), u_2 (x,y,z), u_3 (x,y,z)] . meet the indefinite equations of elastic written using the five elastic constants , but as Volterra , they do not cancel the load over the entire border of the hollow cylinder . In other words, do not give rise to a real distortion in that the action of the shift functions do not carry the cylinder from a natural configuration to a spontaneous through an isothermal transformation in which the load boundary is null but only self- balanced . In Chapter II has been dealt with the analysis of the load acting on the basis only of the cylinder relative to the cable . VI distortion (third component *r* of the vector **k** , that is generated at a rigid rotation of the faces around the z axis of the Cartesian reference assumed coincident with the axis of the cylinder) using the theory of the beam of Saint Venant . the load acting on the bases of the cylinder can be affected in the same way that agent on the basics of a prismatic beam simply connected having a height and length respectively equal to the thickness and the finished height of the hollow cylinder . It was concluded that with respect to the cylindrical surface , lying in the thickness of the hollow cylinder, said " neutral surface", because on its points σ_z is zero, the cylinder is divided into two parts . In particular , after calculating the Resultant and the Resultant Moment with respect to the centroid of each of the two parts it has been observed that the load is equivalent on the inner zone to a conpressive-bending load and for the outdoor zone to a tensile- bending load when $r<0$ with suppression of the matter. Conversely, for $r> 0$, with the inclusion of matter, the inner zone it is a tensile-bending load, while the outdoor zone it is a conpressive-bending load. In Part II (III and IV chapter relating to "New Problems") emerging issues relating to new mathematical models of the phenomena related to the study of phase transitions included as part of the Continuum Thermomechanics . In particular , in Chapter III has been addressed on a macroscopic scale with the technique of the phase transition , due to Ginzburg and Landau, the transition of a hard material to the magnetization from the paramagnetic phase to the ferromagnetic .After observing that this transformation corresponds to a phase transition of II species for whichm while the state variables vary in a continuous manner, it has a discontinuity of some symmetry of the body, such as the crystallographic symmetry . of the body. Introduced the order parameter as a descriptor of the internal structure of the material if it is determined the evolution through a budget bill thus obtaining the Ginzburg-Landau equation .Subsequently, also using Maxwell's equations, it is verified the thermodynamic consistency of the model. For this purpose it is determined the expression assumed in this case by the Law of Thermodynamics where there are two functional of the order parameter F (φ) and G (φ) polynomials respectively the fourth and the second order that characterize the transition to examination . Using the law of heat balance where you

used the classical Fourier's law is obtained by the heat equation .Through this equation is then determined by the expression of the free energy with which it is shown that the model has an expression of the entropy able to verify the II Law of Thermodynamics written using the Clausius-Duhem inequality . In the end , reducing the one-dimensional case in a point of the magnet and numerically integrating the differential system which is obtained by coupling scalar equation G.-L. the constitutive equation . Attach the initial conditions it has been pllotted classic hysteresis cycles (in the plane B, H) with reference to a temperature of less than and greater than the Curie temperature . For temperatures lower than Curie temperature you observe the phase transition from paramagnetic to ferromagnetic material . In Chapter IV was ptoposto a mathematical model to describe the behavior of these alloys Shape Memory . The study, which is a phase transition of the species , examines the transition from a martensitic phase to the austenitic alloys in single crystal such as AuZn , and begins with the determination of the field equations for a 3-D model . In this case, the different variants of martensite are described by the product of the strain tensor for the order parameter φ . This product defines the crystallographic structure of the material. In particular, the austenite phase is characterized by $\varphi = 0$ while the martensitic from $\varphi = 1$ .Following the same procedure followed in the third chapter determines the equation G.-L. using two new functionals F (φ) and G (φ) both of the fourth order for the specific case under examination. Always up for a 3-D model , introduced in the equation of motion and the constitutive law , it is determined by the law of thermodynamics , the internal energy of the alloy that allows you to verify subsequently the thermodynamic consistency of the model through the inequality of Clusius - Duhem entropy. He then examined the 1-D case for which we proceeded to numerical integration of the system is achieved by coupling the scalar constitutive equation G.-L. equation . Introduced in the equations of the new system scalar function of the sixth order, more suitable for the 1-D case and associating appropriate boundary conditions , we derived the gift of stress-strain diagrams (σ ,ε) , for $\theta < \theta_c$ show the typical behavior of a SMA fatigue failure when subjected to a repeated cycle of the load, while , when $\theta > \theta_c$, the diagram shows the phase transition pseudoelastic from a martensitic phase to an austenitic and vice versa.