University of Salerno

Department of Mathematics

Ph.D. in Mathematics, Physics and Applications

Numerical Modeling of Stochastic Differential Problems with Applications

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Abstract

The interest in the study of stochastic differential equations has grown considerably in recent years. The main reason for this interest is that stochastic differential equations are a potential tool for modeling evolutionary problems, especially when the dynamics are affected by stochastic perturbations. Of particular importance is the study of numerics related to these problems since, in the literature, there are a few stochastic differential equations whose solution is known explicitly. Therefore, the spirit of this thesis is to analyze numerical methods for stochastic differential equations and how these can be used for the description of real-life problems. After an introduction to stochastic differential problems and some models in which these are used, we will then move on to some recalls of numerical methods, known in the literature. Subsequently, the focus will be on new research results obtained, divided into three essential parts. In the first part, based on the well-known idea of collocation for Volterra Integral Equations, we obtained continuous numerical methods, which allow us to know the solution not only at the grid points of the numerical discretization but throughout the entire integration interval. Research on this front continued by pointing out how these continuous extensions can be applied to obtain a good estimate of the local truncation error. In fact, as is also known in the deterministic context, this is a first building-block for the development of a variable step-size algorithm, useful especially in the integration of stiff problems. The second part, instead, focused on geometric numerical integration for stochastic Hamiltonian problems. Unlike deterministic Hamiltonian problems, where energy is conserved over time, stochastic Hamiltonian problems of Itô type and driven by the additive Wiener process satisfy the trace equation, i.e. the expected value of the Hamiltonian function grows linearly over time. Interest in this study derives from some results on stochastic Runge-Kutta methods developed by K. Burrage et al. in 2012. These methods, in fact, have a significant error that increases with increasing stochastic noise. Therefore, through a perturbative analysis, the reason for this behaviour was analyzed, concluding that the preservation of the main features of stochastic Hamiltonian problems does not occur directly for any discretization of time. The research was then extended to stochastic Hamiltonian problems with multiplicative noise, first obtaining a characterization of the behaviour of the mean value of the Hamiltonian and then showing that first-order approximations to such systems are unable to maintain such behaviour. In the last part of this thesis, two different models were analyzed in which stochastic differential equations (including stochastic partial differential equations) occur. Specifically, first, the stochastic FitzHugh-Nagumo system for signal propagation in nerve cells was analyzed, in which the voltage variable is the solution of a one-dimensional partial derivative differential equation of a parabolic type with a cubic nonlinearity driven by additive space-time white noise. Splitting methods for temporal integration will then be developed for that model, showing that such schemes admit strong convergence order 1/4. Next, the analysis shifted to modeling the spread out of fake news through the stochastic SIR model, which is widely used in the epidemiological context for the spread of an epidemic. In particular, interest was placed on the stiffness property of the differential problem by pointing out that in a given population, the more stiff the problem, the faster the transit of fake news. Numerical evidence, which will demonstrate the effectiveness of the theoretical results, will be provided in the development of the thesis.