Orbital motion, periastron advance and galaxy rotation curves beyond General Relativity

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Abstract

The Extended Theories of Gravity (ETG) have become one of the most investigated theoretical proposals among the alternative explanations for the observed flatness of galaxy rotation curves, related to the dark matter problem, as well as for the accelerated expansion of the universe, related to the dark energy problem. The reason lies in the fact that ETG's can provide predictions consistent with the observational surveys without implicating invisible matter. In this framework, these phenomena are explained as a physical manifestation of extra-curvature terms of the geometry of the Universe. In the first part, we focus on this class of theories which are a curvature-based extension of GR. Higher order scalar curvature invariants are included in the Einstein-Hilbert action giving rise to the Higher Order Theories and corresponding field equations. For the extension of GR, we consider the Scalar-Tensor-Fourth-Order Gravity (STFOG) in metric formalism as a representative general class for the ETG, obtained from combination of Fourth Order Gravity plus a coupled scalar field. The NonCommutative Spectral Gravity is a special case of STFOG. Other Higher Order Theories, like the $f(R)$-gravity models, are sub-classes of it.

In this scenario to analyse the orbital motion of interacting objects constituting astrophysical gravitating systems, like the Solar System or galaxies, is a very important issue for making new predictions and testing theories. We discuss the fundamentals of the physical regime given by the Weak Field limit, Newtonian and Post-Newtonian limits, and their corresponding expansions as a mathematical procedure to solve the field equations in STFOG. This makes it possible to deal with the problems of motion for a system of many particles and reproduce many physical configurations. We solve the linearized field equations of STFOG stemming from the weak field limit, and this is done in the Standard Post-Newtonian gauge, which is the suitable choice for the purpose. Then we find the space-time metric and the potentials connected to each metric component that give rise to the gravitational field. Their behaviour presents a modification to the Newtonian potential induced by the Yukawa-like potential terms (5th force) of the type $V(r) = \alpha\frac{e^{-\beta r}}{r}$. Finally, in the context of the STFOG, we determine the relativistic Lagrangian leading to the equations of orbital motion for a system of N-body and involving the Post-Newtonian fields. This allows

to find out the equations governing the dynamics of a generic N-body system, like those in the Solar System, binary systems (when $N = 2$) or possibly the S-stars cluster around Sagittarius A*, thus providing a theoretical reference for Relativistic Celestial Mechanics beyond General Relativity and the possibility to study realistic astrophysical models and gravitational tests.

In the second part, we expose the problem of anomalistic precession and deal with the analysis of the periastron shift. We consider the Adkins & MacDonell integrals and making use of the data coming from the precession of planets, we deduce constraints on the parameters of the STFOG, therefore also of Non-Commutative Spectral Gravity (NCSG) (as particular case), including a study for the Quintessence Field (deformation of the Schwarzschild geometry induced by a dark energy) related to a power-law potential. We show that the periastron shift of planets allows us to improve the bounds on the range of interaction β by several orders of magnitude. Then we develop a new resolution method for the determination of the periastron advance by relying on the epicyclic perturbation, which includes also the Post-Newtonian contributions and can be applied to theories beyond GR like the ETG, or models within, without the necessity of numerical integration. Using it, we obtain the final results and then deduce the full analytic expressions for the advance relative to the examined ETG. We carry out the preceding analysis once more, and further improvements on the bounds are achieved.

In the last part, by resorting to the Newtonian limit, we provide the theoretical galaxy rotation curves in the context of the $f(R)$-theory, the more general STFOG and the a NonCommutative Spectral Gravity. Therefore, the first analysis of galaxy rotation curves in NCSG is conducted. Through the parametric fits with observed data, we derive direct predictions on the physical parameters (total mass and mass-to-light ratio) for an unexplored sample of spiral galaxies of the THINGS catalogue. Good reproductions are obtained for these theories as well as numerical predictions on the physical parameters characterizing a galaxy. The predictions are directly comparable with the observations. We compare the numerical outcomes for the metric $f(R)$-theory with those of the Palatini formalism and, in the end, we make a comparison of the results relative to the examined ETG with the observed astronomical estimations.