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FOR OPTIMISTIC MANAGERS

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Abstract. We study an employment contract between an (endogenously) optimistic manager and realistic investors. The manager faces a trade-off between ensuring that effort reflects accurate news and savoring emotionally beneficial good news. Investors and manager agree on optimal recollection when the weight the manager attaches to anticipatory utility is small. For intermediate values investors bear an extra-cost to make the manager recall bad news. For large weights investors renounce inducing signal recollection. We extend the analysis to the case in which anticipatory utility is the manager’s private information and derive testable predictions on the relationship between personality traits, managerial compensation and recruitment policies.

Keywords: Over-optimism, managerial compensation, anticipatory utility.

JEL classifications: D82.

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1. Introduction

It is widely believed that over-optimism is a very common character trait of human beings. It is even more common among businessmen (De Meza and Southey, 1996). For instance, as is suggested by Roll (1986), over-optimism may explain an excess of merger activity. Overconfident CEOs overestimate their ability to generate returns, and so overpay for target companies and effect value-destroying mergers. Malmendier and Tate (2005, 2008) find evidence of CEOs’ over-estimation of their firms’ future performance in their holding of stock options until the expiration date.† For Bénabou (2009) the fact that wishful thinking shows up not only in words but also in deeds, is the proof that reality denial is not, or at least not only, a result of a design to deceive investors.

This paper studies, within a moral hazard framework, an employment contract between a manager with anticipatory emotions and investors who respond strategically to those emotions. Managers with anticipatory emotions have higher current utility if they are optimistic.

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†Also Cooper et al. (1988) find that entrepreneurs see their own chances for success higher than that of their peers.
about the future. But optimism affects decisions, because distorted beliefs distort actions, and exacerbates incentive problems. We thus study how the need to control for both optimism and moral hazard affects the design of managers’ incentives.

Our model has a risk-neutral investor hiring a risk-neutral manager for a project. When the investor offers the contract, the parties are symmetrically informed. If the manager accepts, he will choose a level of effort that affects the project’s probability of success. After signing, but before choosing his unobservable effort, the manager receives a private signal about the profitability of the task. A good signal implies a high return in case of success, a bad signal only intermediate return. Finally, in case failure, there is a low return regardless of type of signal. If the signal is informative about the return from effort, the manager would benefit from having accurate news. However, since he derives utility from the anticipation of his final payoff, the suppression of a bad signal may induce a positive interim emotional effect. We assume that investors cannot observe the agent’s choice, so to induce him to choose the right action, they make compensation contingent on project revenues. More specifically, parties can write a complete contract specifying the rewards contingent on the various outcomes, the effort levels to be exerted contingent on the signals, and the probability that bad news will be remembered accurately. Does the optimal contract always ensure complete recall?

We show that if the manager’s psychological trait – measured by a parameter weighting the anticipatory utility – is sufficiently low, there is no conflict of interest between investors’ and manager’s desired recall. There is a conflict for intermediate values of this parameter, and investors choose to bear the extra cost necessary to have the manager recall the bad signal. Finally, for a sufficiently high weight on anticipatory utility, investors become indifferent between inducing signal recollection and not, and the optimal contract is characterized by a pooling equilibrium reminiscent of adverse selection models.

Why does the optimal contract look like this? Informed managers face a trade-off between ensuring that the level of effort they choose reflects accurate news and savoring emotionally gratifying good news. However, the manager’s preferred level of memory may differ from the investors’. As a result, in writing the contract, investors may want to affect this dimension of the manager’s choice. If in the manager’s total utility the weight of emotions is sufficiently small, accurate news becomes a priority for the manager too, and there is no conflict over information recollection, and a contract can attain the optimal recall at no extra cost. For larger weights of anticipatory utility, the manager’s trade-off tilts away from accurate news towards good news, so that enticing proper information recollection becomes costly. For intermediate values of the parameter, investors choose to move the manager’s trade-off towards accuracy by making it costly, when the true signal is bad, to recall a good signal. This is done by increasing the cost to the manager when he exerts the effort expected for the good rather than the bad signal. But if the weight on emotions instead is sufficiently great, the optimal contract calls for a pooling equilibrium in which the manager exerts the same level of effort and receives the same payments regardless of signal type. Intuitively, when the weight on anticipatory utility is large, the manager will recall the signal accurately only if he does not anticipate a lower payment when the signal is bad.

The common practice of psychological testing for recruitment – including the screening

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2 No extra cost with respect to the resources needed to solve the moral hazard problem.  
3 There also exists an outcome-equivalent equilibrium where investors prefer not to elicit information recollection, the manager never recalls a bad signal, and the level of effort is the same as in the pooling equilibrium.
of emotional aspects – makes one wonder what role play in managerial compensation. The analysis so far implicitly relies on the assumption that investors know what weight the manager attaches to anticipatory utility. But even if psychological testing makes the perfect information assumption an interesting benchmark, we extend the analysis to the opposite extreme, assuming that this parameter is the manager’s private information. In this case, we find that the optimal contract will have a threshold weight: all managers below it, recall the bad signal and above it, none recollect it. Note that with private information the set of managers who will recall the bad signal is larger than in the perfect information second-best benchmark and also includes some managers who, due to the conflict of interest, impose an extra recollection cost on investor.

In addition to the foregoing our analysis has also produced testable predictions on the role played by the managers’ personality traits on hiring and compensation policies. We show that a CEO’s “realism” is always valuable for investors, but more so in less risky sectors. This rather counterintuitive result is accounted for in our setup by the fact that monetary incentives serve the dual scope of eliciting effort and mitigating optimism. Thus, high-risk firms that need to offer high-powered contracts to alleviate moral hazard may care less about the manager’s optimism, as this is already controlled for by the incentive contract; while lower-risk firms, despite the attenuated moral hazard, may be faced with the problem of overoptimism. Since this is costly, they prefer more stable managers.

The paper is organized as follows. Section 2 provides an overview of the economic and psychological literature, in addition to some motivating evidence on the impact of personality traits on hiring and compensation policies. Section 3 presents the model. Section 4 studies the manager’s optimal choices of effort and awareness, and Section 5 sets out the results concerning the conflict between investors’ and manager’s optimal recall. Section 6 characterizes the optimal contract and identifies the type of the manager that investors would choose. Section 7 extends the analysis to the case in which the relevant character trait is the manager’s private information. Section 8 concludes. Proofs are in the Appendix.

2. Related literature and motivating evidence

The paper is related to three strands in the literature that link economics and psychology. One deals with cognitive dissonance, endogenous self-confidence and anticipatory feelings, the second with the role of psychological traits in job performance and earnings, and the third with managerial overoptimism and/or overconfidence. The literature (Heaton, 2002; Goel and Thakor, 2008, among others) defines overoptimism as an overestimation of the probability of favorable future events, and overconfidence as an underestimation of risk. Thus, optimism is modeled as the overestimate of a mean and overconfidence as the underestimate of a variance.

Forward-looking agents care about expected utility flows and may select their beliefs so as to enjoy the greatest comfort or happiness (cognitive dissonance). The channel through which belief management and self-deception operate is imperfect memory. For a person suffering from time-inconsistency (e.g., hyperbolic discounting or anticipatory utility), the current self is interested in the self-confidence of future selves. It is in this context that we model overoptimism as the equilibrium of an intrapersonal game of belief management.

The second strand suggests the importance of personality traits in CEO selection and compensation. Based on this literature, we aim to found managerial optimism (modeled through belief management) directly upon the psychological traits that may give rise to it.

In an extensive review of the wage determination literature, Bowles, Gintis and Osborne (2001) show that the standard demographic and human capital variables explain little of the variance in earning and argue that there may be other factors in performance, namely individual differences in behavioral traits not captured by the usual measures of schooling, professional experience and cognitive performance. The common use of personality tests by employers in recruitment and personnel assessment confirms the importance of character traits to productivity. A number of studies relating personality to performance have attempted to identify the personality dimensions that are good predictors of success in various occupations. The broadest consensus endorses the so-called five-factor model (FFM henceforth). According to this model, personalities are made up of five factors: extraversion, emotional stability, agreeableness, conscientiousness, and openness to experience. Among these, conscientiousness and emotional stability seem to have the most explanatory power (Barrick and Mount, 1991; Salgado, 1997, using meta-analyses). Conscientiousness measures how dependable (trustworthy and reliable), organized, self-disciplined and efficient a person is, rather than unreliable, disorganized, self-indulgent, engaging in fantasy, daydreamer. Conscientiousness and emotional stability are related to our measure of anticipatory utility. People who attach a greater importance to future utility are more emotional, anxious, daydreamer, and so for self-assurance are more prone to distort their assessments of the likelihood of future events (discard bad news). This leads them to expect good outcomes too often, or more often than they ought to, and thus to overoptimism (in the way defined above).

Several studies have tested the validity of the FFM for predicting job performance. Focusing on financial services managers, Salgado and Rumbo (1997) find that both conscientiousness and emotional stability are correlated with performance, in line with previous studies using the FFM (Barrick and Mount, 1991; Salgado, 1997). Using a panel of Australian households containing information on individual’s characteristics, including occupational choices and personality measures conforming with the FFM, Ham, Junankar and Wells (2009) find that personality traits affect occupational choices, and in particular that conscientiousness increases the probability of an individual being in a management position. Kaplan, Klebanov, and Sorensen (2008) study the role that various measures of CEO talent or skills related to the FFM play for the firms’ success and find that conscientiousness is the trait with the most explanatory power.

5 According to McCrae, Costa and Busch (1986), conscientious individuals are less likely to engage in fantasy or daydream.

6 In studying whether over-optimism may explain an excess of merger activity, Malmendier and Tate (2008) use, in addition to measures of optimism based on CEOs’ personal portfolio decisions, a press-based indicator constructed by retrieving all leading business publications articles that characterize sample CEOs as “Confident” (confident, optimistic) versus “Cautious” (cautious, reliable, practical, conservative, frugal, steady, or negating one of the “Confident” terms). Interestingly, the terms negating optimism (the traits described by “Cautious”) resemble closely those described by conscientiousness.

7 The subjects of the study are middle managers in a Spanish financial services organization whose duties are to examine, evaluate and process loan applications, and prepare, file and maintain records of financial transactions.
to it should also help to base future empirical work directly on managers’ personality traits rather than on their likely consequences.

The paper also relates to the literature on the effect that managers with behavioral biases have on corporate decision making. The first paper on this issue is Heaton (2002), who shows how the bias of overoptimism leads managers to believe their firms are undervalued, inducing a preference for internal finance. This lends support to a pecking-order theory of capital structure. Malmendier and Tate (2005) test Heaton’s model finding that investment is more sensitive to internal cash flows in firms with optimistic CEOs.

Hackbart (2009) has studied the effects of both overoptimism and overconfidence on firms’ capital structure and finds that biased managers choose higher debt than unbiased ones, because they perceive their firm as less exposed to financial distress. In a similar vein, Landier and Thesmar (2009) examine the financial contract between rational investors and optimistic entrepreneurs. Because optimists believe good states to be more likely, they may choose short-term debt because it transfers payments and control to the investor in states that they perceive as less likely to occur. Realistic entrepreneurs instead prefer less risky long-term debt. Testing the model on a data set of French entrepreneurs, the authors show that short-term debt is positively correlated with an ex-post measure of optimistic expectations.

Goel and Thakor (2008) study the relationship between managerial overconfidence and corporate governance. In a tournament setting, they show that overconfident managers are more likely to be promoted to CEOs than perfectly rational ones because they perceive less risk and so take more chances. Moreover, a moderate degree of overconfidence is beneficial, mitigating the problem of underinvestment that plagues strictly rational managers. The benefits of overconfidence and moderate overoptimism are also highlighted by Gervais, Heaton, and Odean (2003), who show that both these traits help offset the excessive prudence induced by risk aversion, inducing managers to take investment decisions less hesitantly. This has implications for the design of compensation contracts and calls for less high-powered incentive schemes. Like Gervais et al. (2003), we focus on compensation, but our model of endogenous optimism with moral hazard produces very different results, especially as regards the optimality of overoptimism and the incentive power of the optimal contract; we find that overoptimism may call for more rather than less high-powered incentive schemes.

Finally, in a setting in which the agent undertakes a certain task only if he has sufficient confidence in his ability to succeed, Bénabou and Tirole (2003) inquire into how an informed principal should reward the agent in the awareness that rewards can undermine intrinsic motivation. Similarly, in Fang and Moscarini (2005), wage contracts both provide incentives and affect work morale, by revealing the firm’s private information about workers’ skills. Unlike these two papers, our analysis posits that both parties are ex-ante symmetrically informed.

3. The model

3.1 Players and environment

Consider a setting in which a risk-neutral investor hires a risk-neutral manager for a project that has three possible outcomes, \( \tilde{v} \in \{v_0, v_L, v_H\} \), with \( v_0 < v_L < v_H \). In carrying
out his task, the manager chooses a level of effort \( a \) that affects the probability of success, with \( a \in [0, 1] \). The effort has disutility \( c(a) \), with \( c(0) = 0 \), \( c'(a) \geq 0 \), \( c''(a) > 0 \) and \( c'''(a) \geq 0 \). In order to ensure interior solutions, we also assume that \( c'(0) = 0 \) and \( c'(1) \geq v_H \). After signing the contract but before choosing the effort level, the manager receives a private signal \( \sigma \in \{L, H\} \) correlated with the project’s return \( \tilde{v} \). The probability of a good signal \( H \) is \( q \), that of a bad signal \( L \) is \( (1 - q) \), with \( q \in [0, 1] \). In our setting, good (bad) news means that the outcome is \( v_H \) (or \( v_L \)) with probability \( a \) and \( 1 - a \), respectively. In other words, we assume for simplicity that the signal is perfectly correlated with the return \( \tilde{v} \), implying that

\[
Pr(\tilde{v} = v_0 | \sigma = L) = Pr(\tilde{v} = v_0 | \sigma = H) = 1 - a,
\]

and

\[
Pr(\tilde{v} = v_L | \sigma = H) = Pr(\tilde{v} = v_H | \sigma = L) = 0.
\]

Then, ex-ante \( v_0 \), \( v_L \) and \( v_H \) occur with probabilities \( 1 - a \), \( (1 - q) \cdot a \) and \( qa \), respectively. From now on, for the sake of simplicity and without loss of generality, we normalize \( v_0 = 0 \).

Given that the signal gives information on the return to effort, in choosing its level the manager would benefit from accurate news. But if the manager derives utility from the anticipation of his final payoff, the suppression of a bad signal may induce a positive emotional effect. This is modeled assuming that total utility is a convex combination of the actual physical outcome (at time 3) and the anticipation (at time 2), with weights \( 1 - s \) and \( s \), respectively, where \( s \) is the realization of a random variable distributed over the compact support \( S \equiv [0, 1] \) according to the twice continuously differentiable and atomless cumulative distribution function \( F(s) \), with density \( f(s) \). We generally assume that the parameter \( s \) is observed by investors.\(^8\)

We assume that at the time of the effort decision a bad signal can be forgotten (voluntarily repressed). Suppressing a bad signal determines an overestimation of the mean, and is in line with the definition of overoptimism given in the literature. We then denote by \( \hat{\sigma} \in \{L, H\} \) the recollection, at that time of the news \( \sigma \) and by \( \lambda \in [0, 1] \) the probability that bad news will be remembered accurately, that is, \( \lambda \equiv Pr(\hat{\sigma} = L | \sigma = L) \). We assume that the manager can costlessly increase or decrease the probability of recollection.\(^9\) Finally, we denote by “Manager 1” the manager’s self at time 1 and by “Manager 2” the manager’s self at time 2.

Investors cannot observe the manager’s action directly. Hence, to induce the “right” action, they can only offer the manager rewards contingent on observable, verifiable project revenues. We denote by \( C \equiv \{w_0, w_L, w_H\} \) the contract that investors offer the manager, where \( w_i \) is the reward corresponding to \( v = v_i \), for any \( i = 0, L, H \). We assume that the manager has limited liability, so that \( w_i \geq 0 \) for any \( i \).\(^10\) Finally, we maintain the standard assumption of individuals as rational Bayesian information processors.\(^11\)

The precise sequence of events unfolds as follows:

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\(^8\)This assumption is relaxed in section 7.

\(^9\)Assuming costly recollection would not change our results qualitatively.

\(^10\)All our results generalize to the case of unlimited liability.

\(^11\)By modeling agents as Bayesian, we are treating them as fairly sophisticated. A departure from this assumption though, while seemingly realistic, would be arbitrary, as there may be several ways of being less sophisticated.
$t=0$: Investors offer a contract $C$ to the manager to run a project.

$t=1$: If manager 1 refuses the contract, the game ends. If he accepts, he observes a private signal $\sigma$ and, when the signal is bad ($\sigma = L$), chooses the probability $\lambda$ that bad news will be remembered accurately.

$t=2$: Manager 2 observes $\hat{\sigma}$, updates his beliefs on the outcome $v$ accordingly, selects the effort level $a$ and enjoys the anticipatory utility experienced from thinking about his future prospects.

$t=3$: The project payoff is realized and the payment is executed.

3.2 Equilibrium

To characterize the equilibrium of this game, we first identify the manager’s optimal effort choice $a$, given his beliefs about $\sigma$ and a contract $C$. Then we describe the perfect Bayesian equilibrium of the memory game for any given contract: i) for any realized $\sigma$, manager 1 chooses his message $\hat{\sigma}$ to maximize his expected utility, correctly anticipating the inferences that manager 2 will draw from $\hat{\sigma}$, and the action that he will choose; ii) manager 2 forms his beliefs using Bayes’ rule to infer the meaning of manager 1’s message, knowing his strategy. Finally, we use the manager’s optimal effort choice rule and the equilibrium in the memory game to compute the investors’ contract offer.

4. Effort choice and the memory game

To study the impact of incentives on the manager’s optimal probability of recall and level of effort, we consider the subgame starting in $t = 1$. We take the compensation contract $C \equiv \{w_0, w_L, w_H\}$ as given and analyze the conditions under which the manager prefers perfect recollection of his private signal on the outcome $v$. We study the equilibrium of the memory game in three stages.

4.1 Manager 2’s choice of effort

In the second period, the manager chooses the level of effort that maximizes his intertemporal expected utility. Denoting by $E_2$ the expectation at $t = 2$, the intertemporal utility perceived by the manager, given memory $\hat{\sigma}$ and the compensation contract $C$, is

$$E_2[U_3] = -c(a) + E_2[u(C, a)|\hat{\sigma}],$$

(1)

where $c(a)$ is the disutility of effort and $E_2[u(C, a)|\hat{\sigma}]$ is the sum of the manager’s material payoff, $(1-s)E_2[u(C, a)|\hat{\sigma}]$, and the anticipatory utility experienced by savoring the future material payoff, $sE_2[u(C, a)|\hat{\sigma}]$. When $\hat{\sigma} = L$, the manager is sure that $\sigma = L$, and the expected payoff simplifies to

$$E_2[u(C, a)|L] = aw_L + (1-a)w_0.$$

But when $\hat{\sigma} = H$, the manager is unsure whether he actually received a good signal or instead received a bad signal and censored it. The expected payoff is

$$E_2[u(C, a)|H] = a(rw_H + (1-r)w_L) + (1-a)w_0.$$
where \( r \) is the ex-post probability that the manager attaches to the state \( H \). We denote by \( a(r, C) \equiv \{ a_L, a_H(r) \} \) the manager’s optimal strategy at \( t = 2 \) for any \( r \in [0, 1] \) and for any \( C \in R^2_+ \), where \( a_L \) and \( a_H(r) \) are the levels of effort maximizing the managers’ expected utility when signals \( L \) and \( H \), respectively, are recollected.

4.2 Manager 2’s inference problem

Before choosing the level of effort, the manager observes a recollection of the signal on the distribution of the project’s revenue \( \hat{\sigma} \), which depends on the news received in the previous period \( \sigma \), and on how his previous self processed them, i.e., on \( \lambda \). The manager is aware that there are incentives to manipulate memory when the true state is \( L \), so when he has a memory \( \hat{\sigma} = H \), he has to assess its credibility. If he thinks the bad signal is recalled with probability \( \lambda \), he uses Bayes’ rule to compute the likelihood of an accurate signal recollection as

\[
r(\lambda) \equiv \Pr(\sigma = H | \hat{\sigma} = H, \lambda) = \frac{q}{q + (1 - q)(1 - \lambda)} \in [q, 1].
\]  

(2)

4.3 Manager’s recollection choice

By assumption, when the true signal is good, \( \sigma = H \), manager 1’s recollection will always be accurate, that is, \( \hat{\sigma} = H \). But when it is bad, \( \sigma = L \), he chooses the probability \( \lambda \) that bad news will be remembered accurately so as to maximize the expected utility of his payoff at time \( t = 1 \), that is

\[
\max_{\lambda \in [0,1]} \{ E_1 [U(C, a(r, C), \lambda)] = E_1[-c(a(r, C)) + sE_2 [u(C, a(r, C))] + (1 - s) E_1 [u(C, a(r, C))]] \},
\]  

(3)

where \( E_1 \) denotes expectations at \( t = 1 \) for given \( r \).

Moreover, Bayesian rationality implies that manager 2 knows that manager 1 is choosing the recollection strategy according to (3), and thus uses this optimal \( \lambda \) in his inference problem. A Bayesian Equilibrium of the memory game is a pair \((\lambda^*, r(\lambda^*)) \in [0; 1] \times [q; 1] \) that solves (2) and (3).

It is useful to notice that the manager prefers to remember bad news only when

\[
E_1 [U(C, a(r, C), 1) | \sigma = L] \geq E_1 [U(C, a(r, C), \lambda) | \sigma = L],
\]

so, for given \( r \), the optimal \( \lambda \) is 1 only if\(^{12}\)

\[
[(a_L w_L + (1 - a_L)w_0) - c(a_L)] \geq s (a_H(r)(rw_H + (1 - r)w_L) + (1 - a_H(r))w_0) + (1 - s) (a_H(r)w_L + (1 - a_H(r))w_0) - c(a_H(r)].
\]  

(4)

\(^{12}\)Notice the difference between \( E_1 [u(C, a(r, C))] = a_H(r)w_L + (1 - a_H(r))w_0 \) and \( E_2 [u(C, a(r, C))] = a_H(r)(rw_H + (1 - r)w_L) + (1 - a_H(r))w_0 \), which derives from the different information available at \( t = 1 \), i.e., the true information and the recollection available at \( t = 2 \).
Since at equilibrium $r = r(\lambda)$, condition (4) simplifies to
\[ c(a_H(\lambda)) - c(a_L) \geq sa_H(\lambda)r(\lambda)(w_H - w_L) + (a_H(\lambda) - a_L)(w_L - w_0), \] (5)
where $a_H(\lambda) \equiv a_H(r(\lambda))$. From the above analysis we conclude that the manager has an incentive to remember when, for any $\lambda < 1$, the extra cost he incurs to exert effort $a_H(\lambda)$ rather than $a_L$ exceeds the sum of the emotional gain from forgetting (due to the uncertainty about the true project return in case of success) plus the gain due to obtaining $w_L$ rather than $w_0$ with an increased probability $(a_H(\lambda) - a_L)$. It is clear that the equilibrium of the memory game will depend both on rewards and on $s$.

To simplify the analysis, we assume that, in case of multiplicity, the manager will choose the Pareto-superior equilibrium, i.e., that preferred by investors. Finally, to simplify notation, in what follows we omit the star superscript to denote the optimal $\lambda$.

5. The conflict over optimal recollection between investors and manager

In the previous section, we saw that whenever the weight attached to anticipatory utility $s$ is large, the emotional gain from forgetting (see condition (5)) may induce the manager to forget a bad signal. Now, we will show that, unlike the manager, investors always prefer perfect recollection, pointing to a potential conflict of interest between investor and manager over the memory strategy.

We proceed in two steps. First we solve the investors’ maximization problem under the assumption that $\lambda$ is exogenously given and is common knowledge. Then we find the optimal level of $\lambda$ from the investors’ point of view. Given that the manager’s effort is not observable, investors must offer an incentive-compatible contract that induces the manager to choose the desired level of effort.

Faced with the contract $C$ and the recalled signal $\hat{\sigma}$, the manager chooses a level of effort $a_{\hat{\sigma}}$ such that
\[ a_{\hat{\sigma}} = \arg \max_{a \in [0,1]} \{-c(a) + E_2[u(C, a)|\hat{\sigma}]\}. \] (6)

By the strict concavity of the manager’s objective function, a necessary and sufficient condition for the incentive constraint to be satisfied when $\hat{\sigma} = L$ is
\[ (w_L - w_0) = c'(a_L). \] (7)

Instead, when $\hat{\sigma} = H$, the necessary and sufficient condition is
\[ (r(\lambda))w_H + (1 - r(\lambda))w_L - w_0) = c'(a_H). \] (8)

Finally, we denote by $a(\lambda, C) \equiv \{a_L, a_H(\lambda)\}$ the vector of effort levels that solve problem (6) for $\hat{\sigma} \in \{L, H\}$.\(^{14}\)

When investors make their offer, the manager does not know $\sigma$. So to induce him to accept, the contract has to satisfy the following ex-ante participation constraint
\[ E_0[U(C, a(\lambda, C), \lambda)] = qE_1[U(C, a(\lambda, C), \lambda)|\sigma = H] + (1 - q)E_1[U(C, a(\lambda, C), \lambda)|\sigma = L] \]
\[ = [w_0 + qa_H(w_H - w_0) + (1 - q)((1 - \lambda)a_H + \lambda a_L)(w_L - w_0)] + 
\[ -((1 - \lambda)(1 - q)c(a_H) + \lambda(1 - q)c(a_L)) \geq 0, \] (9)

\(^{13}\)Where $(r(\lambda))w_H + (1 - r(\lambda))w_L - w_0) \in [c'(0), c'(1)]$ for any $r$.

\(^{14}\)Assuming that $c'(0) = 0$ and $c'(1) \geq \nu_H$ ensures interior solutions.
where $E_0$ denotes the expectation at time $t = 0$. Finally, by the limited-liability constraints, the manager’s transfer must always be non-negative, i.e.

$$w_i \geq 0 \forall i \in \{0, L, H\}. \quad (10)$$

If a $\lambda$–type manager accepts contract $C$, the principal’s expected profit in period 0 is

$$E_0 [\Pi (C, a(\lambda, C), \lambda)] = \sum_{i \in \{0, L, H\}} \Pr (v_i | C, a(\lambda, C), \lambda) (v_i - w_i)$$

$$= qa_H (v_H - (w_H - w_0)) + (1 - q) ((1 - \lambda) a_H + \lambda a_L) (v_L - (w_L - w_0)) - w_0, \quad (11)$$

where $qa_H = \Pr_0 (v_H | \lambda)$, and $(1 - q) ((1 - \lambda) a_H + \lambda a_L) = \Pr_0 (v_L | \lambda)$.

The investors’ problem reduces to the choice of effort levels $a_H, a_L$ and payments $w_H, w_L$ and $w_0$ that maximize their expected profits (11) subject to the incentive constraints (7) and (8), the participation constraint (9), and the limited-liability constraints (10). Notice that the limited-liability constraint on $w_0$ is binding. Thus, from now on, we set $w_0 = 0$.

Let us denote by $P^\lambda$ the investors’ programme for given $\lambda$. Notice that moral hazard and limited liability make delegation costly to investors. To be more precise, let us define the $\lambda$-first-best world as a setting where effort is observable and $\lambda$ is exogenous. The limited-liability constraints reduce the set of incentive-feasible allocations and prevent investors from implementing the $\lambda$-first-best level of effort even with a risk-neutral manager.\footnote{Without the limited-liability constraints, the $\lambda$-first-best outcome might be obtained through a contract that rewards the manager in case of success (i.e., $w_H > 0$ and $w_L > 0$) and punishes him in case of failure (i.e., $w_0 < 0$).}

Solving program $P^\lambda$, with the incentive constraints (7) and (8) binding, investors attain their $\lambda$–second-best expected utility $E_0 \Pi^{\lambda SB} (\lambda) = E_0 [\Pi (C^{\lambda SB}, a(\lambda, C^{\lambda SB}), \lambda)]$.\footnote{The constraint on transfers (10) limits the investors’ ability to punish the manager and implies, as shown in the Appendix, that the participation constraint (9) is slack. Thus, we neglect it.}

Next proposition shows that the accuracy of the manager’s information is always valuable to investors.

**Proposition 1.** An increase in the probability that bad news will be remembered accurately $\lambda$ has a positive effect on the investors’ $\lambda$-second-best expected utility, $E_0 \Pi^{\lambda SB} (\lambda)$.

As information becomes more precise, i.e. as $\lambda$ increases, $w_H$ decreases because the manager’s expected benefit from exerting any level of effort when he recalls a good signal increases (see equation (8)). This is a positive indirect effect. However, an increase in $\lambda$ implies a negative direct effect on the probability of success following a bad signal $((1 - q) ((1 - \lambda) a_H + \lambda a_L))$, as the weight of $a_L$ increases and that of $a_H$ decreases. Proposition 1 makes it clear that the positive indirect effect outweighs the negative direct effect, implying that if investors could choose the manager type they would prefer a manager with $\lambda = 1$.

Let us denote by $C^{SB}$ the second-best contract that solved programme $P^\lambda$ when $\lambda = 1$. Then the next proposition shows that managers with sufficiently high anticipatory emotions will prefer to forget bad news when offered contract $C^{SB}$.

**Proposition 2.** If the weight of anticipatory utility $s$ is sufficiently high, the manager always prefers to forget bad news when rewarded with contract $C^{SB}$.\footnote{The constraint on transfers (10) limits the investors’ ability to punish the manager and implies, as shown in the Appendix, that the participation constraint (9) is slack. Thus, we neglect it.}
Taken together, Propositions (1) and (2) highlight a potential conflict in a setting where \( \lambda \) is endogenous: investors always prefer perfect signal recollection, but the manager, if the weight placed on anticipatory utility is great, could prefer to forget bad news.

For simplicity, from now on we assume that \( c (a) = ca^2 / 2 \), with \( c \geq v_H. \) This allows us: first, to find an explicit condition on the parameter \( s \) for the contract \( C^{SB} \) to implement the second-best outcome; and second, to characterize the third-best outcome when there is a conflict between investor and manager over optimal \( \lambda \).

In order to compute the \( \lambda \)-second-best levels of effort and rewards, we solve problem \( P^\lambda \) under the assumption of quadratic costs. We solve (7) and (8) for \( w_L \) and \( w_H \). Substituting into the objective function (11), differentiating with respect to \( a_H \) and \( a_L \), and solving gives

\[
a_H^{SB} (\lambda) = \frac{r(\lambda) v_H + (1 - r(\lambda)) v_L}{2c} \quad \text{(12)}
\]

\[
a_L^{SB} = \frac{v_L}{2c} \quad \text{(13)}
\]

where \( v_L \) and \( r(\lambda) v_H + (1 - r(\lambda)) v_L \) are the gains from effort with a good signal and a bad signal, respectively. Since the marginal benefit of effort is greater when the manager recollects a good signal than a bad one (i.e., \( r(\lambda) v_H + (1 - r(\lambda)) v_L \geq v_L \)), then in the former case the level of effort implemented by investors is higher. Moreover, the marginal benefit of effort when good news is recollected increases with the accuracy of information. As a consequence, \( a_H^{SB} (\lambda) \) increases with \( \lambda \).

Using (12), (13) in the incentive constraints we get

\[
w_H^{SB} = \frac{v_H}{2} \quad \text{(14)}
\]

\[
w_L^{SB} = \frac{v_L}{2} \quad \text{(15)}
\]

Interestingly, the \( \lambda \)-second-best rewards are independent of \( \lambda \). Incentives to induce a \( \lambda \)-type manager to exert effort when good news is recollected depend on \( \lambda \) through the expected rewards in the case of success, not through each state-contingent payment.

Using the \( \lambda \)-second-best payments and effort levels in the investors’ objective function (11) and differentiating with respect to \( \lambda \), in keeping with proposition 1, we find that investors always prefer the manager to remember a bad signal

\[
\frac{\partial E_0 \Pi^\lambda^{SB} (\lambda)}{\partial \lambda} = \frac{q^2 (1 - q) (v_H - v_L)^2}{4c (1 - \lambda (1 - q))^2} \geq 0.
\]

Moreover, using the second-best payments (14), (15) in constraint (5), with \( \lambda = 0 \), and solving for \( s \), we find that the manager would choose \( \lambda = 1 \) only if

\[
s \leq s_1 \equiv \frac{q (v_H - v_L)}{2 (qv_H + (1 - q)v_L)} \leq \frac{1}{2} \quad \text{(16)}
\]

The threshold \( s_1 \) is the greatest weight placed on anticipatory utility that makes the manager indifferent between recalling and forgetting bad news.\(^\text{18}\) Notice that since the ratio in (16) is

\(^{17}\)This condition ensures interior solutions.

\(^{18}\)Under the hypothesis of quadratic costs, the set of values of \( s \) for which this indifference holds is an interval. For any \( s \) in this interval, there are three equilibria: \( \lambda = 0 \), \( \lambda = 1 \) and a partially selective memory \( (0 < \lambda < 1) \). Since we assumed that, in case of multiplicity, the manager chooses the equilibrium preferred by the investor, for any \( s \) lower than \( s_1 \) there will be perfect recollection in equilibrium.
less than 1/2, condition (5) is violated if \( s > 1 - s \), that is whenever the weight attached to anticipatory utility is greater than that of the physical outcome. Finally, it is interesting that the importance of the signal is inversely related to the distance between the intermediate return \( v_L \) and the good return \( v_H \). Indeed, the signal is worthless if \( v_L = v_H \), while it is crucial when \( v_L = 0 \).

As a consequence, the significance of the conflict of interest between investors and manager over the probability of recall \( \lambda \) also depends on the distance between the return for an extremely good project, \( v_H \) and for a business-as-usual result, \( v_L \). In the following, we will interpret this distance as a measure of the riskiness of the industry. For given \( v_H \), as the intermediate outcome \( v_L \) increases, the manager’s tendency to forget bad news increases \((s_1 \text{ is decreasing in } v_L)\). Generally, as the distance between \( v_H \) and \( v_L \) decreases, the manager’s tendency to forget bad news increases \((s_1 \text{ increases in } \Delta v = v_H - v_L)\).

### 6. The optimal contract

In this section we explore the implications of violating condition (16). Earlier, we found that for \( s \leq s_1 \), investors’ and manager’s preferences on \( \lambda \) are perfectly aligned, so that the second-best contract \( C_{SB}^T = \{0; v_L/2; v_H/2\} \) satisfies the non-forgetfulness constraint (5) and implements the second-best level of effort: \(^{20}\)

\[
\alpha_{SB}^H = \frac{v_H}{2c}, \quad \alpha_{SB}^L = \frac{v_L}{2c}. \quad (17)
\]

However, if the manager attaches a large weight to anticipatory utility, that is if \( s > s_1 \), the second-best outcome cannot be achieved because contract \( C_{SB}^T \) fails to induce the manager to recollect his private information correctly. This gives rise to a third-best scenario, in which effort is unverifiable and the manager elects to forget bad news. In such circumstances, the investors’ problem is to choose a vector of effort levels and a contract that solve programme \( \mathcal{P}^\lambda \), with \( \lambda = 1 \), under the further constraint that the manager is indifferent between forgetting and remembering bad news. Let us denote by \( \mathcal{P}^{TB} \) the investors’ programme in this third-best world.

In order to induce the manager to recall the signal, investors offer a contract that satisfies the non-forgetfulness constraint (5) (for \( \lambda = 0 \)) with equality. In this case the non-forgetfulness constraint (5) simplifies to \(^{21}\)

\[
(a_H - a_L) [(1 - 2s) q (a_H - a_L) - 2sa_L] = 0. \quad (19)
\]

Satisfaction of the previous equality produces two possible equilibria: a separating equilibrium, denoted by the superscript \( S \), where \( a_L = \phi(s) a_H \), with \( \phi(s) \equiv \frac{(1-2s)q}{q+2s(1-q)} \), which is possible only if \( s \leq 1/2 \), and a pooling equilibrium, denoted by the superscript \( P \), where \( a_H = a_L \).\(^{22}\) Proposition 3 solves the investors’ problem in these two equilibria.

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19 If \( v_H = v_L, s_1 = 0 \), but this is a degenerate case since there is no memory problem. Instead, when \( v_L = 0, s_1 = 1/2 \, and the manager wants to forget bad news only if the weight of anticipatory utility is greater than that of the physical outcome, that is if \( s > 1/2 \).  
20 Notice that (17) follows from (12) for \( \lambda = 1 \). Moreover, it is easy to verify that the participation constraint (9) is satisfied by these second-best payments and efforts.  
21 See the proof of Proposition 3 for details.  
22 If \( s > 1/2 \), the binding non-forgetfulness constraint (19) would imply \( a_L < 0 \).
Proposition 3. In the separating equilibrium, the optimal levels of effort are

\[ a_H^S = a_H^B + \gamma \frac{(1 - 2s) (1 - q)}{c}, \quad a_L^S = a_L^B - \frac{q + 2s (1 - q)}{c}, \]

and are implemented by the following state-contingent rewards

\[ w_0^S = 0, \quad w_H^S = w_H^B + \gamma (1 - 2s) (1 - q), \quad w_L^S = w_L^B - (q + 2s (1 - q)), \]

where \( \gamma \equiv \frac{qw_H + (1 - q)w_L}{|q + 4s^2 (1 - q)|} (s - s_1) \) is positive for any \( s \geq s_1 \).

In the pooling equilibrium, the optimal level of effort is given by

\[ a_H = a_L = a^P = qa_H^B + (1 - q) a_L^B, \]

and is implemented by the following state-contingent rewards

\[ w_0^P = 0, \quad w_H = w_L = w^P = qw_H^B + (1 - q) w_L^B. \]

The above proposition makes it clear that there are two opposite ways to elicit accurate recollection. In the separating equilibrium, investors increase the cost of forgetting, i.e., the left-hand side of equation (5), by asking for a level of effort higher than second-best when the manager recalls good news, lower when he recalls bad news. In order to implement those effort levels, which are farther apart, a manager recollecting a good (bad) signal must be offered a reward higher (lower) than the second-best.\(^{23}\) In the pooling equilibrium, investors eliminate any incentive to suppress bad news by offering a flat contract paying a constant amount \( w^P \) regardless of result \( \{v_H, v_L\} \) and asking for the same level of effort \( a^P \) following both signals. As neither effort levels nor rewards depend on signal recollection, the manager is indifferent between recalling and forgetting bad news.\(^{24}\)

The manager has a stronger incentive to forget bad news when the weight placed on emotions is large. So it is natural to ask how the distortion of the efforts and payments necessary to induce recollection varies with \( s \). As Proposition 4 shows, the result is ambiguous and depends on the relative importance of intermediate and high return \( (v_L/v_H) \) with respect to the likelihood of the good signal \( q/(1 - q) \).

Proposition 4. There exist a threshold \( s_H \) such that the optimal level of effort following a good signal \( a_H^S \) is increasing for \( s_1 < s \leq s_H \) and decreasing for \( s_H < s \leq 1/2 \), with \( s_H \in (s_1; 1/2) \). It is always decreasing following a bad signal \( a_L^S \). Moreover, there exist a second threshold \( s_\gamma \) such that if \( v_L/v_H \leq q/(1 - q) \), the distance between \( a_H^S \) and \( a_L^S \) is always increasing, while if \( v_L/v_H \geq q/(1 - q) \), the distance between \( a_H^S \) and \( a_L^S \) is increasing for \( s_1 < s \leq s_\gamma \) and decreasing for \( s_\gamma < s \leq 1/2 \), with \( s_\gamma \in (s_1; 1/2) \). Finally, the same comparative statics results hold for rewards \( w_H^S, w_L^S \) and \( w_H^B - w_L^B \).

The intuition for the inverse U-shaped pattern depends on a better understanding of condition (5). Greater distance between \( a_H^S \) and \( a_L^S \) increases both the extra cost of effort, inducing the manager to recollect, and the emotional gain from forgetting, inducing him to forget. The effect on the indirect gain from greater effort is ambiguous, due to the benefit.

\(^{23}\) Indeed, since a separating contract is offered only if \( s_1 \leq s \leq 1/2 \), it is immediate to see that: \( a_H^S \geq a_H^B \) and \( w_H^S \geq w_H^B \), while \( a_L^S < a_L^B \) and \( w_L^S < w_L^B \).

\(^{24}\) It is interesting to observe that in the pooling equilibrium \( a^P \) is the average between \( a_H^B \) and \( a_L^B \), and \( w^P \) is the average between \( w_H^B \) and \( w_L^B \).
of obtaining a lower \( w^S_L \) but with an increased probability \( (a_H^S - a_L^S) \). The way in which the optimal level of efforts and rewards changes with \( s \) depends on which effect prevails. The direct emotional gain from forgetting is weighted by \( s \). Hence, for low \( s \) (i.e., close to \( s_1 \)), it is small relative to the extra cost of effort, and \( a_H^S \) (and \( w^S_H \)) is increasing while \( a_L^S \) (and \( w^S_L \)) is decreasing in \( s \). As \( s \) grows larger than \( s_H \), the weight of the emotional gain from forgetting increases, and both \( a_H^S \) and \( a_L^S \) are then decreasing in \( s \). Indeed, the lower \( a_H^S \) generates a significant reduction in the probability of both direct and indirect gain from forgetting. However, if the magnitude of the low return is large, \( w^S_L \) is large, then the impact on the indirect emotional gain from higher effort is relevant, so that increasing the distance between \( a_H^S \) and \( a_L^S \) is optimal for low \( s \) but detrimental for higher \( s \) (i.e., larger than \( s_s \)). But if the magnitude of the low return is small, the last effect is less important because \( w^S_L \) is small and the distortion of efforts and payments necessary to induce signal recollection is increasing for all \( s \).

This allows us to derive the following empirical prediction.

**Prediction 1** If investors cannot select managers on the basis of the characteristics captured by the parameter \( s \), then the premium for an extremely good result – represented by the distance between payments – is increasing with respect to our inverse measure of emotional stability and conscientiousness \( s \) in relatively riskier firms/industries, while it is inverse U-shaped in less risky firms/industries.

Even if we assume \( s \) to be publicly known, it might be impossible for investors to choose the manager on this basis, perhaps for a shortage of good managers, or because of other features that we have not accounted for in our analysis, but that are nevertheless important to the hiring decision, such as specific expertise.

Proposition 5 states the conditions for parameter \( s \) to generate either the separating or the pooling equilibrium.

**Proposition 5.** The separating equilibrium arises for all \( s \in (s_1; 1/2) \), the pooling equilibrium for all \( s > 1/2 \).

The pooling equilibrium always arises if \( s > (1 - s) \), as in this case the separating equilibrium would entail a negative level of effort when the bad signal is observed. Since by assumption the level of effort has to be positive, we should have \( a_H^S = 0 \). However, in this case, the investors’ expected profit would be greater in the pooling equilibrium. So when facing a manager who attaches a larger weight to anticipatory emotions than to physical utility, investors always offer a pooling contract. On the other hand, if \( s \leq (1 - s) \) the separating equilibrium always arises: the investors’ expected utility under a separating equilibrium \( (E_0 \Pi^S) \) is greater than under a pooling equilibrium \( (E_0 \Pi^P) \), that is \( E_0 \Pi^S - E_0 \Pi^P \geq 0 \) for all \( s \leq 1/2 \).

Thus, depending on the weight placed on anticipatory utility, we get the three possible scenarios depicted in Figure 1. When \( s \) is sufficiently small (\( s \leq s_1 \)), the manager recollects the signals correctly (\( \lambda = 1 \)) and investors design a contract that rewards effort but not memory. Due to moral hazard, investors achieve the second-best. For \( s_1 < s \leq 1/2 \), the emotional impact of bad news may induce the manager to suppress it and instead “recall” good news. To induce accurate memory recollection, investors have to design a separating contract that punishes forgetfulness and rewards memory. This is achieved by setting payments and effort levels farther apart.\(^{25} \) However, when \( s > 1/2 \), investors stop dis-

\(^{25}\)It is immediate to verify that \( \Delta w^S = w_H^S - w_L^S > w_H^S - w_L^S = \Delta a^S \) and \( \Delta (a_H^S - a_L^S) > \)
Figure 1: Optimal contract as a function of $s$. The figure shows that a second-best, separating or pooling contract may arise depending on the weight placed on anticipatory utility.

Depending on $s$, three scenarios are possible. We now analyze the pattern of investors’ utility as $s$ varies.

For $s \in [0; s_1]$, the optimal contract offered and the level of effort the manager chooses are independent of $s$. Indeed, if the parties’ preferences on memory strategy are perfectly aligned, the weight the manager attaches to emotions relative to physical utility does not affect his effort decision, so the investors’ second-best expected utility does not depend on $s$ and is given by

$$E_0 \Pi^{SB} = q \frac{v^2_H}{4c} + (1 - q) \frac{v^2_L}{4c}. \quad (20)$$

Instead, for $s \in (s_1; 1/2]$, investors and manager disagree over the optimal $\lambda$. Both rewards and effort depend on $s$ and the third-best investors’ expected profit in the separating equilibrium, that is $E_0 \Pi^{S}$ (see equation 50 in the Appendix) decreases with $s$.

Last, when $s > 1/2$, investors stop distorting rewards to induce recollection, and neither $a^{SB}_H - a^{SB}_L = \Delta a^{SB}$.  

\[ a^{SB}_H - a^{SB}_L = \Delta a^{SB}. \]
effort nor payment is affected by the weight the manager attaches to emotions. Consequently, in the pooling equilibrium the investors’ third-best expected utility is independent of \( s \) and is given by

\[
 E_0 \Pi^P = E_0 \Pi^{SB} - \frac{q(1-q)(v_H - v_L)^2}{4c}.
\]  

Proposition 6 summarizes these results.

**Proposition 6.** The investors’ expected utility is weakly decreasing in \( s \).

The following figure depicts the pattern of utility as \( s \) varies:

![Figure 2: Investors’ expected utility as a function of \( s \).](image)

The result in Proposition 6 allows us to derive the following prediction.

**Prediction 2** If investors can select managers on the basis of the characteristics captured by the parameter \( s \) they will choose any manager with \( s \leq s_1 \), i.e., relatively more conscientious and emotionally stable. Moreover, these features will be more pronounced where the distance between the return for an extremely good project and that for a business-as-usual result is smaller, i.e., for less risky firms/industries.

Although this may seem counterintuitive, it can be rationalized considering that in our setup incentive contracts play the dual role of inducing effort and eliciting memory. To see this, recall the comparative statics results on \( s_1 \) (equation 16). When the distance between \( v_H \) and \( v_L \) increases, \( s_1 \) shifts rightward (toward \( 1/2 \)), leftward when it decreases. Thus when \( v_H \) and \( v_L \) are distant, to induce effort investors must offer high-powered incentive contracts that also alleviate the manager’s memory problem. Thus, by means of a standard second-best contract, investors manage to resolve even the memory problem of more emotional managers. When \( v_H \) and \( v_L \) are close to each other, a low-powered incentive contract suffices to induce effort. But because \( s_1 \) is smaller, this may conflict with the memory problems of more emotional managers, calling for a high-powered incentive contract, which results in a separating contract.
Thus, riskier firms can “afford” to employ more emotional managers since, by offering high-powered incentives to induce effort, they can also control their “over-optimism” at no extra cost. Less risky firms, instead, have less difficulty in inducing effort, but are confronted with the problem of controlling the manager’s optimism. Since this is costly, they prefer to resort to more stable managers.

7. The manager’s private information

It is common practice in many institutions, industries and businesses to use psychological testing in the recruitment and promotion process, including screening such traits as conscientiousness and emotional stability. If investors (employers) can learn about their managers’ personality traits, one naturally wonders what role these play in managerial compensation. We have studied what happens if investors know the weight that the manager attaches to anticipatory utility. But even if psychological testing is common practice, perfect knowledge of on the part of investors remains a strong assumption.

We now extend the previous analysis to the other extreme case: that in which the weight on is the manager’s private information and we look at the effect of this setup modification on the optimal contract. If only the manager can know , investors can offer no contract (second-best, separating, or pooling) contingent on . For any contract \( C = \{w_0, w_L, w_H\} \), both the preferred recollection strategy and the level of effort chosen will now depend on the manager’s type \( s \).

Let us denote by \( \hat{S} \) a subset of \([0, 1]\) such that all managers with \( s \in \hat{S} \) prefer to recall their private information when offered the contract \( C \). Then the investors’ decision problem, \( P^{AI} \), is to choose the set \( \hat{S} \), the level of effort and the contract \( C \) that maximize their expected profits, subject to the limited-liability constraint, the incentive constraint, the participation constraint (both for accurate recollection and forgetfulness) and the non-forgetful constraint for all managers with \( s \in \hat{S} \). Moreover, let \( a_L \) and \( a_H(1) \) be the levels of effort of a manager with \( s \in \hat{S} \), when he observes bad and good news, respectively. Let \( a_H(0) \) be the level of effort of a manager who prefers to forget bad news whenever he is offered contract \( C \), i.e. a manager with \( s \notin \hat{S} \).

As in the complete-information benchmark, the limited-liability constraint on \( w_0 \) is binding. Substituting \( w_0 = 0 \) into the manager’s incentive constraints (7 and 8) and assuming that the manager prefers to recall the bad signal – i.e., \( \lambda = 1 \), we obtain

\[
w_L(a_L) = ca_L, \tag{22}\]

and

\[
w_H(a_H(1)) = ca_H(1). \tag{23}\]

Then, substituting (22) and (23) into the incentive constraint for a forgetful manager (that is (8) with \( \lambda = 0 \)), we obtain

\[
a_H(0) = \frac{w_H}{c} + (1 - q) \frac{w_L}{c} = qa_H(1) + (1 - q) a_L. \tag{24}\]

Finally, the contract has to be such that all managers with \( s \in \hat{S} \) prefer to recall the signal, which is ensured if we impose the non-forgetfulness constraint (5) for all \( s \in \hat{S} \). But this is equivalent to \( a_L \leq \phi(s) a_H(1) \) for all \( s \in \hat{S} \). Then, defining \( \hat{s} \equiv \sup \hat{S} \) and noticing that \( \phi(s) \) is decreasing in \( s \), this condition is clearly satisfied for all \( s \in \hat{S} \) if and only if

\[
a_L \leq \phi(\hat{s}) a_H(1) \equiv a_L(a_H(1)). \tag{25}\]
To simplify the analysis of the investors’ decision problem, we assume that the weight $s$ is distributed uniformly over $[0, 1]$ and we show in Lemma 1 that $\tilde{s}$ is an interval.

**Lemma 1.** Assume that $s$ is the manager’s private information. Then the optimal contract is such that all managers with $s \leq \tilde{s}$ will recall a bad signal while those with $s > \tilde{s}$ will forget it.

The investors expected profits can be written as

$$E_0 \Pi(a_H(1), a_L, \hat{s}) = [a_H(1)q(v_H - ca_H(1)) + a_L(1 - q)(v_L - ca_L)]\tilde{s} + [a_H(1)q + a_L(1 - q)][q(v_H - ca_H(1)) + (1 - q)(v_L - ca_L)](1 - \tilde{s}).$$

(27)

Substituting (22), (23) and (24) into (26) and rearranging terms we get

$$E_0 \Pi(a_H(1), a_L, \hat{s}) = [a_H(1)q(v_H - w_H) + (1 - q) a_L(v_L - w_L)]ds +$$

$$+ \int_{\hat{s}}^{1} a_H(0) [q (v_H - w_H) + (1 - q) (v_L - w_L)]ds.$$

(26)

Thus, the investors’ problem simplifies to choosing $a_H(1)$, $a_L$ and $\hat{s}$ that maximize (27), subject to constraint (25). Proposition 7 states the main result.

**Proposition 7.** Assume that $s$ is the manager’s private information. Then the optimal contract is such that all managers will accept the investors’ offer and the threshold between remembering and forgetting the bad signal is $\hat{s} \in (s_1; 1/2]$. Moreover, the threshold $\hat{s}$ is equal to 1/2 for all $v_L < r_2(q)v_H$, and is lower than 1/2 for all $v_L \geq r_2(q)v_H$, with $r_2(q) \equiv \frac{(1+q)}{(1+q)}$. Finally, $\partial \hat{s}/\partial v_L \leq 0$.

**Corollary 1.** If $s$ is the manager’s private information, the distance between equilibrium rewards when the good and the bad signals are observed grows relative to the second best.

The intuition behind this result relies on a better understanding of the investors’ expected profits in expression (27). Total investor profits are the average between profits produced by managers who choose to recall the bad signal and that produced by managers who choose to forget it. For the latter group, investors prefer not to elicit information recollection, opting instead for an accommodating strategy that accepts the manager’s forgetfulness ($\lambda = 0$), by neglecting constraint (5). Hence, a threshold $\hat{s} < s_1$ is never optimal because offering the second-best contract $C_{SB}$ to all managers will induce those with $s \leq s_1$ to recall bad news and at the same time maximize the profits generated by the managers who choose to forget. Second, we know from the previous section that for all $s_1 < s \leq 1/2$, the emotional impact of bad news may induce the manager to suppress it and recall good news instead. To induce accurate recollection, investors had to design a costly separating contract that punishes forgetfulness and rewards memory. Thus, it may seem surprising that the optimal contract with imperfect information is such that investors decide to induce recollection also from managers with $s > s_1$. However, for managers whose weight is slightly greater than $s_1$, the extra cost of inducing recollection is small and the increase in profits in switching from accommodating to separating is large. In other words, the direct effect of an increase in the threshold $\hat{s}$ is first-order while the indirect effect via the non-forgetful constraint is second-order. If we interpret private information as a situation in which firms do not make

\footnote{In the Appendix we show that the participation constraints for both types of manager are satisfied and can be ignored.}
use of psychological testing, then these results, along with those in the previous section, allow us to derive the following predictions.

**Prediction 3** For given riskiness, industries and businesses that commonly use psychological testing for recruitment, by comparison with those that do not:

1. will take on more dependable and emotionally stable managers (with an $s$ between 0 and $s_1$);
2. will offer less high-powered incentive schemes;
3. will be less prone to reality denial.

Along with Prediction 2, this suggests that, in less risky sectors in which psychological tests are not used, high-powered incentive schemes may be driven by behavioral causes, rather than by the need to control incentive problems.

**8. Conclusion**

Managers with anticipatory emotions have higher current utility if they are optimistic about the future. We have modeled an employment contract between an (endogenously) optimistic manager and realistic investors. After showing the existence of a potential conflict over memory strategy, we have shown that the manager’s optimism may be affected by monetary incentives. More specifically, we have found that for sufficiently low anticipatory emotions, investors’ and manager’s preferences over optimal recollection are perfectly aligned, so that the second-best contract $C^{SB}$ that solves the moral hazard problem also satisfies a non-forgetfulness constraint. However, if the manager places a large weight on anticipatory utility, the second-best outcome cannot be achieved because contract $C^{SB}$ fails to induce the manager to recall his private information correctly. This gives rise to a third-best world in which investors must distort effort levels and payments to make the manager indifferent between forgetting and remembering bad news.

What happens in our setting if effort is verifiable but the signal is still private information? If payments are contingent on the outcome, so that a better outcome is associated with a higher payment, the manager will always have an incentive to forget bad news. To prevent this, investors can offer a flat contract and obtain the first-best utility. In other words, not only the presence of an emotional manager, but also a second imperfection is required to make our analysis interesting.

To conclude, we think that the interaction between overoptimism and managerial compensation is a significant issue and warrants further investigation, both theoretical and empirical. At the theoretical level, it has been shown that optimism may exacerbate incentive problems. At the empirical level, our analysis derives some interesting predictions on how behavioral traits affect managerial compensation and recruitment. In particular on the relationship between riskiness of an industry and the personality of managers.

**Appendix**

In the analysis to follow, the limited liability constraint on $w_0$ is always binding. Thus, throughout all the proofs, we set $w_0 = 0$. 19
\textbf{Proof of Proposition 1.} In order to solve problem \( P^\lambda \), we solve the incentive constraints for \( w_L \) and \( w_H \), substitute \( w_L(a_L) \) and \( w_H(a_H) \) in the objective function and maximize with respect to \( a_L \) and \( a_H \). By (7),

\[ w_L(a_L) = c'(a_L). \tag{28} \]

Using \( w_L(a_L) \) in (8), we obtain

\[ w_H(a_H; a_L) = \frac{(1 - \lambda(1 - q))c'(a_H) - (1 - \lambda)(1 - q)c'(a_L)}{q}. \tag{29} \]

Substituting (28) and (29) in (11) and rearranging, the objective function becomes:

\[ E_0 \left[ \Pi(a_L, a_H) \right] = qa_H v_H + (1 - q) ((1 - \lambda) a_H + \lambda a_L) v_L + (1 - \lambda(1 - q)) a_H c'(a_H) - \lambda(1 - q) a_L c'(a_L). \tag{30} \]

Substituting (28) and (29) in the participation constraint (9)

\[ E_0 \left[ U(a_L, a_H) \right] = [1 - \lambda (1 - q)] a_H \left[ c'(a_H) - \frac{c(a_H)}{a_H} \right] + \lambda (1 - q) a_L \left[ c'(a_L) - \frac{c(a_L)}{a_L} \right], \]

which, by the convexity of the cost function, is strictly positive and can be neglected.\(^{27}\)

Differentiating (30) with respect to \( a_L \) and \( a_H \) gives the following necessary and sufficient conditions

\[ \frac{\partial E_0 \left[ \Pi(a_L, a_H) \right]}{\partial a_L} = v_L - \left( c'(a_L^{SB}) + a_L^{SB} c''(a_L^{SB}) \right) = 0 \tag{31} \]

\[ \frac{\partial E_0 \left[ \Pi(a_L, a_H) \right]}{\partial a_H} = \frac{q}{(1 - \lambda (1 - q))} \left( v_H - v_L \right) + v_L - \left( c'(a_H^{SB} (\lambda)) + a_H^{SB} (\lambda) c''(a_H^{SB} (\lambda)) \right) = 0 \tag{32} \]

By the envelope theorem, the derivative of the investors’ expected profits with respect to \( \lambda \) is

\[ \frac{\partial E_0 \left[ \Pi(a_L^{SB}, a_H^{SB} (\lambda)) \right]}{\partial \lambda} = (1 - q) \left[ a_L^{SB} (v_L - c'(a_L^{SB})) - a_H^{SB} (\lambda) (v_L - c'(a_H^{SB} (\lambda))) \right], \]

that is positive only if

\[ a_L^{SB} (v_L - c'(a_L^{SB})) > a_H^{SB} (\lambda) (v_L - c'(a_H^{SB} (\lambda))). \tag{33} \]

Define the function \( f(a) \equiv a(v_L - c'(a)), \) with first derivative given by \( f'(a) = v_L - (c'(a) - ac''(a)), \) and observe that

\begin{itemize}
  \item \( f'(a_L^{SB}) = 0 \) by (31) and \( f'(a_H^{SB} (\lambda)) < 0 \) by (32).
  \item \( f''(a) = -2c''(a) - ac'''(a) < 0 \) for any \( a \), since \( c''(a) \geq 0 \) and \( c'''(a) \geq 0 \) by assumption.
\end{itemize}

Hence, \( f(a) \) is decreasing for any \( a \in [a_L^{SB}; a_H^{SB} (\lambda)] \) and condition (33) is satisfied. \( \square \)

\(^{27}\)This result will hold throughout all the proofs in which \( s \) is observable.
Proof of Proposition 2. If the manager strictly prefers to forget the signal, the game has the unique equilibrium $\lambda = 0$. For any given contract $C^{SB}$, by setting $\lambda = 0$ condition (5) becomes

$$c(a_H(0)) - c(a_L^{SB}) \geq s[a_H(0)(qw_H^{SB} + (1-q)w_L^{SB}) - a_L^{SB}w_L^{SB}] + (1-s)(a_H(0) - a_L^{SB})w_L^{SB},$$

where $a_H(0)$ is the solution to the following equation

$$(qw_H^{SB} + (1-q)w_L^{SB}) = c'(a_H(0)).$$

By using (7) and (35) in (34) and rearranging terms, gives

$$c(a_H(0)) - c(a_L^{SB}) \geq s[a_H(0)c'(a_H(0)) - a_L^{SB}c'(a_L^{SB})] + (1-s)(a_H(0) - a_L^{SB})c'(a_L^{SB}).$$

If $s = 1$, (36) becomes

$$a_L^{SB}c'(a_L^{SB}) - c(a_L^{SB}) \geq a_H(0)c'(a_H(0)) - c(a_H(0)),$$

that is never true since $a_H(0) > a_L^{SB}$ and the function $h(a) = a c'(a) - c(a)$ is increasing in $a$ ($h'(a) = ac''(a) > 0$ by assumption). By continuity, the manager also prefers to forget bad news for all $s$ close to 1. □

Proof of Proposition 3. In order to induce the manager to recall the signal, investors offer a contract that satisfies the non-forgetfulness constraint (5) (for $\lambda = 0$) with equality. Substituting (7) and (8) in (5) and recalling that $r(\lambda) = q$ and $a_H(0) = qa_H + (1-q)a_L$ when $\lambda = 0$, the non-forgetfulness constraint (5) simplifies to

$$(a_H - a_L)[(1-2s)q(a_H - a_L) - 2sa_L] = 0.$$ \hspace{1cm} (37)

Satisfaction of the previous equality for $a_H \neq a_L$ gives rise to $a_L(a_H) = \phi a_H$, where $\phi(s) = \phi$ to simplify notation. In order to solve the investors’ problem in the separating scenario when costs are quadratic, we substitute $a_L(a_H)$ in the incentive constraints and solve with respect to $w_L$ and $w_H$. We substitute $w_L(a_H), w_H(a_H)$ and $a_L(a_H)$ in the objective function and maximize with respect to $a_H$.

Substituting $a_L(a_H)$ in (7) and solving for $w_L$, we obtain

$$w_L(a_H) = ca_L(a_H).$$ \hspace{1cm} (38)

Substituting $\lambda = 1$ in (8) and solving for $w_H$, we have

$$w_H(a_H) = ca_H.$$ \hspace{1cm} (39)

Substituting $\lambda = 1$, $a_L(a_H)$, (38) and (39) in (11) and rearranging terms, the objective function becomes

$$E_0[\Pi(a_H)] = qa_H[v_H - c a_H] + (1-q)\phi a_H[v_L - c \phi a_H].$$ \hspace{1cm} (40)

Differentiating with respect to $a_H$ gives the following necessary and sufficient condition

$$\frac{\partial E_0[\Pi(a_H)]}{\partial a_H} = q[v_H - 2ca_H] + (1-q)(v_L - 2c\phi a_H) = 0$$ \hspace{1cm} (41)

Solving (41) with respect to $a_H$ gives

$$a_H^S = \frac{qv_H + (1-q)\phi v_L}{2c(q + (1-q) \phi)}.$$ \hspace{1cm} (42)
that is lower than 1 since \( c > v_H > v_L \) by assumption. Substituting (42) in \( a_L(a_H) \), in (38) and in (39) and rearranging terms we obtain the effort levels and payments in the proposition.

In order to solve the investors’ problem in the pooling scenario, we impose \( a_L = a_H \). Substituting \( a_L = a_H = a \) in (8) and in (7), we have

\[
w_H(a) = w_L(a) = c a.
\]

Substituting \( \lambda = 1, a_L = a_H = a \), and (43) in (11) and rearranging terms, the objective function becomes

\[
E_0[\Pi(a)] = qa v_H + (1 - q) a v_L - c a^2.
\]

Differentiating with respect to \( a \) gives the following necessary and sufficient condition:

\[
\frac{\partial E_0[\Pi(a)]}{\partial a} = q v_H + (1 - q) v_L - 2ca = 0
\]

Solving (45) for \( a \) and considering that \( a_L^p = a_H^p = a \), we obtain

\[
a_L^p = a_H^p = \frac{(qv_H + (1 - q)v_L)}{2c}.
\]

Substituting (46) in (43) and rearranging terms we obtain the effort level and payments in the proposition. \( \square \)

**Proof of Proposition 4.** i) The derivative of \( a_H^S \) with respect to \( s \) is

\[
\frac{\partial a_H^S}{\partial s} = \frac{1 - q}{c} [(1 - 2s) \frac{\partial \gamma}{\partial s} - 2 \gamma] = \frac{(1 - q)(qv_H + (1 - q)v_L)}{c(q + 4s^2(1 - q))^2} \cdot f(s),
\]

where \( f(s) \equiv -4s^2(1-q)(1+2s_1) - 4s(q-2s_1(1-q)) + q(1+2s_1) \) is a concave parabolic function equal to zero when \( s = \frac{-d - \sqrt{d^2 + (1-q)d}}{2(1-q)} < 0 \) and \( s = s_H \equiv \frac{-d + \sqrt{d^2 + (1-q)d}}{2(1-q)} > 0 \), with \( d \equiv q - \frac{2s_1}{1+2s_1} \). Simple algebra shows that \( s_H \in (s_1; 1/2) \). This implies that \( \partial a_H^S / \partial s \leq 0 \) for all \( s \in (s_1; s_H] \) and \( \partial a_H^S / \partial s < 0 \) for all \( f(s) \geq 0 \) for all \( s \in (s_1; s_H] \) and \( f(s) < 0 \) for all \( s \in (s_H; 1/2) \). This proves the first part of the proposition since \( \partial a_H^S / \partial s \propto f(s) \).

ii) The non-forgetfulness constraint implies that in the separating equilibrium \( a_L^S = \phi a_H^S \). Since \( \partial \phi / \partial s \leq 0 \) for all \( s \in [s_1; 1/2] \) and \( \partial a_H^S / \partial s \leq 0 \) for all \( s \geq s_H \), then \( \partial a_L^S / \partial s < 0 \) for all \( s \in [s_H; 1/2] \). Moreover, from Proposition 3, \( a_L^S = a_L^S \phi - \gamma q + 2s(1-q) \) and then

\[
\frac{\partial a_L^S}{\partial s} = -\left[ \frac{\partial \gamma q + 2s(1-q)}{c} + \gamma \frac{2(1-q)}{c} \right].
\]

Simple algebra shows that

\[
\frac{\partial \gamma}{\partial s} = \frac{qv_H + (1 - q)v_L}{(q + 4s^2(1 - q))^2} h(s),
\]

where \( h(s) \equiv [-4(1-q)s^2 + 8(1-q)s_1 s + q] \) is a concave parabolic function positive for all \( s \in [s_1; s_\gamma] \) and negative for all \( s > s_\gamma > 0 \), with \( s_\gamma \equiv s_1 + \sqrt{s_1 + (q/4(1-q))} \geq s_1 \) and such that \( h(s_\gamma) = 0 \). Equation (47) implies that the derivative of \( a_L^S \) with respect
to \( s \) is negative whenever the derivative of \( \gamma \) with respect to \( s \) is positive. Since \( \partial \gamma / \partial s \propto \partial h(s)/\partial s \), we can conclude that \( \partial a_L/\partial s < 0 \) for all \( s \in [s_1; s_\gamma] \). By solving (47) we get

\[
\frac{\partial a_L^S}{\partial s} = -\frac{qvH + (1 - q)v_L}{c (q + 4s^2(1 - q))^2} g(s),
\]

where \( g(s) \equiv \left[ 4(1 - q)(2(1 - q)s_1 - q)s^2 + 4(1 - q)q(q + 2s_1)s + q(2(1 - q)s_1 - q) \right] \) is a parabolic function with a unique positive zero, \( s_L \). This means that either \( g(s) \geq 0 \) for all \( s \in [0; s_L] \) and \( g(s) < 0 \) for \( s > s_L \), or \( g(s) \leq 0 \) for all \( s \in [0; s_L] \) and \( g(s) > 0 \) for \( s > s_L \). Since \( \partial a_L/\partial s \propto -\partial g(s)/\partial s \), \( \partial a_L^S/\partial s < 0 \) for all \( s \in [s_H; 1/2] \), and \( \partial a_L/\partial s < 0 \) for all \( s \in [s_1; s_\gamma] \), then it is not possible that \( s_L \in (s_1; 1/2) \). Indeed, this would imply either \( \partial a_L/\partial s \leq 0 \) for all \( s \in [s_1; s_L] \) and \( \partial a_L/\partial s > 0 \) for all \( s \in (s_L; 1/2] \), by contradicting \( \partial a_L^S/\partial s < 0 \) for all \( s \in [s_H; 1/2], \) or \( \partial a_L/\partial s \geq 0 \) for all \( s \in [s_1; s_L] \) and \( \partial a_L/\partial s < 0 \) for all \( s \in (s_L; 1/2] \), by contradicting \( \partial a_L^S/\partial s < 0 \) for all \( s \in [s_1; s_\gamma] \).

As a consequence, either \( s_L \leq s_1 \) or \( s_L \geq 1/2 \) and \( \partial a_L^S/\partial s < 0 \) for all \( s \in [s_1; 1/2] \). This proves that \( a_L^S \) is always decreasing.

iii) From Proposition 3, \( \Delta a^S = (v_H - v_L)/2c + \gamma/c \) and then \( \partial(\Delta a^S)/\partial s = (1/c) \cdot \partial(\gamma)/\partial s \). From the proof of point ii) of this proposition we know that \( \partial \gamma/\partial s \geq 0 \) for all \( s \in [s_1; s_\gamma] \) and \( \partial \gamma/\partial s < 0 \) for all \( s > s_\gamma \). Notice that

\[
s_\gamma \geq 1/2 \iff s_1 \geq \frac{1 - 2q}{4(1 - q)}. \tag{49}
\]

Using (16) in (49), after some manipulations, gives

\[
s_\gamma \geq 1/2 \iff v_L \leq \frac{q}{1 - q} v_H.
\]

As a consequence, \( \partial \gamma/\partial s > 0 \) for all \( s \in [s_1; 1/2] \) if \( v_L \leq (q/(1 - q))v_H \) and \( \partial \gamma/\partial s < 0 \) for all \( s \in [s_1; s_\gamma] \) and \( \partial \gamma/\partial s < 0 \) for all \( s \in (s_\gamma; 1/2] \) if \( v_L > (q/(1 - q))v_H \).

The comparative statics for rewards is proved almost identically and then omitted. \( \square \)

**Proof of Proposition 5.** Substituting out \( a_H^P \) in (44) and \( a_H^S \) in (40) we obtain the investors expected profit in the pooling (see equation 21) and separating equilibrium respectively, i.e.,

\[
E_0 \Pi^S = \frac{qvH + (1 - q) \phi v_L}{2c (q + (1 - q) \phi)}.
\]

\[
\left\{ q \left[ v_H - \frac{qvH + (1 - q) \phi v_L}{2(q + (1 - q) \phi)} \right] + (1 - q) \phi \left[ v_L - \phi \frac{qvH + (1 - q) \phi v_L}{2(q + (1 - q) \phi)} \right] \right\}.
\]

By equating \( E_0 \Pi^S \) and \( E_0 \Pi^P \) (21) and solving for \( s \), after some tedious algebra we obtain

\[
s_2 = \frac{(v_H - v_L)(qvH + (1 - q) v_L)}{v_L \cdot v_H + (v_H - v_L)(qvH + (1 - q) v_L)}.
\]

Recalling that a separating equilibrium arises only for \( s \leq 1/2 \) (otherwise \( a_L^S < 0 \)) and noticing that \( s_2 > 1/2 \), we conclude that the separating equilibrium arises for all \( s \in (s_1; 1/2] \), whilst the pooling equilibrium arises for all \( s > 1/2 \). \( \square \)

**Proof of Proposition 6.** Observe that:

1. from (20), \( \partial E_0 \left[ \Pi^{SE} \right] / \partial s = 0 \) for each \( s \in [0; s_1] \);
Using (22) and (23) it becomes
\[ E_s^{a} = \text{manager with} \]
(22), (23) and (24) satisfy also the participation constraints. The participation constraint of efforts chosen by each manager do not depend on his type. Then, a manager with the same for all managers, from the incentive constraints (22), (23) and (24), we know that the constraint of a manager with \( S = [0; 1] \) is always positive. Similarly, by substituting (22), (23) and (24) in the participation constraint, we have
\[ v_L - 2c a_H^\phi \geq 0 \iff v_L \geq -2w_L^S \iff v_L \geq v_L - 2\gamma (q + 2s (1 - q)), \]
that is true for all \( s \geq s_1 \);
3. from (21), \( \partial E_0 \left[ \Pi^P \right] / \partial s = 0 \) for each \( s \in [1/2; 1] \).

The proof is completed noting that \( E_0 \left[ \Pi^P \right] = E_0 \left[ \Pi^S(s) \right] \) and \( E_0 \left[ \Pi^S(1/2) \right] > E_0 \left[ \Pi^P \right] \). \( \square \)

**Proof of Lemma 1.** Suppose, by contradiction, that there exists a manager with \( s = s' < \hat{s} \) which prefers to forget bad news. Since the contract offered by the investors is the same for all managers, from the incentive constraints (22), (23) and (24), we know that the efforts chosen by each manager do not depend on his type. Then, a manager with \( s = s' \) prefers to forget bad news only if \( a_L \geq \phi(s') a_H(1) \), which is possible only if \( s' \geq \hat{s} \) since \( a_L \leq \phi(\hat{s}) a_H(1) \) and \( \phi \) is decreasing. This contradicts our assumption and implies that \( \hat{S} = [0; \hat{s}] \). \( \square \)

**Proof of Proposition 7.** We start by showing that all contracts that satisfy constraints (22), (23) and (24) satisfy also the participation constraints. The participation constraint of a manager with \( s \in \hat{S} \) is
\[ q \left( a_H(1) w_H - \frac{ca_H^2(1)}{2} \right) + (1 - q) \left( a_L w_L - \frac{ca_L^2(1)}{2} \right). \]
Using (22) and (23) it becomes
\[ q \frac{ca_H^2(1)}{2} + (1 - q) \frac{ca_L^2}{2}, \]
which is always positive. Similarly, by substituting (22), (23) and (24) in the participation constraint of a manager with \( s \notin \hat{S} \) gives
\[ a_H(0) (qw_H + (1 - q) w_L) - \frac{ca_H^2(0)}{2} = \frac{c}{2} (qa_H(1) + (1 - q) a_L), \]
that is always positive.

Next, we show that constraint (25) is binding at equilibrium. Suppose, by way of obtaining a contradiction, that this is not true. If (25) is not binding, the expected profit of investors (27) is linear in \( s \) and
\[ \frac{\partial E_0 \left[ \Pi(a_H(1), a_L, \hat{s}) \right]}{\partial \hat{s}} = q (1 - q) (a_H(1) - a_L [(v_H - ca_H(1)) - (v_L - ca_L)]). \quad (51) \]
The optimal \( \hat{s} \) is 1 if (51) is positive and 0 otherwise. However, \( \hat{s} = 1 \) is not possible since \( a_L \) cannot be negative and constraint (25) would require \( a_L \leq \phi(1) a_H(1) < 0 \). On the other hand, if \( \hat{s} = 0 \), the first order conditions on \( a_H(1) \) and \( a_L \) would imply \( a_H(0) = (q w_H + (1 - q) w_L) / 2c \). It is easy to verify that the second best level of efforts (17) and (18) which satisfy the first order conditions are such that (51) is positive. Thus, \( \hat{s} \in (0; 1/2] \) and constraint (25) is binding at equilibrium.
In the following we show that the optimal \( \hat{s} \) is greater than \( s_1 \). Substituting (25) in (27) and rearranging terms gives

\[
E_0[\Pi(a_H(1), \hat{s})] = a_H(1)[q(v_H - ca_H) + (1 - q)\phi(\hat{s})(v_L - c\phi(\hat{s})a_H(1))]\hat{s} + \\
+ a_H(1)[q + (1 - q)\phi(\hat{s})][q(v_H - ca_H(1)) + (1 - q)(v_L - c\phi(\hat{s})a_H(1))](1 - \hat{s}).
\]

(52)

The investors’ problem simplifies to the choice of \( a_H(1) \in [0; 1] \) and \( \hat{s} \in (0; 1/2] \) that maximize (52). Differentiating (52) with respect to \( a_H(1) \) gives the following necessary and sufficient condition for an interior solution

\[
\begin{align*}
\frac{\partial E_0[\Pi(a_H(1), \hat{s})]}{\partial a_H(1)} &= -2qa_H(1)(q + 4\hat{s}^3(1 - q))c + \\
&+ \frac{q2(1 - q)(v_H - v_L)\hat{s}^2 + qv_H + (1 - q)v_L(q + 2(1 - q)\hat{s})}{(q + 2s(1 - q))^2} = 0.
\end{align*}
\]

Solving for \( a_H(1) \), we obtain

\[
a_H(1) = \frac{(2(1 - q)(v_H - v_L)\hat{s}^2 + qv_H + (1 - q)v_L)(q + 2(1 - q)\hat{s})}{(q + 4(1 - q)\hat{s}^3)c}.
\]

(53)

Substituting (53) in (52), deriving with respect to \( \hat{s} \) and rearranging terms gives the following necessary and sufficient condition for an interior solution

\[
\begin{align*}
\frac{\partial E_0[\Pi(a_H(1), \hat{s})]}{\partial \hat{s}} &= q(1 - q)\hat{s}2(2q + (1 - q)\hat{s}^3)(v_H - v_L) - 3(qv_H + (1 - q)v_L)\hat{s}, \\
&+ \frac{(2(1 - q)(v_H - v_L)\hat{s}^2 + qv_H + (1 - q)v_L)}{(q + 4(1 - q)\hat{s}^3)c} = 2a_H(1)\hat{s} - \frac{q(1 - q)\hat{s}}{c(q + 4(1 - q)\hat{s}^3)} \varphi(\hat{s}) = 0,
\end{align*}
\]

(54)

with \( \varphi(\hat{s}) = 2(q + (1 - q)\hat{s}^3)(v_H - v_L) - 3(qv_H + (1 - q)v_L)\hat{s} \). Since at equilibrium \( a_H(1) > 0 \), the first order condition (54) for an interior solution reduces to \( \varphi(\hat{s}) = 0 \).

Observe that

\[
\varphi'(\hat{s}) = 6(1 - q)\hat{s}^2(v_H - v_L) - 3(qv_H + (1 - q)v_L) \leq 0 : \iff -s_3 \leq \hat{s} \leq s_3,
\]

with \( s_3 = \sqrt{\frac{qv_H + (1 - q)v_L}{2(qv_H + (1 - q)v_L) - 3(qv_H + (1 - q)v_L)}} \). Hence, the function \( \varphi(\hat{s}) \) is decreasing for all \( \hat{s} \in [0; s_3] \), increasing for all \( \hat{s} > s_3 \) and the point \( \hat{s} = s_3 > 0 \) is a local minimizer of \( \varphi(\hat{s}) \). Depending on the parameters of the model, it can be \( \varphi(s_3) \leq 0 \). If \( \varphi(s_3) \geq 0 \), then \( \varphi(\hat{s}) \) is positive for all \( \hat{s} \in [0; 1/2] \) and the maximizer of \( E_0[\Pi(a_H(1)(\hat{s}), \hat{s})] \) in this interval is \( \hat{s} = 1/2 \). If \( \varphi(s_3) < 0 \), then \( \varphi(\hat{s}) \) is positive for all \( \hat{s} \in [0; s_M] \), negative for all \( \hat{s} \in [s_M; s_m] \) and positive for all \( \hat{s} \in [s_m; \infty) \), with \( \varphi(s_M) = \varphi(s_m) = 0 \) and \( s_m > s_3 > s_M > 0 \). This means that the function \( E_0[\Pi(a_H(1)(\hat{s}), \hat{s})] \), in the interval \([0; 1/2] \), has at most one interior local maximizer, \( s_M \) (when \( s_M < 1/2 \)). Moreover, \( s_M \) is strictly larger than \( s_1 \). Indeed,

\[
\varphi(s_1) = \frac{1}{2} + \frac{q^3(1 - q)}{4(qv_H + (1 - q)v_L)} \geq 0
\]

and since \( \varphi(\hat{s}) \) is decreasing for all \( \hat{s} < s_1 \) because \( s_3 > s_1 \), then \( \varphi(\hat{s}) > \varphi(s_1) > 0 \) for all \( \hat{s} < s_1 \) and \( s_1 < s_M \). Since the global maximizer of \( E_0[\Pi(a_H(1)(\hat{s}), \hat{s})] \) in the interval \([0; 1/2] \) can be either \( s_M \) or 1/2, we can conclude that the optimal \( \hat{s} \) is always greater than \( s_1 \).

To prove the last part of the proposition, note that
1. \( \varphi(\hat{s}) > 0 \) for all \( \hat{s} \in [0; 1/2] \) if and only if \( v_L < r_1(q)v_H \), with \( r_1(q) \equiv \frac{(2q^2(1-q))^{1/4} - q}{(1-q + 2q^2(1-q))^{1/4}} \).

Indeed, tedious algebra shows that

\[
\varphi(s_3) = 2q(v_H - v_L) - 2(qv_H + (1-q)v_L) \sqrt{\frac{qv_H + (1-q)v_L}{2(v_H - v_L)(1-q)}} > 0 : \iff \frac{v_L}{v_H} < r_1(q).
\]

This implies for all \( v_L < r_1(q)v_H \), the optimal \( \hat{s} \) is the border maximizer 1/2.

2. \( \varphi(\hat{s}) \) is a strictly decreasing function of \( v_L \) for all \( \hat{s} \in [0; 1/2] \).

Indeed, \( \partial \varphi(\hat{s})/\partial v_L = \frac{2q + (1-q)s^3 - 3(1-q)s}{\varphi'(s)} < 0 \) for all \( \hat{s} \in [0; 1/2] \). This implies for all \( v_L \geq r_1(q)v_H \), \( \varphi(s_3) \leq 0 \) and the local interior maximizer \( s_M > 0 \) exists.

3. \( s_M \) is a strictly decreasing function of \( v_L \) for all \( v_L \geq r_1(q)v_H \).

Indeed, from the implicit function theorem

\[
\partial s_M/\partial v_L = \left. \frac{-\partial \varphi(\hat{s})/\partial v_L}{\partial \varphi(\hat{s})/\partial \hat{s}} \right|_{s_M} = \frac{2(q + (1-q)s_M^3) + 3(1-q)s_M}{\varphi'(s_M)} < 0
\]

since \( 0 < s_M \leq s_3 \) and then \( \varphi'(s_M) \leq 0 \) for all \( v_L \geq r_1(q)v_H \).

4. \( \varphi(1/2) \leq 0 \) if and only if \( v_L \geq r_2(q)v_H \), with \( r_2(q) \equiv \frac{(1+q)}{(7+q)} \). Moreover, \( s_M = 1/2 \) if \( v_L = r_2(q)v_H \).

Indeed, algebraic calculus shows that

\[
\varphi(1/2) = \frac{v_H(1+q) - v_L(7+q)}{4} \leq 0 : \iff \frac{v_L}{v_H} \geq r_2(q).
\]

This implies that for all \( v_L \geq r_2(q)v_H \) (\( v_L < r_2(q)v_H \)), \( \hat{s} = 1/2 \) is a local border minimizer (maximizer) of \( E_0[\Pi(a_H(1)(\hat{s}), \hat{s})] \).

5. \( v_L \geq r_2(q)v_H \) implies that \( v_L \geq r_1(q)v_H \), whilst the converse is not true.

Indeed, algebraic calculus shows that

\[
r_2(q) - r_1(q) \propto 1 + 7q - 6(2q^2(1-q))^{1/4} \geq 0 \quad \text{for all } q \in [0; 1].
\]

This means that for all \( v_L \geq r_2(q)v_H \), the local interior maximizer \( s_M > 0 \) exists.

By combining points 3, 4 and 5, it follows that

\[
s_M > \frac{1}{2} \quad \text{for all } v_L \in [\max\{0; r_1(q)v_H\}; r_2(q)v_H], \quad (55)
\]

and

\[
s_M \leq \frac{1}{2} \quad \text{for all } v_L \geq r_2(q)v_H. \quad (56)
\]

By combining (55) and points 1, 4 and 5, it follows that for all \( v_L < r_2(q)v_H \), the optimal \( \hat{s} \) is the border maximizer 1/2. Finally, by combining (56) and point 4, it follows that for all \( v_L \geq r_2(q)v_H \), the optimal \( \hat{s} \) is the interior maximizer \( s_M \leq 1/2 \). \( \square \)
Proof of Corollary 1. Let us define the distance between equilibrium rewards when the good and the bad signals are privately observed by the manager as \( \Delta w(\hat{s}) = w_H(1)(\hat{s})(1 - \phi(\hat{s})) \) and the distance between second best rewards as \( \Delta w^{SB} \equiv \frac{v_H}{v_L}(1 - \frac{v_L}{v_H}) \). In order to show that \( \Delta w(\hat{s}) \) is always larger than \( \Delta w^{SB} \) we will prove the following claims.

Claim 1: \( \inf_{\hat{s} \in [s_1, 1]} w_H(1)(\hat{s}) \geq \frac{v_L}{v_H} \), for all \( \frac{v_L}{v_H} \in [0, 1] \).

The claim is immediately proved by noticing that \( \partial w_H(1)(\hat{s})/\partial \hat{s} \geq 0 \) for all \( \hat{s} \in (s_1, 1/2] \) and, then, \( \inf_{\hat{s} \in (s_1, 1/2]} w_H(1)(\hat{s}) = w_H(1)(s_1) = \frac{v_L}{v_H} \).

Claim 2: \( \inf_{\hat{s} \in [s_1, 1/2]} (1 - \phi(\hat{s})) \geq (1 - \frac{v_L}{v_H}) \), for all \( \frac{v_L}{v_H} \in [0, 1] \).

Observe that: \( \inf_{\hat{s} \in [s_1, 1/2]} (1 - \phi(\hat{s})) \geq (1 - \frac{v_L}{v_H}) \) if \( \sup_{\hat{s} \in (s_1, 1/2]} \phi(\hat{s}) \leq \frac{v_L}{v_H} \). Since \( \partial \phi(\hat{s})/\partial \hat{s} \leq 0 \) for all \( \hat{s} \in (s_1, 1/2] \), then \( \sup_{\hat{s} \in (s_1, 1/2]} \phi(\hat{s}) = \phi(s_1) = \frac{v_L}{v_H} \). □

Proof of the accommodating equilibrium. In the accommodating scenario, the investors problem \( \mathcal{P}^A \) is to choose the levels of effort, \( a_L^A \) and \( a_H^A \), and the contract, \( C^A = \{0, w_L, w_H\} \), that maximize their expected profits (11), subject to the limited liability constraints and the incentive constraints, given \( \lambda = 0 \). From (7), \( w_L = ca_L \). From (8) and \( \lambda = 0 \), we obtain

\[
qw_H + (1 - q)w_L = ca_H. \tag{57}
\]

Substituting out \( \lambda = 0 \) and (57) in (11), we have

\[
E_0[\Pi(a_H)] = a_H[q(v_H - w_H) + (1 - q)(v_L - w_L)] = a_H[(qv_H + (1 - q)v_L) - ca_H], \tag{58}
\]

which does not depend on \( a_L \). Indeed, in the accommodating scenario, the manager never recollects the bad signal, hence \( a_L \) is out of the equilibrium path. Differentiation of (58) with respect to \( a_H \) gives the following necessary and sufficient condition

\[
\frac{\partial E_0[\Pi(a_H)]}{\partial a_H} = (qv_H + (1 - q)v_L) - 2ca_H = 0, \tag{59}
\]

whence, solving for \( a_H \):

\[
a_H^A = \frac{(qv_H + (1 - q)v_L)}{2c}. \tag{60}
\]

Substituting out (60) in (57), we have

\[
qw_H^A + (1 - q)w_L^A = \frac{(qv_H + (1 - q)v_L)}{2}, \tag{61}
\]

Hence, the accommodating equilibrium is characterized by any contract \( C^A = \{0, w_L^A, w_H^A\} \), such that \( qw_H^A + (1 - q)w_L^A \) satisfies (61), and by the levels of effort \( a_H^A \) given by (60) and \( a_L^A \) given by (28).

Finally, since \( a_H^A = a_P \) and \( qw_H^A + (1 - q)w_L^A = w_P \) (see (57) and (43)), the accommodating equilibrium is welfare equivalent to the pooling equilibrium. □

References


