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IDENTIFIABILITY FOR MIXTURES OF DISTRIBUTIONS FROM A LOCATION-SCALE FAMILY WITH UNIFORMS

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Abstract

In this paper we study the indentifiability of a class of mixture models where a finite number of one-dimensional location scale distributions is mixed with a finite number of uniform distributions on an interval. We define identifiability and we show that, under certain conditions, the afore-mentioned class of distributions is identifiable.

Keywords

Mixture models, identifiability, robustness

1 Introduction

Mixture models are widely used in many contexts. In cluster analysis and classification, mixture distributions have the role to model data with group-structures. Banfield and Raftery (1993) called this approach “model based clustering”. Several different mixture models have been studied and estimation methods proposed over the years. Normal mixtures and the related theory of maximum likelihood estimation played an important role in the research. However, maximum likelihood estimates for a wide class of location-scale mixtures are not robust (see Hennig, 2004). Fraley and Raftery (1998) treats outliers as points coming from some unknown mixture component interpreted as “noise”. They proposed a model where a uniform distribution is mixed with the Gaussians in order to account for noise. Hennig (2004) defined robustness measures for clusters, and he studied the properties of several robust model based clustering methods including Gaussian mixtures with uniform noise, uniform-normal mixture from now onward. However, identifiability for this model has not been studied. Teicher (1963) shows that univariate normal mixtures are identifiable, while in general mixture of uniform distributions are not. The proposal of Banfield and Raftery (1993) consists of a mixture with arbitrary number of normals with the addition of a uniform component, the authors proposed estimation methods as well as a computational procedures to get estimates.

The aim of this paper is to define and show identifiability for a general class of models which includes the afore-mentioned uniform-normal mixture. In a nutshell, in the context of parametric family of distributions, identifiability means that different parameter values lead to different distributions. If a distribution is not identifiable, the same parameter can define more than one distribution; this would affect any consistency statement about estimators. Identifiability is also relevant from the practical point of view. In fact, if a population is represented by some non-identifiable parametric distribution this means that the population under study can be represented by more than one parameter. Here we do not constraint the number of uniform components, and we do not confine ourself to the case of Gaussian mixtures. Our model of interest will be a finite mixture of location-scale distributions mixed with a finite number of uniform distributions. This means that we have a mixture with components belonging to different families. The original definition of identifiability for mixtures was given by Teicher (1961). This definition does not take into account the situation where distributions from different families are mixed and it is required that the family memberships of the components are identified. For the class of models under study we want to identify the parameters, the number of components and their family memberships. Thus we will propose a definition of identifiability which extends the one proposed by Teicher (1961). Yakowitz and Spragins (1968) and Atienza, Garicia-Heras, and Munoz-Pichardo (2006) provided sufficient conditions to show identifiability under Teicher's definition. However, The afore-mentioned sufficient conditions are not applicable in our situation. Heuristically, the idea under our identifiability proof is simple. The distribution function of the class of mixture models under study is not differentiable at a set of points which coincides with the set of the uniform parameters. Thus the points where the mixture distribution is not differentiable will identify the uniform parameters; then we remove the uniform components and we will identify the remaining components. Even though the idea behind this identifiability results is simple, the formal implementation is not easy.

The paper is organized as follows: first we introduce motivations for the class of models under study; in section 2 we introduce the general definition of identifiability given by Teicher (1961); in section 3 we define the class of models under study and in section 4 we extend the Teicher's definition of identifiability; in section 5 we state and prove some identifiability results for the class of models under study; finally in section 6 we draw conclusions and outline some future works.

2 *Identifiability of mixture distributions*

Let $\mathcal{P}_\theta$ be a family of probability measures indexed by some unknown – finite or infinite dimensional – index $\theta \in \Theta$ which we call parameter. We observe an experiment generated by some member of $\mathcal{P}_\theta$. The main problem of statistical inference is to infer θ based on observed data. Instead, identification is a pre-inferential problem which is devoted to assess whether with data at hand it is possible to state that different parameter values correspond to different probability measures $P \in \mathcal{P}_\theta$, where the meaning of the word "different" has to be specified. Roughly speaking indentifiability means that there exists a sort of one-to-one correspondence between the indexes $\theta \in \Theta$ and $P \in \mathcal{P}_\theta$. The first account of identification of mixture models was given by Feller (1943) and since then many results extended that work in several directions (we shall review those results in the following paragraphs).

Identifiability is a general concept that has to be carefully defined depending on the context. The very first definition of identifiability for finite mixtures was formalized first by Teicher (1961). Let $\mathscr{F} := \left\{F(x;\theta) : \quad x \in \mathbb{R}^q, \theta \in \mathbb{R}^k\right\}$ be a family of distribution functions over $\mathbb{R}^q$ indexed by a point θ in a Borel subset of $\mathbb{R}^k$ such that $F(\cdot;\theta)$ is measurable on $\mathbb{R}^q \times \mathbb{R}^k$. Let $G \in \mathscr{G}$ be a k-dimensional distribution function with the underlying measure assigning total mass to $\mathbb{R}^k$. Let $\mathscr{H}$ be a family of distribution functions. We consider a map $Q : \mathscr{G} \longrightarrow \mathscr{H}$, where its image is defined as $Q(G) = H$, $H(x) = \int_{\mathbb{R}^k} F(x;\theta) dG(\theta)$. Following Teicher (1961), the mixture model generated by the family $\mathscr{F}$ with mixing distribution in $\mathscr{G}$ is said to be identifiable if given $F \in \mathscr{F}$, then Q is a one-to-one map of $\mathscr{G}$ onto $\mathscr{H}$. As we have already noticed, when G is discrete, the set of all finite mixtures $\mathscr{H}$ of the family $\mathscr{F}$ is simply the convex hull of $\mathscr{H}$. Identifiability of the mixture models means that the convex hull $\mathscr{F}$ has a uniqueness representation property which can be translated into following:

Definition 1 (Single Family Identifiability)**.** Let $\mathscr{H}$ be the class of finite mixtures generated by the class $\mathscr{F}$ with discrete mixing distribution. Given

$$H(x,\eta)=\sum_{j=1}^{s}\pi_j F(x;\theta_j), \quad \pi_j>0, \theta_j\neq\theta_r \quad \forall j,r=1,2,\dots,s, \quad j\neq r,$$

and

$$H(x,\eta^*)=\sum_{i=1}^{t}\pi_i^* F(x;\theta_i^*), \quad \pi_i^*>0, \theta_i^*\neq\theta_k^* \quad \forall i,k=1,2,\dots,t, \quad i\neq k;$$

if $H(\cdot,\eta)=H(\cdot,\eta^*)$ implies that $s=t$, and there is some permutation $\bar{j}$ of the indexes $j=1,2,\dots,s$ such that $\pi_j=\pi_{\bar{j}}^*$ and $\theta_j=\theta_{\bar{j}}^*$, then we say that $\mathscr{F}$ generates identifiable finite mixture distributions.

The definition above has been used to study the identification of a number of models. The wording "single family identifiability" will be clearer thereafter. Feller (1943) started the literature about identifiability of mixtures studying gamma mixture models. Teicher (1961) formalized the definition of identifiability for general mixture models. He extended the results in Feller (1943) showing the identifiability of finite mixtures generated by Poisson distributions. He also showed that models based on mixtures of uniform and binomial distributions are not identifiable. Teicher (1963) gave a sufficient condition for identifiability of a general class of finite mixture models and showed that mixtures based on univariate Gaussian distributions are identifiable. Yakowitz and Spragins (1968) defined identifiability for classes of finite mixtures (Definition 1) and gave a necessary and sufficient condition for the identifiability of such models. The main theorem in Yakowitz and Spragins (1968) states that given a discrete mixing distribution the class $\mathscr{F}$ generates identifiable mixtures if and only if $\mathscr{F}$ is a linearly independent set over the field of the real numbers. They apply their theory showing that exponential distributions, multivariate Gaussian distributions, Cauchy distributions and negative binomials generate identifiable mixture models. Atienza, Garicia-Heras, and Munoz-Pichardo (2006) weakened the assumptions of the sufficient conditions given by Teicher (1963) and showed that mixtures of Log-Gamma distributions and mixtures of Lognormal, Gamma and Weibull distributions are identifiable with the respect to the Definition 1.

3 Model definitions

In this section we introduce the notation and the main assumptions about the general model under study. Let $0 < s < \infty$ be the number of components in our mixture distribution, and let q be the number of uniform components $0 < q < s$ in the mixture. Let X be a real valued random variable distributed according to the following distribution function:

$$G(x;\eta) = \sum_{k=1}^{q} \pi_k U(x;\theta_k) + \sum_{l=q+1}^{s} \pi_l \Phi(x;\theta_l), \tag{3.1}$$

where $\eta = (\pi, \theta)$, $\pi = (\pi_1, \pi_2, \ldots, \pi_s)$, $0 < \pi_j < 1$, $\sum_{j=1}^{s} \pi_j = 1$. Here $\theta = (\theta_1, \theta_2, \ldots, \theta_s)$, where $\theta_k = (a_k, b_k)$, a_k and b_k take values on the real line, and $-\infty < a_k < b_k < +\infty$ for each $k = 1, 2, \ldots, q$. Thus $\pi \in (0,1)^s$, $\theta_k \in \Theta_1 := \mathbb{R}^{2q}$ for $k = 1, 2, \ldots, q$. The parameter θ_l lies in some finite dimensional space Θ_2 for each $l = q+1, q+2, \ldots, s$. Furthermore the parameter space is denoted by $\Gamma := (0,1)^s \times \mathbb{R}^{2q} \times \Theta_2^{s-q}$. U is the uniform distribution function, i.e.

$$U(x;\theta_k) = \frac{x - a_k}{b_k - a_k} \mathbf{1}_{[a_k, b_k]}(x) + \mathbf{1}_{(b_k, +\infty)}(x),$$

$k = 1, 2, \ldots, q$, with $\mathbf{1}_A$ being the indicator function of the set A. The distribution function U has the density

$$u(x;\theta_k) = \frac{\mathbf{1}_{[a_k, b_k]}(x)}{b_k - a_k}.$$

The distribution function Φ belongs to a family of distributions satisfying

Assumption 1. $\Phi(x;\theta)$, $\theta \in \Theta_2$, is absolutely continuous with the respect to Lebesgue measure. It has density $\phi(x;\theta), \theta \in \Theta_2$, which is continuous both with the respect to $x \in \mathbb{R}$ and $\theta \in \Theta_2$.

For notational convenience we will often rewrite the model in (3.1) as

$$G(x, \eta) := \sum_{j=1}^{s} \pi_j F_{v_j}(x;\theta_j) \tag{3.2}$$

where $v_j = \{1, 2\}$ for $j = 1, 2, \ldots, s$, when $v_j = 1$ then $F_{v_j} = U$, whenever $v_j = 2$ then $F_{v_j} = \Phi$. Moreover $g(x; \eta)$ will denote the density of $G(x; \eta)$.

4 Identifiability of "heterogeneous" mixtures

In section 2 we introduced the identifiability problem for general finite mixtures. In this section we define and study the identifiability of a class of models which consists of a mixture of distributions coming from different families. We are in a situation where a finite number of distributions belonging to a general class of continuous distributions is mixed with a finite number of uniform distributions. We call such a mixture distribution: heterogeneous. Here the wording heterogeneous mixtures means that the components in the mixture belong to different families of distributions. Such a statistical model can be very attractive in all those situations where the underlying heterogeneity in the data generating process is strong enough to let us consider that groups of observations come from populations with completely different features. In fact the uniform distribution here is introduced as a probabilistic model for noise, while Φ should represent the probabilistic structure of the population under study. Here we do not require that the number of components is known, nor do we require that the number of components belonging to each of the families of distributions is known. This situation is somehow more general than that of the model proposed by Fraley and Raftery (1998), where the number of uniform components is considered as fixed and known. In fact in the uniform–normal mixture model proposed by Fraley and Raftery (1998) the number of uniform components is fixed to be one.

We now refer to section 2 where we presented the definition of identifiability as given by Teicher (1961). Let us assume that $\mathscr{F}_k$, with $k = 1, 2, \ldots, m$, are all families of probability distribution functions. For each k, $F_k(x; \theta) \in \mathscr{F}_k$, $\theta \in \Theta_k$, and Θ_k is some finite dimensional parameter space. A general element of the set of finite mixtures generated by the class $\mathscr{E} = \cup_{k=1}^{m} \mathscr{F}_k$ will be called heterogenous mixture distribution. Teicher's definition of identifiability does not require that the number of components in each family is identified. However this is relevant in a situation where membership to different population components have different meaning. In our model for example we want to distinguish between noise components and non-noise components, and we want that the number of distributions belonging to each of the family composing the mixture is identified. To see why definition 1 does not take into account the identifiability of family memberships let us consider some results in the paper by Atienza, Garicia-Heras, and Munoz-Pichardo (2006). The authors studied the identifiability a model proposed by Marrazzi, Paccaud, Ruffieux, and Beguin (1998) in the context of fitting the length of stay in a hospital; the model is a mixture of three components: one Lognormal, one Gamma and one Weibull distribution. Atienza, Garicia-Heras, and Munoz-Pichardo (2006) gave a new sufficient condition for identifiability of finite mixtures following Teicher's definition, and based on this they showed the identifiability of the afore–mentioned class of mixtures. However, following the proof of their theorem 3 it is clear that for some value of the parameter space, a component having Gamma distribution cannot be distinguished from a component having Weibull distribution, thus the number of components belonging to each family cannot be identified.

Here we will give a definition of identifiability which is similar to the one given by Teicher (1961) but it adds some more restrictions so that the family membership of components is taken into account in the sense explained above. It should be now clear why we named the identifiability defined by Teicher as "single family identifiability". Before we give our definition, let us introduce some more notations.

We will consider the set of all heterogenous finite mixtures generated by $\mathscr{E}$ with discrete mixing distribution. Let $s < +\infty$ be the number of components of the heterogenous mixture, and let $c = \{n_1, n_2, \ldots, n_m\}$ be a set of natural numbers where n_k, $k = 1, 2, \ldots, m$, indicates the number of distributions belonging to $\mathscr{F}_k$ being present in the mixture. From now onward it is understood that c is finite, and of course it must be $s = \sum_{k=1}^{m} n_k$. We will denote c as the "composition" index. $\mathscr{H}$ is the family of all the finite mixtures generated from $\mathscr{E}$ with discrete mixing distribution. A general element of $\mathscr{H}$ will be $H_c(x; \eta) = \sum_{j=1}^{s} \pi_j F_{k_j}(x; \theta_j)$, where $k_j \in \{1, 2, \ldots, m\}$

for $j = 1, 2, \ldots, s$, is the index which expresses the "family membership" of the jth component (e.g. $k_2 = 1$ means that the distribution of the second mixture component belongs to $\mathscr{F}_1$). The parameter η lies in the parameter set Ω, and $\eta = (\pi_1, \ldots, \pi_s, \theta_1, \ldots, \theta_s)$. We will consider the following definition:

Definition 2 (Global Identifiability)**.** Let $\mathscr{H}$ the class of finite mixtures generated by the class $\mathscr{E}$. Let $\mathscr{H}^\star \subseteq \mathscr{H}$, and $H_c \in \mathscr{H}^\star$. Given

$$H_c(x, \eta) = \sum_{j=1}^{s} \pi_j F_{v_j}(x; \theta_j), \pi_j > 0, \theta_j \neq \theta_r \quad \forall j, r = 1, 2, \ldots, s, \quad j \neq r,$$

and

$$H_{c^*}(x, \eta^*) = \sum_{j=1}^{z} \pi_j^* F_{v_j}(x; \theta_j^*), \pi_j^* > 0, \theta_j^* \neq \theta_k^* \quad \forall j, k = 1, 2, \ldots, z, \quad j \neq k;$$

if $H_c(\cdot, \eta) = H_{c^*}(\cdot, \eta^*)$ implies $s = z$, and that there exists a permutation $\bar{j}$ of the indexes $j = 1, 2, \ldots, s$ such that $\pi_j = \pi^*_{\bar{j}}$, $\theta_j = \theta^*_{\bar{j}}$, $k_j = k_{\bar{j}}$, for $k_j, k_{\bar{j}} \in \{1, 2, \ldots, m\}$, and $c = c^*$, then we say that $\mathscr{E}$ generates globally identifiable finite mixture distributions in $\mathscr{H}^\star$.

As highlighted before, we use the wording *global identifiability* to make a distinction between the notion of identifiability given in definition 2 with the one given in definition 1 which refers to Teicher's definition. With reference to definition 2, we require that the permutation of the component label (the index j) is constructed so that for each family $\mathscr{F}_k$ we identify the parameters, obtaining $\pi_j F_{k_j}(x; \theta_j) = \pi^*_{\bar{j}} F_{k_{\bar{j}}}(x; \theta^*_{\bar{j}})$, and at the same time we require that the number of distributions identified in the family $\mathscr{F}_k$ is consistent with the composition index c. To see the relevance of this argument let us refer to the the model proposed by Banfield and Raftery (1993). In that case we require that not only the uniform parameters, the Gaussian parameters and all proportions are identified but we also require that it is also possible to identify the number of noise components and Gaussian components.

5 The identifiability of the model with uniform noise

First, we will introduce some notations and assumptions and we also reconcile the exposition here with the notation used in the previous sections. We consider the model defined in section 3 with the addition of the following definitions: (i) $\mathscr{F}_1$ is the family of all uniform distributions with support on an interval; (ii) $\mathscr{F}_2$ is a family of one dimensional distributions satisfying assumption 1 and finite mixtures generated by $\mathscr{F}_2$ are indentifiable in the sense of the definition 2.

Let $n_1 = q, n_2 = s - q$ and let $c = \{q, s-q\}$ be the composition index. $\mathscr{H}$ is the family of finite mixtures generated by $\mathscr{E} = \mathscr{F}_1 \cup \mathscr{F}_2$, obtained by mixing q distributions from $\mathscr{F}_1$ and $s-q$ distributions from $\mathscr{F}_2$. The function $g_c(x;\eta)$ will denote the density of $G_c(x;\eta)$ likewise the model defined in section 3. $G_c(x;\eta)$ is an element of $\mathscr{H}$, with $\eta \in \Gamma$. $\mathscr{H}^\star \subset \mathscr{H}$ is the set of mixtures generated by $\mathscr{E}$ such that if $G_c(x;\eta)$ belongs to $\mathscr{H}^\star$, then $[a_t, b_t] \cap [a_r, b_r] = \emptyset$ for all $r, t = 1, 2, \ldots, q$ and $r \neq t$.

To show identifiability here we will make use of arguments based on derivatives so that it is necessary to introduce some more notations before to state and prove the next result. We notice that the density $g_c(x;\eta)$ it is discontinuous at a finite number of points, namely at $x \in W := \{a_1, b_1, a_2, b_2, \ldots, a_q, b_q\}$. Thus by properties of the Riemann integral, $dG_c(x;\eta)/dx = g_c(x;\eta)$ at all $x \in \mathbb{R} \backslash W$. However, right and left derivatives of G_c at all points in W exist and can be found by taking right and left limits of derivative quotients. The notation $D^-_y(\eta)$ and $D^+_y(\eta')$ stands for the left and right derivative of G_c respectively, and these derivatives are evaluated at a point y when the parameter vector is η', i.e.

$$D^-_y(\eta') = \lim_{t \uparrow 0} \frac{G_c(y+t;\eta') - G_c(y;\eta')}{t},$$

$$D^+_y(\eta') = \lim_{t \downarrow 0} \frac{G_c(y+t;\eta') - G_c(y;\eta')}{t}.$$

Computing these derivatives for the model (3) and for $h = 1, 2, \ldots, q$ will give us

$$D^-_{a_h}(\eta) \;=\; \sum_{l=q+1}^{s-q} \pi_l \phi(a_h;\theta_l);$$

$$D^+_{a_h}(\eta) \;=\; \frac{\pi_h}{b_h - a_h} + \sum_{l=q+1}^{s-q} \pi_l \phi(a_h;\theta_l);$$

$$D^-_{b_h}(\eta) \;=\; \frac{\pi_h}{b_h - a_h} + \sum_{l=q+1}^{s-q} \pi_l \phi(b_h;\theta_l);$$

$$D^+_{b_h}(\eta) \;=\; \sum_{l=q+1}^{s-q} \pi_l \phi(b_h;\theta_l).$$

Theorem 1. *The class $\mathscr{E} = \mathscr{F}_1 \cup \mathscr{F}_2$ generates globally identifiable heterogeneous mixtures in $\mathscr{H}^\star \subset \mathscr{H}$.*

Proof. Let us assume that $G_c(x;\eta) = G_{c^*}(x;\eta^*)$, i.e.

$$\sum_{j=1}^{s} \pi_j F_{v_j}(x;\theta_j) = \sum_{j=1}^{z} \pi^*_j F_{v_j}(x;\theta^*_j), \tag{5.1}$$

for every x, $v_j \in \{1,2\}$ and $j = 1,2,\dots,s,\dots z$, i.e. without loss of generality we assume that $s \le z$. For a given function $f(y,z)$ differentiable at least on a subset of its own domain, we define the set

$$S_f(z) := \left\{ y : \; \frac{\partial^-}{\partial y} f(y,z) \neq \frac{\partial^+}{\partial y} f(y,z) \right\};$$

provided that all at points in $S_f(z)$ left and right partial derivatives of f exist. The assumption that $G_c(x;\eta) = G_{c^*}(x;\eta^*)$ implies that $S_{G_c}(\eta) =$

$S_{G_{c^*}}(\eta^*)$. If $\#(A)$ stands for the cardinality of the set A, then $\#(S_{G_c}(\eta)) = \#(S_{G_{c^*}}(\eta^*)) = 2q$ which means that the number of the uniform components q is uniquely identified. Given a finite set $A := \{y_1, y_2, \ldots, y_n\}$, with $y_i \in \mathbb{R}$ all $i = 1, 2, \ldots, n$, $\mu(A) \in \mathbb{R}^n$ denotes a vector where the components are all the elements of A, furthermore $\bar{\mu}(A)$ is defined as $\bar{\mu}(A) = (y_{(1)}, y_{(2)}, \ldots, y_{(n)})$ where $y_{(i)}$ is such that $y_{(i)} \leq y_{(i+1)}$ all $i = 1, 2, \ldots, n-1$. Now, $\bar{\mu}(S_{G_c}(\eta)) = (x_{(1)}, x_{(2)}, \ldots, x_{(2q)}) = \bar{\mu}(S_{G_{c^*}}(\eta^*)) = (x^*_{(1)}, x^*_{(2)}, \ldots, x^*_{(2q)})$. We take a set of pairwise different indexes $r_i \in \{1, 2, \ldots, s\}$ with $i = 1, 2, \ldots, q$ and we fix

$$\begin{aligned}
\theta_{r_1} &= (a_{r_1}, b_{r_1}) = (x_{(1)}, x_{(2)}),\\
\theta_{r_2} &= (a_{r_2}, b_{r_2}) = (x_{(3)}, x_{(4)}),\\
&\vdots\\
\theta_{r_q} &= (a_{r_q}, b_{r_q}) = (x_{(2q-1)}, x_{(2q)}).
\end{aligned}$$

Let us take another set of pairwise different indexes $t_i \in \{1, 2, \ldots, s\}$ with $i = 1, 2, \ldots, q$ and we fix

$$\begin{aligned}
\theta^*_{t_1} &= (a^*_{t_1}, b^*_{t_1}) = (x^*_{(1)}, x^*_{(2)}),\\
\theta^*_{t_2} &= (a^*_{t_2}, b^*_{t_2}) = (x^*_{(3)}, x^*_{(4)}),\\
&\vdots\\
\theta^*_{t_q} &= (a^*_{t_q}, b^*_{t_q}) = (x^*_{(2q-1)}, x^*_{(2q)}).
\end{aligned}$$

It is clear that $\bar{\mu}(S_{G_c}(\eta)) = \bar{\mu}(S_{G_{c^*}}(\eta^*))$ implies that $\theta^*_{r_i} = \theta^*_{t_i}$ all $i = 1, 2, \ldots, q$. Let us consider the equation

$$\left(D^+_{a_{r_i}}(\eta) - D^-_{a_{r_i}}(\eta)\right)(b_{r_i} - a_{r_i}) = \left(D^+_{a^*_{t_i}}(\eta^*) - D^-_{a^*_{t_i}}(\eta^*)\right)(b^*_{t_i} - a^*_{t_i}),$$

for all $i = 1, 2, \ldots, q$. These equations imply that $\pi_{r_i} = \pi^*_{t_i}$ all $i = 1, 2, \ldots, q$. Hence, we have that there exists a permutation $\bar{j}$ of the indexes $j = 1, 2, \ldots, s, \ldots z$ such that if $j = r_i$ then $\bar{j} = t_i$, for which

$\theta_j = (a_j, b_j) = \theta^*_{\bar{j}} = (a^*_{\bar{j}}, b^*_{\bar{j}})$, $\pi_j = \pi^*_{\bar{j}}$ and $v_j = v_{\bar{j}} = 1$. By this we have identified the number of uniform components, and all their parameters. Without loss of generality let us assume that $r_i = t_i$ for all $i = 1, 2, \ldots, q$ and that $r_1, r_2, \ldots, r_q = 1, 2, \ldots, q$.

For $j = q+1, q+2, \ldots, s, \ldots, z$ all the mixture components belong to $\mathscr{F}_2$ and q is identified as well. We consider the one-to-one transformation $\tilde{\pi}_j = \pi_j/(1 - \sum_{j=1}^{q} \pi_j)$ for $j = q+1, q+2, \ldots, s, \ldots, z$; and $\tilde{\pi}^*_j$ is defined analogously. Note that the denominator of $\tilde{\pi}_j$ is identified, in fact it depends on $\pi_1, \pi_2, \ldots, \pi_q$ which have been already identified. By (5.1) and the previous results we can write

$$\sum_{j=q+1}^{s} \tilde{\pi}_j F_{v_j}(x, \theta_j) = \sum_{j=q+1}^{z} \tilde{\pi}^*_j F_{v_{\bar{j}}}(x, \theta^*_j). \tag{5.2}$$

By assumption the class of finite mixtures over $\mathscr{F}_2$ is identifiable with respect to definition 1, thus we have that: (i) $s = z$ and these indexes are identified; (ii) there exists some permutation $\bar{j}$ of indexes $j = q+1, q+2, \ldots, s$ such that $\tilde{\pi}_j = \tilde{\pi}^*_{\bar{j}}$ and $\theta_j = \theta^*_{\bar{j}}$. But, $\tilde{\pi}_j = \tilde{\pi}^*_{\bar{j}}$ implies $\pi_j = \pi^*_{\bar{j}}$. Thus the $s - q$ components belonging to $\mathscr{F}$, their parameters and their mixing proportions are identified. The proof is completed noting that having identified q and s it also results that $c = c^*$. □

Given the proposition above we can easily get the next result.

Corollary 1. *Let $\mathscr{F}_2$ be the class of Gaussian distributions, then the class $\mathscr{F} = \mathscr{F}_1 \cup \mathscr{F}_2$ generates globally identifiable mixtures in $\mathscr{H}$.*

Proof. The result follows easily by noting that: (i) Gaussian distributions clearly satisfy assumption 1; (ii) they are partially identifiable by theorem 3 in Yakowitz and Spragins (1968). □

We defined $\mathscr{F}_1$ so that it contains uniform distributions having not intersecting support. The reason for this should be clear. Let us assume that $\mathscr{F}_1$ contains all uniform distribution with support on a real interval, and let us consider the following mixture distribution

$$\frac{1}{3}U(x; 0, 2) + \frac{1}{3}U(x; 2, 4) + \frac{1}{3}F(x; \theta),$$

where F is some distribution function satisfying assumption 1. We notice that $1/3U(x;0,2) + 1/3U(x;2,4) = 2/3U(x;0,4)$ so that identifiability does not hold. In fact, not only the parameters of the uniform distributions are not identifiable but also the composition index referring to the uniform components would not be identifiable.

6 Conclusions

In this paper we studied the identifiability for some class of mixture distributions. We gave a new definition of identifiability which take into account heterogeneity in the mixture, and we showed that a wide class of mixture with uniform components are identifiable. In the literature the model consisting of a Gaussian-uniform mixture have been proposed to overcome problems of robustness (see Fraley and Raftery, 1998). In this proposal the number of uniform components is fixed to be one, but we could be interested in determining whether the number of noise components is one or zero. The estimation of the number of noise components requires that it is identified. We extended this model to the case when more then one uniform components is added to a mixture of a class of continuous location-scale mixtures and we showed that the resulting mixture is identifiable. In a future paper we will study the estimation of such a model and we will explore the related computational issues.

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