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A Locally Adaptive Bandwidth Selector For Kernel Based Regression

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Abstract

The selection of the smoothing parameter represents a crucial step in the local polynomial regression, because of the implications on the consistency of the nonparametric regression estimator and because of the difficulties in the implementation of the selection procedure. Moreover, to capture the complexity of the unknown regression curve, a local variable bandwidth is needed, which determines an increase in the efficiency and computational costs of such algorithms. This paper focuses on the problem of the automatic selection of a local bandwidth. We propose a slightly different approach with respect to the traditional ones, which does not require additional computational effort. The empirical performance of the method is shown in the paper through a simulation study.

Keywords: *nonparametric regression; variable bandwidth selection; derivative estimation; neural networks; local polynomials; dependent data.*

1 Context and aims

Nonparametric estimation is particularly useful in the context of model selection. Volatility analysis, trend estimation, forecasts, represent only some of the objectives of statistical analysis which imply the selection of a suitable parametric model for the data generating process. As a consequence, reliable considerations about the risk of misspecification and robustness of the results are generally required. In order to validate the analysis through the empirical evidence, the selection of the parametric model is usually done through the comparison with some nonparametric estimation.

Kernel based estimators represent, maybe, the most used nonparametric tool for the estimation of a regression function. Among them, the Nadaraya-Watson estimator and the local polynomial estimator of order p (linear, quadratic, etc...). The good properties of local polynomial estimators (which include the Nadaraya-Watson estimator as a special case) have been analyzed several times. See, above all, the book of Fan & Gijbels (1996). However, such good properties are often challenged by the misspecification of the tuning parameter, the *bandwidth* of the kernel function, which regulates the smoothness of the estimated function. The difficulties in specifying such tuning parameter may do vanish at all the advantages in using these nonparametric tools. Therefore, many automatic data-driven bandwidth selection procedures have been proposed so far in the literature. We refer to Fan & Gijbels (1996), Loader (1999) and references therein for a deep review on the topic.

The aim of this work is to propose a new procedure for the automatic selection of the smoothing parameter, based on a slightly different approach with respect to the traditional ones. Just as an example of application, we focuses on the problem of estimating the volatility function of dependent data through the local polynomial estimator. However, our procedure can be easily adapted to other contexts, such as the estimation of a generic conditional moment or the estimation of some derivative function, both for dependent and independent data. For this reason, we tried to expose the theory by using a notation the more general and standard as possible, in order to make comparisons easier.

The rationale of our proposal is the following. Consider a local polynomial estimation of a regression function. It is usually suggested to choose the bandwidth by minimizing some estimated measure of the mean squared error (MSE). We start from the *plug-in* point of view, that is to consider the asymptotic MSE and then to estimate the unknown functionals which appear therein. Then we adopt the approach proposed in Giordano & Parrella (2007), that is to estimate such functionals through the neural network technique, in order to avoid the use of pilot bandwidths. Finally, we extend here the result in two directions. From the one hand, we consider the problem of local adaptation of the bandwidth; for this problem we base on the idea of Fan & Gijbels (1995) of splitting the support of the function into subintervals and then estimating global bandwidths on each subinterval. On the other hand, we remove the assumption of homoschedasticity in the model, which is instead assumed by Giordano & Parrella (2007) and Fan & Gijbels (1995). This represents an interesting extension of our previous setup, and it is a must when analyzing economic and financial time series.

In the following two sections we give the assumptions which have to be considered in this paper, together with the notation involved with local polynomial regression. Section 4 gives the definition of the asymptotical optimal bandwidth for the local polynomial estimator. Our procedure is described in detail in section 5. Finally, section 6 reports the results of a simulation study devoted to the assessment of the empirical performance of our procedure and some concluding remarks.

2 Setup

Consider the process $\{Y_t, X_t\}$, where $X_t \in \mathbb{S}_X \subseteq \mathbb{R}$ and $Y_t \in \mathbb{R}$, and define the following nonparametric regression model

$$Y_t = m(X_t) + \sigma(X_t)\varepsilon_t, \qquad t = 1, 2, \ldots. \tag{1}$$

The errors ε_t are real random variables independent from X_t, for which $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = 1$, $\forall t$. Given model (1), we consider the generic problem of estimating the conditional regression function

$$m_\phi(x) = E\left\{\phi(Y_t)|X_t = x\right\}, \tag{2}$$

which includes several special cases by suitable defining the function $\phi(\cdot)$. Note, for example, that the conditional mean function is given by considering $\phi_1(x) = x$; the conditional second moment function is given by considering $\phi_2(z) = x^2$; the indicator function $\phi_3 = \mathbb{I}(X_t \leq x)$ can be used, instead, in order to define the conditional distribution function, and so on. Finally, model (1) can be extended to the nonparametric ARCH model by putting $X_t = Y_{t-1}$. Given a realization of the process $\{Y_t, X_t; t = 1, \ldots, n\}$, the unknown function $m_\phi(\cdot)$ and its derivatives can be estimated nonparametrically using the local polynomial estimators (LPE).

Here we present the assumptions we have to consider for the process.

(a1) The errors have continuous and positive density function f_ε, and

$$E(\varepsilon_t^2) = 1, \quad E(\varepsilon_t) = E(\varepsilon_t^3) = 0, \quad E|\varepsilon_t|^\delta < \infty,$$

for some $\delta \geq 4$.

(a2) The function $m(\cdot)$ has continuous second derivative on $\mathbb{S}_X$. The function $\sigma(\cdot)$ is positive and it has continuous second derivative on $\mathbb{S}_X$.

(a3) For dependent data, there exist the constants $M_1 > 0$ e $M_2 > 0$ such that, for $y \in \mathbb{S}_X$

$$|m(y)| \leq M_1(1 + |y|), \; |\sigma(y)| \leq M_2(1 + |y|), \; M_1 + M_2\left[E|\varepsilon_t|^\delta\right]^{1/\delta} < 1.$$

(a4) The density function $f_X(\cdot)$ of the probability measure of the process μ exists, and it is bounded, continuous and positive on $\mathbb{S}_X$.

Remark 1: Under the assumptions (a1)-(a4), it can be shown that the process is geometrically ergodic (see Härdle & Tsybakov, 1997).

Remark 2: The value of δ must be fixed also considering the particular function $\phi(\cdot)$ in the (2), in order to guarantee that $E|\phi(Y_t)|^2 < \infty$.

3 The fundamentals of local polynomial regression.

Let $K(\cdot)$ be a *kernel function*, supposed to be a density function defined on the interval $[-1, 1]$. Assuming that the derivative of order $(p+1)$ of the function $m_\phi(X_t)$ exists at the point x, we can use the taylor expantion to approximate $m_\phi(X_t)$ around the point x through the following polynomial of order p

$$m_\phi(X_t) \approx \sum_{j=0}^{p} \frac{1}{j!} m_\phi^{(j)}(x)(X_t - x)^j \tag{3}$$

where $m_\phi^{(v)}(x)$ is the derivative of order v of the function m_ϕ evaluated at the point x, for $v = 1, \ldots, p$. Define $\beta_j(x) = \frac{1}{j!} m_\phi^{(j)}(x)$, so that $m_\phi^{(j)}(x) = j!\beta_j(x)$, for $j = 0, \ldots, p$. Now consider the vector $\boldsymbol{\beta}(x) = \{\beta_0(x), \beta_1(x), \ldots, \beta_p(x)\}^T$ of length $p+1$. Using the (3) and the least squares method, the local polynomial estimator derives an estimate of the vector $\boldsymbol{\beta}(x)$ by solving the following weighted least squares problem:

$$\min_{\boldsymbol{\beta}} \sum_{t=1}^{n} \left\{ \phi(Y_t) - \sum_{j=0}^{p} \beta_j(x)(X_t - x)^j \right\}^2 K\left(\frac{X_t - x}{h_n}\right).$$

The positive real number h_n represents the *bandwidth* of the kernel estimator. It regulates, through the kernel function, the amount of local averaging and so the smoothness of the estimated function. It also affects the consistency of the estimator, so its selection seems to be very crucial. For simplicity of notation, later on we will omit in the notation the dependence of β on x and h and the dependence of h on n. Moreover, generally we will indicate explicitly if the bandwidth h depends also on the point x, although it will be clear from the context. Using the matrix notation, it is easy to show that

$$\hat{\boldsymbol{\beta}}(x) = \left(\mathbf{X}^T\mathbf{W}\mathbf{X}\right)^{-1} \mathbf{X}^T\mathbf{W}\mathbf{Y} \tag{4}$$

$$\hat{m}_\phi^{(v)}(x) = v!\mathbf{u}_v^T\, \hat{\boldsymbol{\beta}}(x), \quad v = 0, \ldots, p, \tag{5}$$

where $\mathbf{Y} = (\phi(Y_1), \ldots, \phi(Y_n))$, $\mathbf{u}_v$ is a column vector of length $p+1$ with all elements equal to zero except for a one in position v, and

$$\mathbf{X} = \begin{pmatrix} 1 & (X_1 - x) & \ldots & (X_1 - x)^p \\ \vdots & \vdots & \ddots & \vdots \\ 1 & (X_n - x) & \ldots & (X_n - x)^p \end{pmatrix}, \; \mathbf{W} = diag\left(K\left(\frac{X_i - x}{h_n}\right)\right)_{n \times n}.$$

The (4) and the (5) represent the most general configuration of the non-parametric local polynomial estimator of a regression function, and it includes several different setups by considering different values for p, v and $\phi(\cdot)$. For example, for $p = 0$ we have the Nadaraya-Watson estimator, which is usually considered as the classic kernel regression estimator. For $p = 1$ we have the local linear estimator and for $p = 2$ the local quadratic estimator. By fixing $v = 0$ we estimate the regression function $m_\phi(x)$. The derivatives of the function $m_\phi(x)$ are estimated by considering $v > 0$, but in this case we must fix $p \geq v$. Moreover, as shown before, by considering a particular function $\phi(\cdot)$ we can obtain estimators of several conditional moments of the process.

As concerning the order of the polynomial fit p, it is known that there is a high correlation between p and the size of the smoothing parameter h. It is difficult (and also useless) to optimize the local polynomial estimation with respect to both this tuning parameters. Generally, one is fixed and the optimization is performed with respect to the other. We choose to fix the parameter p at a value such that $p - v$ is odd. This because of the advantages shown in Ruppert & Wand (1994).

The asymptotic properties of the local polynomial estimator reported in (5) have been derived by several researchers. For mixing processes, see Masry & Fan (1997) and Härdle & Tsybakov (1997).

4 The Mean Squared Error and the optimal bandwidth.

Most of the bandwidth selectors given in the literature are based on the minimization of some measure of the estimation error, for example the mean squared error (MSE). The optimal bandwidth is then the bandwidth which minimizes the MSE of the estimator

$$h^{opt}_{MSE}(x) = \arg\min_h MSE\{\hat{m}^{(v)}_\phi(x;h)\}. \tag{6}$$

The (6) is considered the theoretical optimal bandwidth, that is the one toward which each bandwidth estimator must converge to. The main difficulty is how to estimate the mean squared error $MSE\{\hat{m}^{(v)}_\phi(x;h)\}$. The usual approach uses the asymptotic expansion of the MSE, as a polynomial in the variable h, in order to evaluate the main asymptotic components and to derive the asymptotic mean squared error ($AMSE$). As usual, the $AMSE$ of the local polynomial estimator can be decomposed into the sum of the asymptotic variance and squared bias

$$\begin{aligned} AMSE\{\hat{m}^{(v)}_\phi(x;h)\} &= bias^2_A\{\hat{m}^{(v)}_\phi(x;h)\} + Var_A\{\hat{m}^{(v)}_\phi(x;h)\} \\ &= \mathbb{B}^2(x)h^{2(p+1-v)} + \mathbb{V}(x)\frac{1}{nh^{(2v+1)}} \end{aligned} \tag{7}$$

where

$$\mathbb{B}^2(x) = \left\{C_1 m^{(p+1)}_\phi(x)\right\}^2 \qquad \mathbb{V}(x) = \frac{C_2\sigma^2_\phi(x)}{f_X(x)} \tag{8}$$

Note that the only unknown components in the (8) are the variance function $\sigma^2_\phi(x) = Var\{\phi(Y_t)|X_t = x\}$, the derivative function $m^{(p+1)}_\phi(x)$ and the design density $f_X(x)$. The constant values C_1 and C_2 are known and they depends, obviously, on the kernel function, on the degree of the polynomial p and on the order of the derivative v. In particular, $C_1 = v!B_v/(p+1)!$ and $C_2 = v!^2V_v$, where the values V_v and B_v are derived as follows. Denote with

$$\mu_j = \int_{-\infty}^{+\infty} u^j K(u)du, \quad \nu_j = \int_{-\infty}^{+\infty} u^j K^2(u)du, \qquad j = 1,\dots,2p+1$$

the equivalent moments of the Kernel. Arrange these moments into the $(p+1)\times(p+1)$ matrices $\mathbf{S}$ and $\tilde{\mathbf{S}}$, whose (i,j)-th elements are μ_{i+j-2} and ν_{i+j-2} respectively. Consider also the vector $\mathbf{r} = (\mu_{p+1}, \mu_{p+2}, \dots, \mu_{2p+1})^T$. The numbers V_v and B_v are, respectively, equal to the $(v+1)$th diagonal element of the matrix $\mathbf{S}^{-1}\tilde{\mathbf{S}}\mathbf{S}^{-1}$ and the $(v+1)$th element of the vector $\mathbf{S}^{-1}\mathbf{r}$.

The asymptotically optimal bandwidth is the bandwidth which minimizes the right-hand part of the (7). Denote such bandwidth with h^{opt}_{AMSE}. It is given by

$$h^{opt}_{AMSE}(x) = \left\{ \frac{(2v+1)\mathbb{V}(x)}{2n(p-v+1)\mathbb{B}^2(x)} \right\}^{1/(2p+3)}. \tag{9}$$

The *plug-in* method derives an estimation of the optimal bandwidth by estimating the unknown functionals $\mathbb{V}(x)$ and $\mathbb{B}^2(x)$, and plugging them into equation (9). Note that these functionals and the optimal bandwidth $h^{opt}_{AMSE}(x)$ depend on the degree of the estimated derivative v by simple and known constants, so the optimal bandwidth for the estimation of the function $m_\phi(x)$ or for the estimation of some derivative $m^{(v)}_\phi(x)$ can be computed in the same way. Note also that $h^{opt}_{AMSE}(x)$ represents an optimal local bandwidth, given its dependence on the point x, so that we have a local bandwidth function. One way to simplify the problem of estimating the optimal bandwidth is to consider the optimal global bandwidth, that is the bandwidth which minimizes a global measure of the estimation error. In this way one has to estimate a single value instead of a bandwidth function. For example, consider the integrated value of the asymptotical mean squared error, for a given weight function $w(x)$,

$$\begin{aligned} AMISE\{\hat{m}^{(v)}_\phi\} &= \int_{\mathbb{S}_X} \left[Bias^2_A\{\hat{m}^{(v)}_\phi(x;h)\} + Var_A\{\hat{m}^{(v)}_\phi(x;h)\} \right] \omega(x) dF_X \\ &= \mathbb{B}^2_\omega h^{2(p+1-v)} + \mathbb{V}_\omega \frac{1}{nh^{(2v+1)}} \end{aligned} \tag{10}$$

where

$$\mathbb{B}^2_\omega = C_1^2 R_f\left(m^{(p+1)}_\phi\right), \qquad \mathbb{V}_\omega = C_2 R(\sigma_\phi) \tag{11}$$

$$R_f\left(m^{(p+1)}_\phi\right) = \int_{\mathbb{S}_X} [m^{(p+1)}_\phi(x)]^2 \omega(x) dF_X \quad R(\sigma_\phi) = \int_{\mathbb{S}_X} \sigma^2_\phi(x)\omega(x)dx. \tag{12}$$

Note that the two integrals $R_f\left(m^{(p+1)}_\phi\right)$ and $R(\sigma_\phi)$ involves different measures, because in the second the density $f_X(x)$ cancels with the denominator in the (8). The use of the weigth function $\omega(x)$ is useful in order to generalize the selection problem to a wide range of cases, for example to select a bandwidth variable on the support, or to control border effects. It can be shown that in this case the optimal global bandwidth is given by

$$h^{opt}_{AMISE} = \left\{ \frac{(2v+1)\mathbb{V}_\omega}{2n(p-v+1)\mathbb{B}^2_\omega} \right\}^{1/(2p+3)}. \tag{13}$$

Here, as expected, the functionals $\mathbb{V}_\omega$, $\mathbb{B}^2_\omega$ and h^{opt}_{AMISE}, do not depend on the point of estimation x, because they are derived by integration on the support $\mathbb{S}_X$ with respect to the weight function $\omega(x)$. Again, an estimate of the functionals $\mathbb{V}_\omega$, $\mathbb{B}^2_\omega$ must be plugged into the (13) in order to derive the plug-in estimate of the optimal global bandwidth.

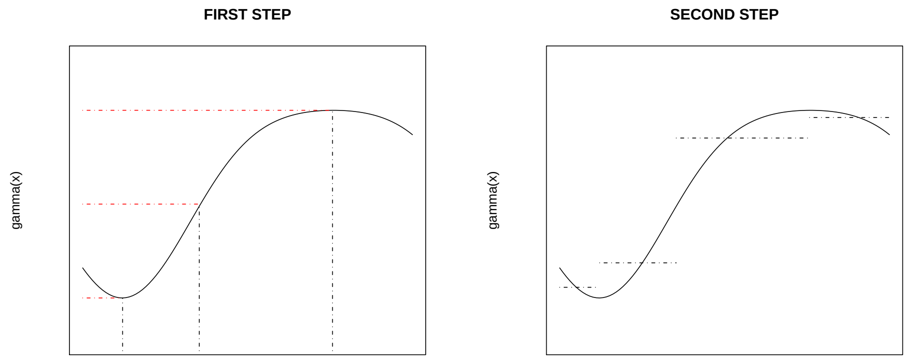

Figure 1: The two steps of the procedure proposed for the estimation of the *locally global bandwidth*. The plots report the function $\gamma(x) = \mathbb{V}(x)/\mathbb{B}^2(x)$.

As said before, the estimation of the (9) is more difficult than the estimation of the (13), because the first is a function while the second is a number (=mean value). Anyway, at least asymptotically, it is always true that

$$AMSE\{\hat{m}_\phi^{(v)}(x; h_{AMSE}^{opt}(x))\} \leq AMSE\{\hat{m}_\phi^{(v)}(x; h_{AMISE}^{opt})\}, \forall x \in \mathbb{S}_X,$$

so one could ask in what measure it is convenient to consider the local bandwidth $h_{AMSE}^{opt}(x)$ instead of the global bandwidth h_{AMISE}^{opt}, since the kernel estimators are particularly used for local estimations. Moreover, the estimation of the function h_{AMSE}^{opt} would raise some other questions. For example,

- how to smooth the estimated $\hat{h}_{AMSE}^{opt}(x)$ function?
- what is the effective gain in using the (estimated) local bandwidth?
- is it always convenient to afford a more complicated bandwidth estimation problem?
- how to deal with pilot bandwidths?

These will remain as open questions, although some considerations will be made in the last section of this paper.

5 The Neural Network bandwidth selector for the estimation of the volatility function.

We point to deriving a procedure based on two steps. In the first step we should evaluate the effective gain in using a local bandwidth for the estimation of the function $m_\phi(x)$ instead of a global bandwidth. Then we should split in some way the support $\mathbb{S}_X$ of the function $m_\phi(x)$ into a number of compact subsets $I_X \in \mathbb{S}_X$. In the second stage we estimate the "*locally global bandwidths*", that are global bandwidths (=constant) which are estimated on each one of the homogeneous subsets by means of a global optimization. Figure 1 shows the rationale of the method. For the moment, we concentrate on the second step of the procedure.

Suppose we want to estimate the function $m_\phi(x)$ on a given compact subset $I_X \subseteq \mathbb{S}_X$. We are interested in the estimation of a global bandwidth which could be considered optimal on that subset. As seen before, we have to estimate the two functionals in (12) conditionally on the subset I_X. Specific problems follow concerning the integration, since we have to consider a conditioned probability measure. In particular, denoting with $f_{X|I_X}$ the density of the process conditioned on the event $X \in I_X$, we have

$$\begin{aligned}
AMISE\{\hat{m}_\phi\} &= \int_{I_X} AMSE\{\hat{m}_\phi(x;h)\} f_{X|I_X}(x)dx \\
&= C_1^2 h^{2(p+1)} \int_{I_X} \left[m_\phi^{(p+1)}(x)\right]^2 \frac{f_X(x)}{\mu(I_X)} dx + \frac{C_2}{nh} \int_{I_X} \sigma_\phi^2(x) \frac{dx}{\mu(I_X)} \\
&= C_1^2 h^{2(p+1)} \int_{I_X} \left[m_\phi^{(p+1)}(x)\right]^2 f_X(x) d\omega_I + \frac{C_2}{nh} \int_{I_X} \sigma_\phi^2(x) d\omega_I \\
&= \mathbb{B}_{\omega_I} h^{2(p+1)} + \mathbb{V}_{\omega_I} \frac{1}{nh}
\end{aligned}$$

where $d\omega_I = \frac{dx}{\mu(I_X)}$ and

$$\mathbb{B}_{\omega_I} = C_1^2 R_f^I\left(m_\phi^{(p+1)}\right), \qquad \mathbb{V}_{\omega_I} = C_2 R^I(\sigma_\phi) \tag{14}$$

$$R_f^I\left(m_\phi^{(p+1)}\right) = \int_{I_X} [m_\phi^{(p+1)}(x)]^2 f_X(x) d\omega_I \quad R^I(\sigma_\phi) = \int_{I_X} \sigma_\phi^2(x) d\omega_I. \tag{15}$$

As usual, the optimal bandwidth can be derived by estimating the functionals in (14) and (15) and plugging them into the (13). In order to do that, the problem is now how to estimate the functions $\sigma_\phi^2(x)$ and $m_\phi^{(p+1)}(x)$ on the subset I_X. We propose to use a global estimator based on the neural network technique. For simplicity, we consider here one particular example of application but the procedure can be easily extended to other cases.

Suppose we want to estimate the volatility function $\sigma^2(x)$ for model (1) and suppose that $m(x) = 0$. This is a very typical set up when analyzing financial data. It results that we have to consider $\phi(Y_t) = Y_t^2$ in (2), (4), (5) and following equations. Let the notation $m_r(x)$ denote the conditional moment function $E(Y^r|X_t = x)$. Note that $\sigma^2(x) \equiv m_2(x)$ and

$$\sigma_\phi^2(x) = Var\{Y_t^2|X_t = x\} = m_4(x) - m_2^2(x). \tag{16}$$

Generally, the nonparametric estimation of the (16) implies two simultaneous (but different) nonparametric estimations of the functions $m_4(x)$ and $m_2(x)$, as, for example, in Yao & Tong (1994), Härdle & Tsybakov (1997), Fan & Yao (1998) and Franke & Diagne (2006). In particular, denoting with $\hat{\boldsymbol{\eta}}_i$ a generic nonparametric estimator, we should have

$$\hat{\boldsymbol{\eta}}_i = \arg\min_{\boldsymbol{\eta}} \sum_{t=1}^{n} \left[g_i(Y_t) - q(X_t; \boldsymbol{\eta})\right]^2, \qquad i = 1, 2 \tag{17}$$

where

- $g_1(z) = z^4$ for the estimation of $m_4(x)$;
- $g_2(z) = z^2$ for the estimation of $m_2(x)$;
- $q(X_t; \boldsymbol{\eta})$ is some approximation function (neural network function, local polynomial function, etc...).

This procedure is somehow inefficient. For example, when the estimation is (again) based on the use of a kernel estimator, it is necessary to select two different *pilot bandwidths*. To avoid the problem of the pilot bandwidths, we could use the neural network technique, as in Franke & Diagne (2006), but still we would have to consider the two estimations in (17), which is particularly inefficient with neural networks. For this reason, we propose an alternative approach based on the following riparametrization:

$$\sigma_\phi^2(x) = m_4(x) - m_2^2(x) = m_2^2(x)\left[m_{4\varepsilon} - 1\right] \tag{18}$$

where $m_{4\varepsilon} = E(\varepsilon_t^4)$. Then we use a *Feedforward Neural Network* (FNN), with one input layer and one hidden layer, as approximation function in the (17). It is defined as

$$q(X_t; \boldsymbol{\eta}) = \sum_{k=1}^{d} c_k \Gamma\left(a_k X_t + b_k\right) + c_0, \tag{19}$$

where

- $\boldsymbol{\eta} = (c_0, c_1, \ldots, c_d, a_1, \ldots, a_d, b_1 \ldots b_d)$ is the vector of parameters of the FNN, to be estimated;
- d is the number of nodes in the hidden layer, such that $d = O\left(\sqrt{n/\log n}\right)$;
- $\Gamma(\cdot)$ is the *logistic activation function*.

Now using (17), (18) and (19), we define the following estimator of the function $\sigma_\phi^2(x)$ based on the neural network approach:

$$\hat{\sigma}_\phi^2(x) = \hat{m}_2^2(x)\left[\hat{m}_{4\varepsilon} - 1\right] \quad \hat{m}_2(x) = q(x, \hat{\boldsymbol{\eta}}_2) \quad \hat{m}_{4\varepsilon} = \frac{\sum_{t=1}^{n} X_t^4}{\sum_{t=1}^{n} [\hat{m}_2(X_t)]^2}. \tag{20}$$

The appeal of the estimator proposed in (20) is that we use only one neural network estimator $\hat{\boldsymbol{\eta}}_2$ in (17). This is, in our opinion, a novel approach for the estimation of the function $\sigma_\phi(x)$ which has some interests in its own.

Secondly, we need to estimate the derivative function $m_\phi^{(p+1)}(x)$. Considering that

$$m_\phi^{(p+1)}(x) \equiv m_2^{(p+1)}(x),$$

we can use the same previous NN estimate in order to get the estimate of the derivative function:

$$\hat{m}_2^{(p+1)}(x) = q^{(p+1)}(x; \hat{\boldsymbol{\eta}}_2). \tag{21}$$

Finally, given the (20), the (21) and the ergodicity of the process, we propose the following two estimators for the functionals in (15):

$$\hat{R}_f^I\left(m_\phi^{(p+1)}\right) = \frac{\sum_{t=1}^n \left[\hat{m}_2^{(p+1)}(X_t)\right]^2}{\sum_{t=1}^n \mathbb{I}(X_t \in I_X)}, \tag{22}$$

$$\hat{R}^I(\sigma_\phi) = \frac{\sum_{i=1}^{n^*} \hat{\sigma}_\phi^2(x_i)}{n^*}, \tag{23}$$

In the (23), the points $\{x_1, x_2, \ldots, x_{n^*}\}$ are uniformly spaced values from the interval I_X, with $n^* < n$, such as $n^* = \lfloor n/2 \rfloor$ ($\lfloor x \rfloor$ is the integer part of x). Once again, note that we use only one neural network to estimate both the functionals $R_f^I\left(m_\phi^{(p+1)}\right)$ and $R^I(\sigma_\phi)$.

6 Numerical performance of the NNB bandwidth selector.

Consider an application of LPE of degree $p = 1$ for the estimation of the volatility function of the following time series models (here $\psi(\cdot)$ denotes the density function of the normal $N(0,1)$, and $\mathbb{I}(A)$ is the indicator function, which is equal to one if condition A is satisfied and zero elsewhere)

Models		*errors*
1:	$Y_t = 0.7\varepsilon_t$	$\varepsilon_t \sim N(0,1)$
2:	$Y_t = [\psi(Y_{t-1} + 1.2) + 1.5\psi(Y_{t-1} - 1.2)]\,\varepsilon_t$	$\varepsilon_t \sim N(0,1)$
3:	$Y_t = \sqrt{0.1 + 0.3Y_{t-1}^2}\varepsilon_t$	$\varepsilon_t \sim N(0,1)$
4:	$Y_t = \sqrt{0.1 + 0.15Y_{t-1}^2}\varepsilon_t$	$\varepsilon_t \sim t_{(10)}$
5:	$Y_t = \sqrt{0.01 + 0.1Y_{t-1}^2 + 0.2Y_{t-1}^2\mathbb{I}(Y_{t-1} < 0)}\varepsilon_t$	$\varepsilon_t \sim N(0,1)$

Model 1 is a simple white noise with variance $\sigma_\varepsilon^2 = 0.49$, $Y_t \sim WN(0, 0.47)$. Models 2-4 are autoregressive and heteroschedastic models, $Y_t \sim ARCH(1)$, where the number 1 in the brackets denotes the lags in the model. The last model is a threshold autoregressive model, $Y_t \sim TARCH(1)$. As you can note, all of these models suppose that the conditional mean function is equal to zero. Moreover, only model 1 is homoschedastic. Some of these models are modified versions of models considered in other papers. In particular, Härdle & Tsybakov

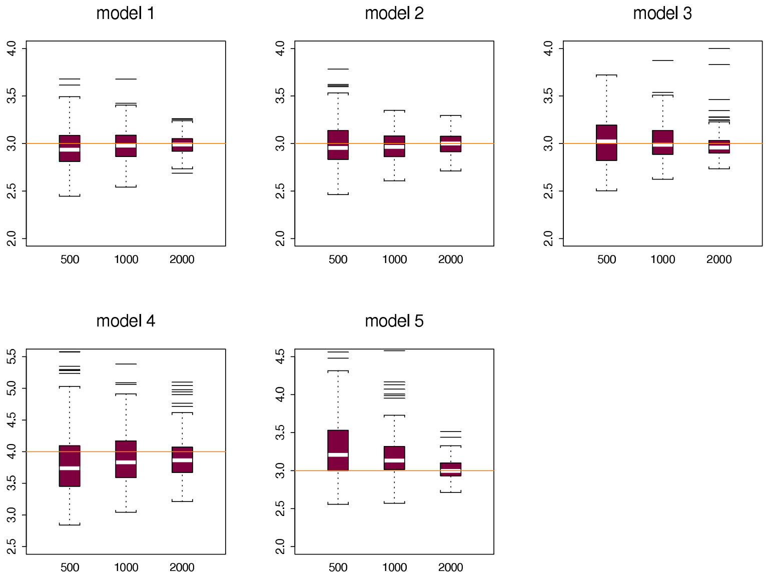

Figure 2: Boxplots of the neural network estimates of the error moment $m_{4\varepsilon} = E(\varepsilon)^4$, for models 1-5.

(1997) for model 2; Franke & Diagne (2006) and Franke, Neumann & Stockis (2004) for model 5.

We performed a Monte Carlo simulation study with 100 replications for each model, with three different lengths: $n = (500, 1000, 2000)$. We considered the estimation of the volatility function $\sigma^2(x)$ onto two different subsets ($\equiv$ intervals): the first is denoted with ${}_1I_X = (x_{11}, x_{21})$ and it was selected in such a way that it includes for each model about the 90% of the central points; the second is indicated with ${}_2I_X = (x_{12}, x_{22})$, and it includes about the 50% of the central points. It follows that ${}_2I_X \subset_1 I_X$.

We implemented the procedure described in section 5 on the simulated data. As concerning the neural network estimations, the number of nodes in the hidden layer was selected by following an automatic procedure (BIC). See Giordano & Parrella (2007) for further remarks on this topic.

Figure 2 shows empirically the consistency of the method. The plots report the neural network estimations of the error moment $m_{4\varepsilon} = E(\varepsilon)^4$, for all the models. In our opinion, they show a clear and strong good result for our procedure, if we consider the (20) and if we remember that all the estimations are based on the same neural network estimated approximation function. In particular, note how the estimations improve when $n = 2000$, even for those models which cause some trouble when $n = 500$ (models 4 and 5). More specifically, note that model 4 is equivalent to model 3, but it presents t-student innovations instead of normal innovations. This entails asymmetry and fat tails in the box-plots of the estimations. For model 5, note that the assumption (a2)

	MISE			*MEDISE*			*SDISE*		
$n =$	500	1000	2000	500	1000	2000	500	1000	2000
Model 1									
${}_1I_X$	**0.03**	-0.52	-0.56	**0.02**	-0.46	-0.51	**0.07**	-0.74	-0.71
${}_2I_X$	**0.01**	-0.58	-0.54	**0.01**	-0.64	-0.49	**0.01**	-0.60	-0.58
Model 2									
${}_1I_X$	**0.05**	-0.50	-0.51	**0.03**	-0.56	-0.29	**0.11**	-0.50	-0.81
${}_2I_X$	**0.01**	-0.47	-0.44	**0.01**	-0.46	-0.44	**0.01**	-0.46	-0.54
Model 3									
${}_1I_X$	**0.30**	-0.85	-0.54	**0.05**	-0.46	-0.47	**2.77**	-0.95	-0.80
${}_2I_X$	**0.04**	-0.63	-0.67	**0.02**	-0.69	-0.58	**0.04**	-0.46	-0.78
Model 4									
${}_1I_X$	**0.06**	-0.32	-0.47	**0.04**	-0.44	-0.35	**0.09**	-0.05	-0.55
${}_2I_X$	**0.02**	-0.35	-0.42	**0.01**	-0.22	-0.44	**0.04**	-0.49	-0.42
Model 5									
${}_1I_X$	**0.08**	-0.27	-0.65	**0.08**	-0.12	-0.86	**0.04**	-0.15	-0.19
${}_2I_X$	**0.04**	-0.37	-0.74	**0.03**	-0.37	-0.84	**0.03**	-0.31	-0.60

Table 1: Results from the simulation study for the integrated squared errors (ISE) of models 1-5. *MISE* is the mean of ISE; *MEDISE* is the median of ISE; *SDISE* is the standard deviation of ISE. ${}_1I_X$ is the interval including about the 90% of the central points; ${}_2I_X$ is the subset including about the 50% of the central points. Numbers in bold refer to the observed values; the other numbers show the relative variation with respect to the previous value of n.

is not completely satisfied, since the second derivative of the volatility function does not exist for $x = 0$. Anyway, the box-plot in figure 2 shows that there is some robustness of the procedure against the violation of the assumption (a2), at least when the set of points interested has zero probability measure.

In table 1 we report some numerical results concerning the integrated squared errors (*ISE*) of models 1-5, which empirically confirm the consistency of the procedure. In particular, the mean value of the integrated squared error (*MISE*) is reported in the first column, for the three different time series lengths; the median value of the integrated squared error (*MEDISE*) is reported in the second column, while the standard deviation value of the integrated squared error (*SDISE*) is reported in the last column. On the rows, for each model, we report the results concerning the two intervals ${}_1I_X$ (about the 90% of the central points) and ${}_2I_X$ (about the 50% of the central points). The numbers written in bold style represent the observed values. The other numbers show the relative variation with respect to the previous value of n, so they indicate the relative variation observed when the length of time series increases. Note that all these numbers are negative, which means that all the values (mean, median and standard deviation of ISE) tend to zero when n increases, so that the procedure is consistent. As expected, models 4 and 5 show the lowest rate of convergence. Also note that the second row of each model (concerning the interval

${}_2I_X$) generally reports the best results.

In order to better clarify things, we present some plots of the true functions. In figure 3 we report the true optimal bandwidth function $h^{opt}(x)$ and the true asymptotic mean squared error function for the estimation of the volatility function of models 2 and 3. In particular, the first and third rows of plots are devoted to the optimal bandwidth function $h^{opt}(x)$, while the second and fourth rows show the mean squared error. On the columns there are the three different time series lengths, $n = 500$, $n = 1000$ and $n = 2000$. In all the plots of the bandwidth (first and third rows), the black line refers to the optimal local bandwidth (optimum), the red line denotes the global bandwidth (usually used) and the dashed blue line indicates the locally global bandwidth (our proposal). In all the plots of the mean squared error (second and fourth rows), the black line refers to the true MSE function evaluated for the true optimal bandwidth (optimum); the red line denotes the function evaluated for the global bandwidth (usually used); the dashed blue line indicates the function evaluated for the locally global bandwidth (our proposal). Figure 6 is organized in the same way, but they refer to models 4 and 5. From these plots we can note that the locally global bandwidth performs always better than the global bandwidth and always quite as well as the local bandwidth. So generally there is a convenience in using the locally global bandwidth. Anyway, for some model we can see a very little gain, like for models 2 and 4. It seems, therefore, that even the use of the local bandwidth sometimes does not produce significantly better results than the use of a simple global bandwidth. If we consider, moreover, that the (unknown) local bandwidth function must be estimated from the realized time series, we can imagine how furthermore this gain vanish.

Finally, figures 6, 6 and 6, report the boxplots of the local polynomial estimations of the volatility function for models 1-5, when $n = 500$. The red line is the true function. The estimations are made for 20 points in the interval ${}_1I_X$. The bandwidth used is our locally global bandwidth, derived on the interval ${}_1I_X$. The blue box in the middle shows the position of the (included) interval ${}_2I_X$. The plot in the second line shows the same as in the first plot, but this time the estimations are based on the interval ${}_2I_X$. The plots in the third and fourth lines are organized as in the first two lines, but they refer to another model. For the problematic model 5 in figure 7, we report also the plots for the case of $n = 2000$, to show how the estimations improves for longer time series.

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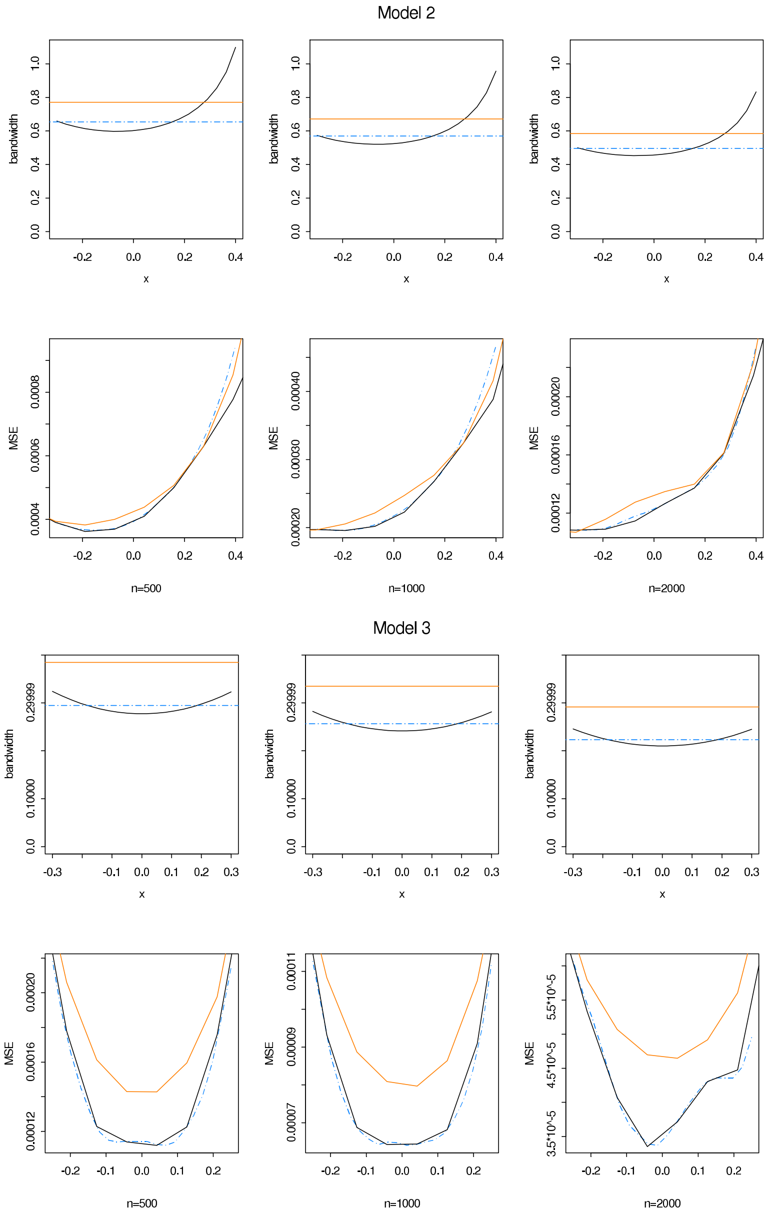

Figure 3: First row of plots: the optimal bandwidth function $h^{opt}(x)$ for the estimation of the volatility function of model 2, respectively for $n = 500$, $n = 1000$ and $n = 2000$ (on the columns). The black line refers to the true function; the red line denotes the global bandwidth; the dashed blue line indicates the locally global bandwidth (our proposal). Second row: the asymptotic Mean Squared Error function, for the three different lengths. The black line refers to the true function evaluated for the true optimal bandwidth (optimum); the red line denotes the function evaluated for the global bandwidth; the dashed blue line indicates the function evaluated for the locally global bandwidth (our proposal). Third and fourth rows: the same as the first two rows, but for model 3.

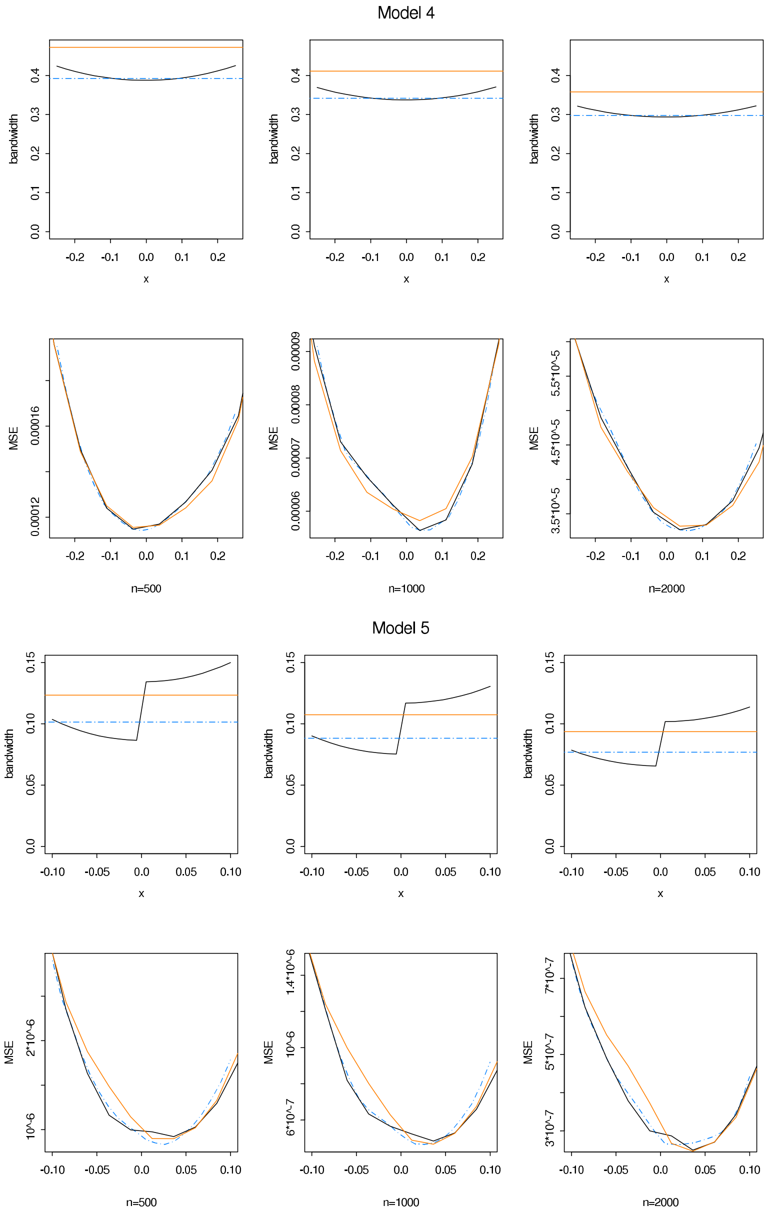

Figure 4: First row of plots: the optimal bandwidth function $h^{opt}(x)$ for the estimation of the volatility function of model 4, respectively for $n = 500$, $n = 1000$ and $n = 2000$ (on the columns). The black line refers to the true function; the red line denotes the global bandwidth; the dashed blue line indicates the locally global bandwidth (our proposal). Second row: the asymptotic Mean Squared Error function, for the three different lengths. The black line refers to the true function evaluated for the true optimal bandwidth (optimum); the red line denotes the function evaluated for the global bandwidth; the dashed blue line indicates the function evaluated for the locally global bandwidth (our proposal). Third and fourth rows: the same as the first two rows, but for model 5.

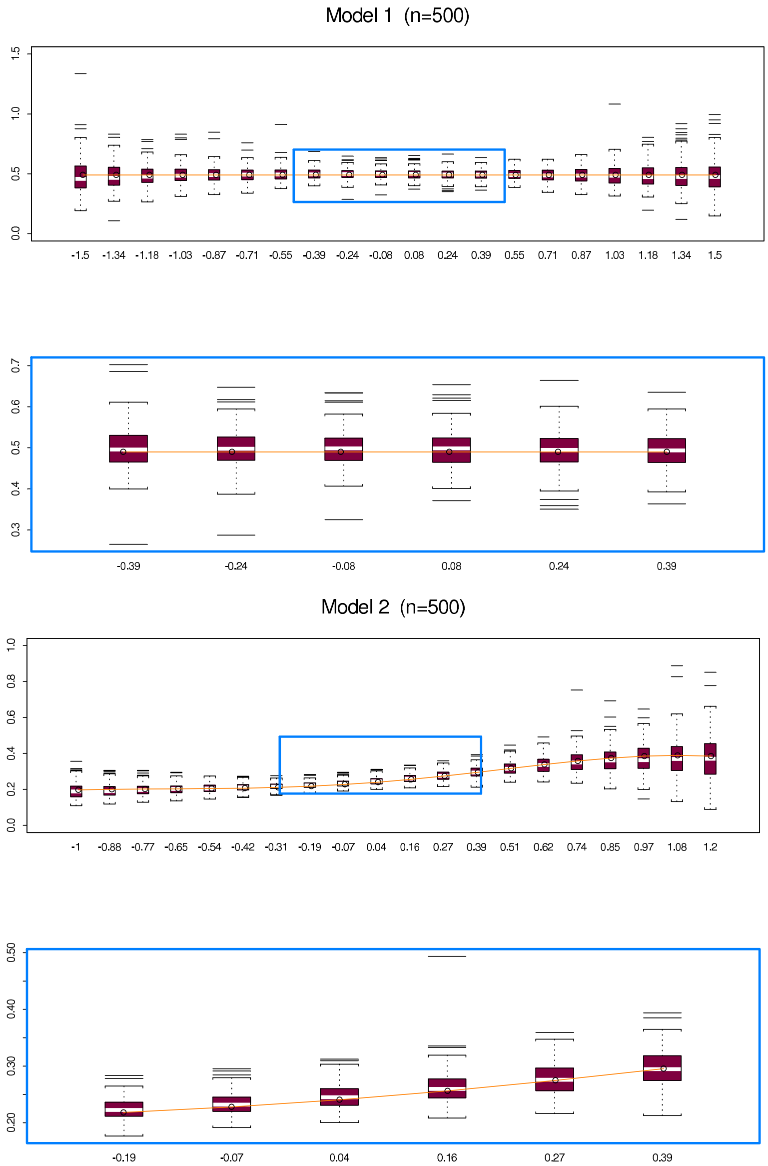

Figure 5: First plot on the top: box-plots of the local polynomial estimations of the volatility function for model 1 ($n = 500$). The red line is the true function. The estimations are made for 20 points in the interval ${}_1I_X$. The bandwidth used is our locally global bandwidth, derived on the interval ${}_1I_X$. The blue box in the middle shows the position of the (included) interval ${}_2I_X$. Plot in the second row: the same as above, but this time the estimations are based on the interval ${}_2I_X$. The plots in the third and fourth rows are organized as in the first two rows, but they refer to model 2.

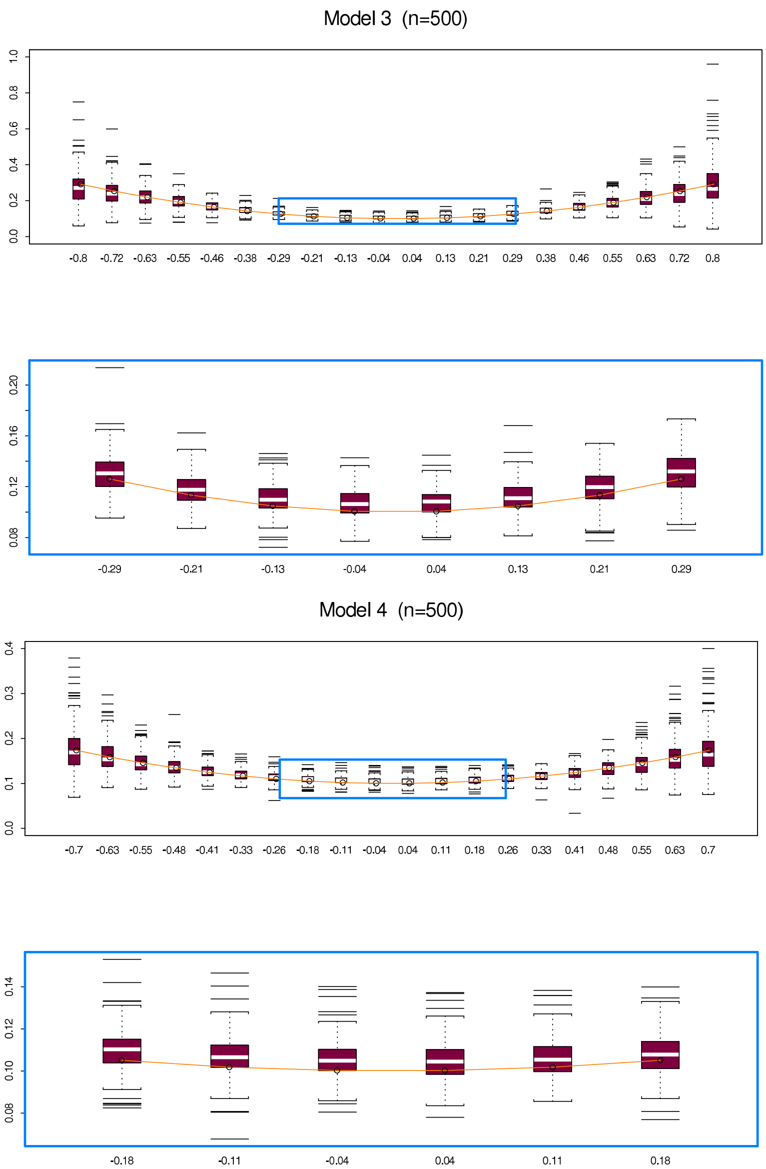

Figure 6: First plot on the top: box-plots of the local polynomial estimations of the volatility function for model 3 ($n = 500$). The red line is the true function. The estimations are made for 20 points in the interval ${}_1I_X$. The bandwidth used is our locally global bandwidth, derived on the interval ${}_1I_X$. The blue box in the middle shows the position of the (included) interval ${}_2I_X$. Plot in the second row: the same as above, but this time the estimations are based on the interval ${}_2I_X$. The plots in the third and fourth rows are organized as in the first two rows, but they refer to model 4.

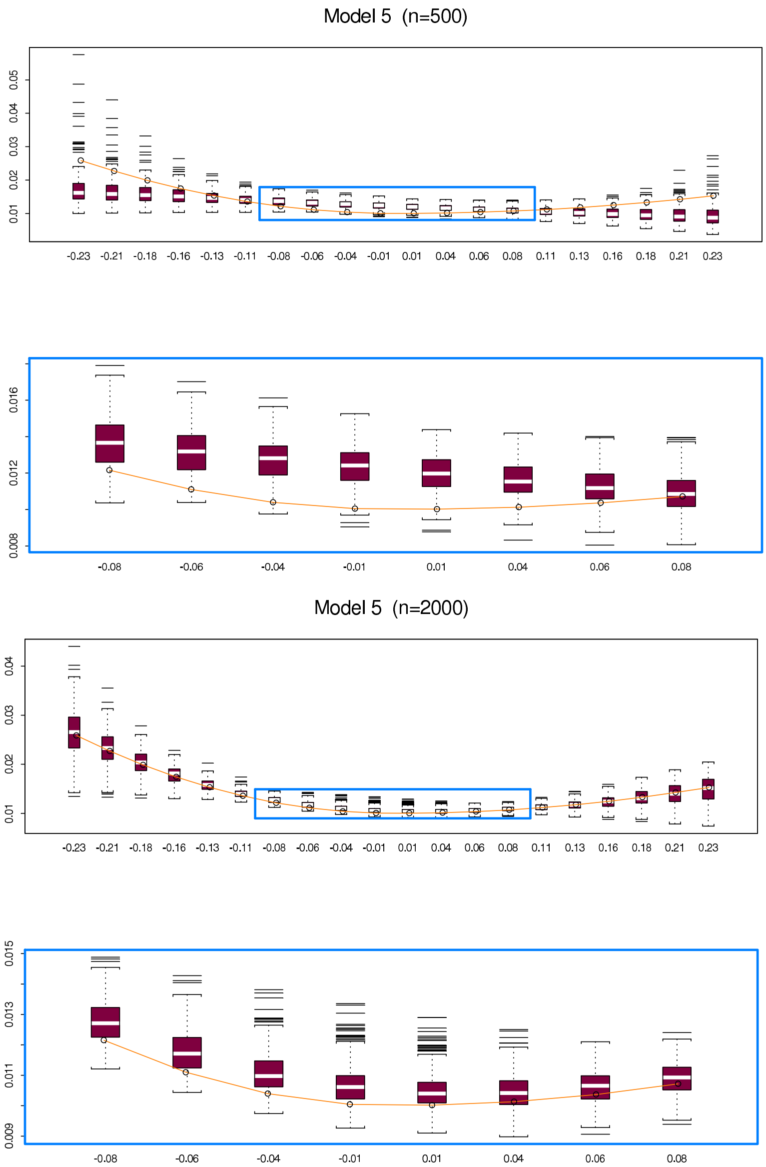

Figure 7: First plot on the top: box-plots of the local polynomial estimations of the volatility function for model 5 ($n = 500$). The red line is the true function. The estimations are made for 20 points in the interval ${}_1I_X$. The bandwidth used is our locally global bandwidth, derived on the interval ${}_1I_X$. The blue box in the middle shows the position of the (included) interval ${}_2I_X$. Plot in the second row: the same as above, but this time the estimations are based on the interval ${}_2I_X$. The plots in the third and fourth rows are organized as in the first two rows, but for $n = 2000$.

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