Università degli Studi di Salerno

DIPARTIMENTO DI SCIENZE ECONOMICHE E STATISTICHE

On the Minty and Stampacchia scalar variational inequalities

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Abstract

The paper aims to deepen the analysis of the two different variational inequalities e and to provide a short statement of fundamental results. Subsequently it intends to clarify the role of the scalar Stampacchia and Minty solutions in optimization problem.

Introduction

Variational inequalities are known to be a very useful and powerful tool for investigation and formulation of solutions of many equilibrium problems in Economics, Optimization, Engineering, Operations Research and Mathematical Physics [1], [5]. There are many classical examples: for istance, equilibrium problems in a financial or a traffic network and equilibrium problems in a oligopolistic market . Many variational formulations of these problems, have been presented in recent years since these problems, whose study has emerged as an interesting branch of applicable mathematics, possess a partitionable structure which usually enables one to weaken conditions in existence and uniqueness theorems and to propose more powerful solution methods.

Two very useful tools in the study of equilibrium solutions and their stability are Stampacchia variational inequalities and Minty variational inequalities. In this paper, we study just these inequalities because they allow a unified treatment of equilibrium problems and optimization problems.

The paper is organized as follows. In section 1 are defined the scalar variational inequalities of Stampacchia and Minty in finite-dimensional Euclidean space R^n.

In section 2 are provided conditions for the existence and uniqueness of solutions for two problems while in section 3 will present the connections between the two variational inequalities and an optimization problem.

1 Finite-dimensional variational inequalities: definitions

Let K be a nonempty, closed and convex subset of R^n, and let $F : K \to R^n$ be a mapping from R^n into itself. The *Stampacchia variational inequality*, denoted by SVI and introduced by Philip Hartman and Guido Stampacchia in [6] , is the problem to find a vector $x^* \in K$ such that:

$$SVI\ (F, K) \qquad < F(x^*), x - x^* > \geq 0 \qquad \forall\, x \in K$$

where $< \cdot, \cdot >$ denotes the inner product defined on R^n.

The vector x^* solution of $SVI\ (F, K)$ is called *Stampacchia equilibrium point* of the map F on K.

An equivalent geometric formulation of $SVI(F, K)$, when K is a convex and closed set, can be given introducing the concepts of normal cone and of generalized equation.

Definition 1.1: *If $K \subseteq R^n$ is a convex and closed set, the normal cone to K at a point $x^* \in K$ is*

$$N_K(x^*) = \{x \in R^n : < x, y - x^* > \leq 0 \quad \forall\, y \in K\}$$

If $x^* \notin K$, then $N_K(x^*) = \emptyset$. It is easily seen that the normal cone is a closed convex cone.

In geometric terms, $SVI(F, K)$ states that $F(x)$ is orthogonal to K in the point $x^* \in K$; then, if K is convex, $x^* \in K$ is a solution of $SVI(F, K)$ if and only if

$$-F(x^*) \in N_K(x^*),$$

and that is

$$0 \in F(x^*) + N_K(x^*)$$

so the variational inequality $SVI(F, K)$ is equivalent to a generalized equation.

If instead the domain K is an open set, the solution of a variational inequality $SVI(F, K)$ is equivalent to that of a system of equations, as shows the following proposition:

Proposition1.1: *Let $K \subseteq R^n$ be an open set and let $F : K \to R^n$. The vector $x^* \in K$ is a solution of $SVI(F, K)$ if and only if x^* it solves the system of equations*

$$F(x^*) = 0$$

Proof: If $F(x^*) = 0$, then SVI holds with equality

$$< F(x^*), x - x^* > = 0 \qquad \forall x \in K$$

Conversely if x^* is a solution of $SVI(F, K)$ and K is an open set , exists $\delta > 0$ such that $\beta(x^*, \delta) \subset K$ and so, by supposition,

$$< F(x^*), x - x^* > \geq 0 \qquad \forall x \in \beta\ (x^*, \delta)$$

But $\forall x \in \beta(x^*, \delta)$, also $(2x^* - x) \in \beta(x^*, \delta)$ and then

$$< F(x^*), 2x^* - x - x^* > \ = \ < F(x^*), x^* - x > \geq 0 \qquad \forall x \in \beta\ (x^*, \delta)$$

Therefore

$$< F(x^*), x - x^* > = 0 \qquad \forall x \in \beta\ (x^*, \delta)$$

and that is equivalent to condition x^* solves $F(x^*) = 0$.

An alternative formulation of the Stampacchia variational inequality (equivalent only under monotonicity and continuity hypotheses) has been proposed, in 1962, by G.J. Minty and is known as Minty variational inequality.

Let the function $F : K \to R^n$ be given, with K nonempty subset of R^n. The *Minty variational inequality* is the problem to find a vector $x^* \in K$ such that

$$MVI(F, K) < F(y),\ x^* - y > \leq 0 \qquad \forall\ y \in K$$

This problem is denoted, for short, with $MVI(F, K)$ and any solution of $MVI(F, K)$ is called a *Minty equilibrium point* of the map F over K. In the Minty variational inequality is

considered the value assumed by F in every $y \in K$, while in the Stampacchia variational inequality the function F is estimated only in the given point $x^* \in K$; therefore, $SVI(F, K)$ express a local condition for function at one point.
A well-known Lemma, formulated by Minty in 1967, states the equivalence of the two alternative formulations (the one presented by Stampacchia and the one introduced by Minty) under continuity and monotonicity assumptions of involved operator. In other words, Minty's lemma establishes relationships between the solutions of $SVI\ (F, K)$ and the solutions of $MVI\ (F, K)$, that is it gives a complete characterization of the solution $MVI(F, K)$ in terms of the solution of $SVI(F, K)$, when the set K is convex and the operator F is continuous and monotone. In fact, it asserts

$$M(F, K) \subseteq S(F, K)$$

where $M(F, K)$ denotes the set of solutions of the Minty variational inequality while $S(F, K)$ the solution set of Stampacchia variational inequality. The reverse inclusion holds if F is pseudomonotone.
Before, however, it is necessary to recall some definitions.

Definition 1.2: *The mapping $F : R^n \to R^n$ is said to be*

a) *monotone over a set K if*

$$< F(y) - F(x), x - y > \geq 0 \qquad \forall\, x.y \in K$$

b) *pseudomonotone over K if*

$$< F(x), x - y > \leq 0 \;\Rightarrow\; < F(y), x - y > \leq 0 \qquad \forall x, y \in K$$

Obviously a monotone function is too pseudomonotone but not holds the reverse implication.

Minty Lemma: *Let $F : K \to R^n$ be given with $K \subseteq R^n$ closed and nonempty set.*

i) *If* F *is continuous on* K *and* K *is convex, then each* $x^* \in K$ *which solves* $MVI(F,K)$ *is also a solution of* $SVI(F,K)$*.*

ii) If, instead, F *is monotone on the convex set* K*, then every* $x^* \in K$ *which solves* $SVI(F,K)$*, is also a solution of* $MVI\ (F,K\ )$*.*

Remark: The hypotesis contained in the point ii) can be weakened with the concept of pseudomonotonicity: the implication $SVI \Rightarrow MVI$ is still true if F it is pseudomonotone.

2. Existence and uniqueness of the solutions

A central problem in the study of a variational inequalities is the problem of giving sufficient conditions for the existence of a solutions of a variational inequality. The research of these existence conditions has played a very important role in theory and pratical applications since this question has been at the origin of many contributions during the last years. In particular, after the works of G. Stampacchia on monotone variational inequalities, Hartmann-Stampacchia (1966), Crouzeix (1997), Luc (2001) ecc., have obtained existence results for variational inequalities under weaker monotonicity assumptions or rather a substantial number of papers on existence results for solving equilibrium problems is based on different relaxed monotonicity notions and various compactness assumptions. Because it is not possible to list all results on the existence and uniqueness of solutions to variational inequalities, we consider the most fundamental results and some of their consequences.
The most basic result on the existence of a solution to the variational inequality $SVI(F,K)$, the following well Known Hartman-Stampacchia theorem, requires the set K to be compact and convex and the mapping F to be continuous:

Theorem 2.1 [6]: *Let K be a nonempty, compact and convex subset of R^n and let F be a continuous mapping from K into R^n. Then there exists a solution $x^* \in K$ to the problem $SVI(F,K)$, that is $S \neq \emptyset$*

In general the variational inequality problem $SVI(F,K)$ can have more solution. If F is strongly monotone on K, then the problem $SVI(F,K)$ can have at most one solution, as stated by following proposition.

Proposition 2.1: *Let K be a nonempty, closed, convex subset of R^n and let F be a continuous mapping from K into R^n. If F is strongly monotone on K, then there exists a unique solution to the problem $SVI(F,K)$.*

Since strong monotonicity implies strict monotonicity it is possible derive the following existence and uniqueness result for variational inequality problems of the strictly monotone type.

Proposition 2.2: *If F is strictly monotone on K, then the problem $SVI(F,K)$ has at most one solution.*

If F is pseudomonotone, the solution set of a variational inequality is convex.

Proposition 2.3 [9]: *Let K be a nonempty, closed and convex subset of R^n and let F be a continuous, pseudomonotone mapping from K into R^n.*
Then x^* solves the $SVI(F,K)$ if and only if $x^* \in K$ and $< F(x^*), x - x^* > \geq 0 \ \forall x \in K$. In particular, the solution on set of $SVI(F,K)$ is convex if it is nonempty.

If the mapping F is pseudomonotone or monotone, the variational inequality $SVI(F,K)$ need not have a solution. However, if a certain constraint qualification hold, then pseudomonotonicity is sufficient to establish existence:

Proposition 2.4: *Let K be a nonempty, closed, convex subset of R^n and let F be a continuous mapping from R^n into itself. If F is pseudomonotone on K and if there exists a vector $x^* \in K$ such that $F(x^*) \in \operatorname{int}(K^*)$, where K^* is the dual cone of an arbitrary set K while int$(\cdot)$ denotes the interior of the set, then the problem $SVI(F,K)$ has a nonempty, compact, convex solution set.*

The hypotheses of continuity of F and of compactness of K do not ensure, instead, the existence of a solution for a Minty variational inequality; they don't ensure, that is, that M, the solution set of MVI, is a nonempty set. In other words even if F is continuous and is defined on a compact and convex set, a Minty variational inequality solution may not exists.

In the case in which some solution exists to $SVI(F,K)$ o to $MVI(F,K)$, that is in the case in which $S \neq \emptyset$ or $M \neq \emptyset$, to calculate such solutions we can use the so-called gap-functions. More precisely, we define:

$$H(x,y) = < F(y), x-y > \qquad \forall\, x,\ y \in K$$

and we consider the followings functions

$$s(x) = \sup\{H(x,y) : x \in K\}$$

$$m(y) = \inf\{H(x,y) : y \in K\}$$

The functions $s(x)$ and $m(y)$ are called *gap functions*, respectively, for $SVI(F,K)$ and $MVI(F,K)$. It is easy to verify that:

$s(x) \geq 0 \quad \forall\, x \in K$ while $m(y) \leq 0 \quad \forall\, y \in K$

The following proposition characterizes the solution sets S and M in terms of gap functions $s(x)$ and $m(y)$.

Proposition 2.5:

$$S=\{a\in K : s(a)=0\} \text{ and } M=\{a\in K : m(a)=0\}.$$

Proof : It is obvious that $s(a)=0$ is equivalent to

$$\max\{< F(a), a-y >: y\in K\}=0$$

from whose $\quad < F(a), a-y > \leq 0 \qquad \forall\, y\in K$
which is equivalent to the condition a is solution of $SVI(F,K)$, that is $a\in S$.

It is easy to show that $s(x)$ is a convex function and that important existence results on $SVI(F,K)$ are obtained by J.P. Crozeix studing the compactness property of $s(x)$ [4] .
The solutions of variational inequalities $SVI(F,K)$ and $MVI(F,K)$ give saddle points of a particular function defined on K x K:

$$H(x,y)=< F(y), x-y > \qquad \forall (x,y)\in K \text{ x } K$$

Is known, indeed, the following result.

Proposition 2.6:

1. (x_0, y_0) *is a saddle point for* $H \Leftrightarrow \begin{cases} x_0 \text{ solves the } MVI(F,K) \\ y_0 \text{ solves the } SVI(F,K) \end{cases}$

2. (x_0, y_0) *is a saddle point for* $H \Rightarrow H(x_0, y_0)=0$

From the previous proposition it follows that a method to find the solutions of $SVI(F,K)$ and $MVI(F,K)$ can be based on the search of the saddle points of the function $H(x,y)$. Furthermore, the knowledge of one solution of $SVI(F,K)$ can be useful to search the solutions of $MVI(F,K)$ and reverse. In fact, supposing to know a solution y_0 of $SVI(F,K)$ with $F(y_0) \neq 0$, for the point 2 the set $\{x \in K : H(x_0, y_0) = 0\}$ contains the solutions of $MVI(F,K)$. A similar condition can be obtained for the solutions of $SVI(F,K)$, starting from the solution x_0 of $MVI(F,K)$.

3. Variational inequalities and optimization problems

An optimization problem is characterized by a specific objective function, typically single, that is to be minimized or maximized since, generally, represent profits, costs, portfolio risk etc. This problem can be formulated as variational inequality problem. To identify the relationship between an optimization problem and a variational inequality problem, we focus on the particular case in which K is a convex set and the function F has a primitive $f : R^n \to R$, defined and differentiable on an open set containing K, i.e. the operator F is a gradient of a function f. It can consider, then, the following variational problems:

- find a point $x^* \in K$ such that

$$SVI(f',K) \qquad < f'(x^*), y - x^* > \geq 0 \qquad \forall y \in K$$

- find a point $x^* \in K$ such that

$$MVI(f',K) \qquad < f'(y), x^* - y > \leq 0 \qquad \forall y \in K$$

where f' is derivative of the function $f : R^n \to R$.

These problems are called usually and respectively *Stampacchia and Minty variational inequality of differential type.*

The variational inequalities $SVI(f',K)$ and $MVI(f',K)$ have been widely studied mainly in relation with the minimization of the function f over the set K. They can be related, that is, to minimization problem:

$P(f',K)$ $\qquad\qquad \min_{x \in K} f(x)$

where the objective function to minimize over the set K is a primitive of operator involved in the inequality itself.

In particular if $x^* \in K \subseteq R^n$, with K convex and nonempty, is a solution of $P(f,K)$ for some function $f: R^n \to R$, differentiable on an open set containing the convex set K, then x^* solves the Stampacchia differential variational inequality as stated by the following result.

Preposition 3.1 [3]: *Let K be a convex subset of R^n and let $f: R^n \to R$ differentiable on an open set containing K.*

i) If $x^* \in K$ is a solution of $P(f,K)$, then x^* solves $SVI(f',K)$.

ii)If f is convex and $x^* \in K$ solves $SVI(f',K)$, then x^* is a solution of $P(f,K)$ and that is, is a minimum point of $f(x)$ on K.

In other word if $F(x)$ is gradient of the differentiable function $f: R^n \to R$ and if K is convex , then $SVI(f',K)$, is a necessary optimality condition for the minimization of the function f over the set K, condition which, if f is convex, becomes also sufficient.

If, instead, $x^* \in K \subseteq R^n$, with K convex and nonempty, is a solution of a Minty differential variational inequality then x^* is also solution of $P(f,K)$. More precisely, $MVI(f',K)$ is a sufficient optimality condition, condition which becomes necessary if f is convex.

Proposition 3.2 [3]: *Let K be a convex subset of R^n and let $f : R^n \to R$ be differentiable on a open set containing K.*

i) If $x^* \in K$ is a solution of $MVI(f', K)$, then x^* is a solution of $P(f, K)$.

ii) If f is convex and x^* is a solution of $P(f, K)$, then x^* solves $MVI(f', K)$.

Remark 1: If, in point i) of Proposition 3.2, we suppose that x^* is a "strict solution" of $MVI(f', K)$, i.e.:

$$< f'(y), y - x^* > < 0 \qquad \forall y \in K \ , \ y \neq x^*$$

then it is possible to prove that x^* is the unique solution of $P(f, K)$.

Remark 2: In both propositions the convexity of f is necessary to prove only one of the implications. Such hypothesis can be weakened with the pseudo-convexity.

Therefore, in the case that the variational inequality formulation of the equilibrium conditions underlying a specific problem is characterized by a function with a symmetric jacobian, then the solution of equilibrium conditions and the solution of a particular optimization problem are one and the same.

$MVI(f', K)$ has a geometric interpretation that is at the base of so-called Minty variational principle: it means that, for the function f, regardless in which point of K one is, if one moves toward a minimum point x^*, then the directional derivative must be nonpositive.

It is Known that $MVI(f',K)$ characterize a Kind of equilibrium more qualified than Stampacchia variational inequalities: it seems that an "equilibrium" modelled through a $MVI(f',K)$ is more regular that one modelled through a $SVI(f',K)$. This conclusion leads to argue that when a $MVI(F,K)$ admits a solution and the operator F admits a primitive f (i.e. the function f to minimize is such that $F = f'$), then f has some regularity property, i.e. convexity or generalized convexity. In other word the solution of $MVI(f',K)$ exists if the considered function f obeys some generalized convexity property. In the case of one variable ($n = 1$) we observe that it must be quasi-convex.

Proposition 3.4: *Let $K \subseteq R^n$ be convex and $f : R \to R$. If there exists a solution x^* of $MVI(f',K)$, then f is quasi-convex.*

Proof: In this case $MVI(f',K)$ is $f'(y)(x^* - y) \le 0$ $\forall\, y \in K$. This involves that the function is nondecreasing on each half-line with origin at x^*, and hence is quasi-convex.

In the case of several variables $(n \ge 2)$ the existence of a solution does not necessarily imply quasi-convexity of the function but implies star-shapedness of the level sets of the function f at a point which is a solution of $MVI(f',K)$.

Proposition 3.5: *Let $f : K \subseteq R^n \to R$ be. If x^* is a solution $MVI(f',K)$ and K is star-shaped at x^*, then all the nonempty level sets of f :*

$$lev_c f := \{x \in K : f(x) \le c\} \qquad \forall\, c \in R$$

are star-shaped at x^*.

This proposition can be regarded as a convexity-type condition: recall that, by definition, a function $f : K \to R$ is quasi-convex if and only if its level sets are convex.

Definition 3.1: *A set* K *is said to be star-shaped at* $x \in K$ *if and only if* $z = x + t(y - x) \in K \quad \forall\, y \in K$ *and* $\forall\, t \in [0,1]$.

Definition 3.2: *Let* K *be a subset of* R^n.
i) the set $Ker\ K = K^{\Diamond} = \{x^* \in K : [x^*, x] \subseteq K,\ \forall x \in K\}$ *is called the Kernel of* K;
ii) the nonempty set K *is said star-shaped if* $Ker\ K \neq \emptyset$.

Ubder convexity assumption of the set K, a necessary condition for existence of a solution of $MVI(f', K)$ is that the intersection of the Kernels of the level sets is nonempty.

Proposition 3.6: *If* $K \subseteq R^n$ *is convex and nonempty and* $\cap_{c > \bar{c}} (lev_c f)^{\Diamond} = \emptyset$, *with* $\bar{c} = \inf_K f(x)$, *then* $MVI(f', K)$ *has not solutions.*

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